2PN LIGHT PROPAGATION AND MEASUREMENT IN THE SOLAR SYSTEM

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ABSTRACT. As a sensitive and useful tool in gravitational physics, especially for some high order effects, the propagation of light carries lots of information about the nature of spacetime and plays an important role in high-precision experiments and measurements. Three methods can be used in this issue. First is mainly developed by Kopeikin & Schäfer (1999) and Kopeikin & Makarov (2007) and so on. Second is mainly developed by Brumberg (1991) and Klioner & Kopeikin (1992) and so on. Recently, Linet & Teyssandier (2002) and Le Poncin-Lafitte & Teyssandier (2008) and others used Synge's world function to investigate the light propagation avoiding the integration of geodesic equations. We adopt the second one. In this paper, the second post-Newtonian (2PN) framework for light propagation is developed with two additional parameters ς and η besides the two parameterized post-Newtonian (PPN) parameters γ and β. For a precision level of a few microarcsecond for space astrometry missions in the near future, started from the definition of a measurable quantity, a gauge-invariant angle between the directions of two incoming photons for a differential measurement in astrometric observation is discussed.

1. SECOND POST-NEWTONIAN FRAMEWORK FOR LIGHT PROPAGATION

We parameterize the 2PN metric of Klioner & Kopeikin (1992) in BCRS (Soffel et al., 2003) as follows

$$g_{00} = -1+\epsilon^2 2\sum_A\left(\frac{Gm_A}{r_A}+\frac{3}{2}\frac{G}{r_A^5}J_A^{<ik>}r_A^i r_A^k\right)-\epsilon^4 2\beta\frac{G^2m_\odot^2}{r_\odot^2}+\mathcal{O}(5), \quad (1)$$

$$g_{0i} = -\epsilon^3 2(1+\gamma)\sum_A\frac{Gm_A}{r_A}v_A^i-\epsilon^3 2(1+\gamma)\sum_A\frac{G}{r_A^3}\varepsilon^i_{jk}S_A^j r_A^k+\mathcal{O}(5), \quad (2)$$

$$g_{ij} = \delta_{ij}+\epsilon^2\delta_{ij}2\gamma\sum_A\left(\frac{Gm_A}{r_A}+\frac{3}{2}\frac{G}{r_A^5}J_A^{<kl>}r_A^k r_A^l\right) +\epsilon^4\left\{\delta_{ij}\varsigma\frac{G^2m_\odot^2}{r_\odot^2}+\eta\frac{G^2m_\odot^2}{r_\odot^4}r_\odot^i r_\odot^j\right\}+\mathcal{O}(5), \quad (3)$$

where $\epsilon=1/c$ and $\mathcal{O}(n)$ means of order ϵ^n, ς and η are two 2PN parameters and have different values and dependences in different gravitational theories (see Table below).

Parameter	General Relativity	Scalar-Tensor theory Damour & Esposito-Farèse (1996)	Einstein-aether theory Xie & Huang (2008)
ς	1	$2\gamma^2-\frac{1}{2}\gamma+2\beta-\frac{5}{2}$	$1+\frac{1}{2}c_{14}$
η	1	$\frac{1}{2}(1+\gamma)$	$1-\frac{1}{2}c_{14}$

For a photon propagating in a spacetime in which Einstein Equivalence Principle (EEP) is valid, we obtain the equations of light propagation (ELP) based on the basic equations of light. ELP includes four parts: the 1PN monopole components coupled with orbital motions, the influences of quadrupole moments of the bodies, the effects from their spins which are also called by gravitomagnetic fields, and the 2PN monopole component of the Sun. The trajectory of a light ray in BCRS can then be obtained through integrating ELP by adopting an iterative method used by Brumberg (1991), Klioner & Kopeikin (1992) and Klioner (2003).

2. ANGULAR MEASUREMENT

Now, we construct a gauge-invariant angle θ between the directions of two incoming photons based on Brumberg (1991) and Will (1993). Then, we obtain

$$\begin{aligned}\theta(t) &= \vartheta_0 + \overset{(1)}{\vartheta}_{obs} + \left(\overset{(2)}{\vartheta}_{obs} + \overset{(2)}{\vartheta}_{1PN} + \overset{(2)}{\vartheta}_{Q}\right) + \left(\overset{(3)}{\vartheta}_{obs} + \overset{(3)}{\vartheta}_{OM} + \overset{(3)}{\vartheta}_{S}\right) \\ &\quad + \left(\overset{(4)}{\vartheta}_{obs} + \overset{(4)}{\vartheta}_{2PN}\right), \qquad (4)\end{aligned}$$

where $\vartheta_0 \in (0, \pi)$ is the angle between the unperturbed light paths from two given sources

$$\vartheta_0 = \arccos(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2), \qquad (5)$$

and $\overset{(n)}{\vartheta}_{obs}$ is the deflection angle due to the observer's motion in terms of order ϵ^n, $\overset{(2)}{\vartheta}_{1PN}$ is the 1PN deflection angle due to the spherically symmetric field of each body, $\overset{(2)}{\vartheta}_{Q}$ is the deflection angle due to quadrupole moment, $\overset{(3)}{\vartheta}_{OM}$ is the deflection angle due to the orbital motions of N-body, $\overset{(3)}{\vartheta}_{S}$ is the deflection angle due to the spin of the bodies and $\overset{(4)}{\vartheta}_{2PN}$ is the 2PN deflection angle due to the spherically symmetric field of the Sun.

2. DISCUSSION

For LATOR-like missions, with the angle between the initial emitting directions of two light signals 1 and 2

$$\vartheta_0 = \hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2 \approx 1^\circ, \qquad (6)$$

it means that the distance between these two spacecrafts is about 5.22×10^9 m. With the cut-off precision of ~ 1 μas: the 1PN monopole moment of the Sun causes $1.75''$ and $0.47''$ deflections for two light rays respectively; the 1PN monopole moment of Mercury causes 83.06 μas; the deflections caused by the 1PN monopole of Venus are 492.76 μas and 4.13 μas . If we assume that the velocity of the observer in BCRS is the same as the orbital velocity of the Earth. The first order aberration is about 3.14 mas. The second and the third ones are respectively 18.2 μas and 3.05 μas. The coupling term of solar monopole and aberration leads to 1.53 μas. The 2PN solar monopole causes 11.32 μas and 7.44 μas. However, for the Sun, quadrupole moments ($J_2 \approx 10^{-7}$)and its spin lead to light deflection are beyond 1 microarcsecond. They are respectively 0.35 μas and 0.72 μas. It needs further analytical and numerical studies.

3. REFERENCES

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