A DYNAMICAL STUDY OF PHOEBE’S ROTATION

L. COTTEREAU
Observatoire de Paris, Systèmes de Référence Temps Espace (SYRTE), CNRS/UMR8630
61 avenue de l’Observatoire, 75014 Paris, France
e-mail: laure.cottereau@obspm.fr

ABSTRACT. With a fast non synchronous rotation, Phoebe is a particular satellite in the Saturnian system (most of the satellites are synchronously rotating) and must show a very different rotational evolution. We propose for the first time to determine the combined motion of the precession and nutation of Phoebe considered as a rigid body by numerical and analytical integration. We further compare our results with those obtained by Kinoshita for the Earth (1977), emphasizing their astonishing similarities (obliquity, value of the precession, nutation amplitudes and arguments). Moreover we show that a pure analytical accurate model of the nutation is not easy to construct due to the fact that the orbital motion of Phoebe is far from being Keplerian. At last we present the prospect for future studies among which are the effect of the Sun, Titan and the dynamical ellipticity of Saturn on the precession-nutation motion.

1. PHOEBE A NON KEPLERIAN MOTION

One of the important steps in our study was to describe the orbital motion of Phoebe by fitting the curves of the temporal variations of the orbital elements $a, e, M$ and $L_s$ (Emelyanov, 2007) in the same way as in Simon et al.(1994) for the planets. for the Earth, the semi-major axis is nearly constant with relative variation about $10^{-5}$. In comparison, for Phoebe, approximating $a$ by a constant is not a good approximation. The semi major axis of the satellite shows periodic variations with a relative amplitude of $10^{-3}$ around a mean value of $a = 0.0864273$. The presence of the relatively large periodic components on the mean elements of Phoebe shows that the motion of this satellite is not close to a Keplerian one, as it is the case for the planets of the Solar system. This departure may be due to the attraction of the Sun, Jupiter and the other satellites of Saturn on Phoebe. The individual study of the other orbital elements confirms this result. Indeed, the residuals obtained after subtraction of a polynomial functions at 6th order fitted to the mean anomaly, the mean longitude and the eccentricity, have periodic components which reach an amplitude of $4^\circ$ for the mean anomaly.

2. PRECESSION NUTATION OF PHOEBE : NUMERICAL AND ANALYTICAL RESULTS

Applying the theoretical framework already used by Kinoshita (1977) for the Earth, the precession and the nutation motion of Phoebe are determined both analytically and from numerical integration of the equations of motion. We found that the precession-nutation motion of Phoebe undergoing the gravitational perturbation of Saturn is quite similar to that of the Earth undergoing the gravitational effect of both the Moon and the Sun. Thus our value for the precession of Phoebe, that is to say $5580.65$ cy, is very close to the corresponding value for the Earth ($5081''/cy$) and the nutation in longitude and in obliquity (see Figs.) of Phoebe with peak to peak variations of $26''$ and $8''$ are of the same order of amplitude as the nutation of the Earth (respectively $36''$ and $18''$ peak to peak). Moreover Phoebe obliquity ($23.95^\circ$) is roughly the same as the Earth’s one ($23.43^\circ$). Notice that the physical dissymmetry characterized by the large value of the dynamical flattening $0.06465$ and of the triaxiality $-0.0111$ (Aleshkina, 2010) and the large eccentricity of Phoebe which directly increase the amplitude of the precession and the nutation is compensated by its slow revolution and fast rotation ($0.386d$) (Bauer et al., 2004).

We also investigated the possibility to construct analytical tables of Phoebe nutation, as was done for the Earth. After fitting the curves of temporal variations of the orbital elements given by Emelyanov (2007) with linear expressions and replacing them in the equations motion (Cottereau et al, 2010), the precession-nutation of Phoebe is determined by analytical integration. Although the amplitude of the
nutation motion is close to the Earth one, we demonstrated that the analytical model used by Kinoshita (1977) for the Earth does not describe the nutation motion of a disturbed body like Phoebe with the same accuracy ($10^{-5}$ for the Earth). This analytical model does not take into account the large perturbing effects of the celestial bodies on the orbit of the satellite. To describe the nutation motion of Phoebe a FFT approach is better than a pure analytical integration done with linear expressions for $e, M$ and $L_s$. The FFT analysis is better fitted to describe the periodic variations characterizing the nutation signals of Phoebe. For the calculation of the precession, the discrepancies between the two models is decreased because these periodic variations are averaged.

To conclude we have shown that the analytical model set by Kinoshita (1977) gives a good first approximation of the precession-nutation of Phoebe but further analytical developments are needed to reach the same accuracy than for the terrestrial planets. We think that this work can be a starting point for further studies such as the elaboration of another very precise analytical model of the rotation of Phoebe by taking into account effects ignored in this paper, as the direct effects of the Sun, of Titan and of Saturn dynamical flattening. Such a model is required to develop the long term ephemerides of Phoebe’s rotation, which should require long term orbital ephemerides, not still available.

3. REFERENCES

Kinoshita H. 1977, Celestial Mechanics, 15, 277

Figure 1: Nutation of Phoebe in obliquity for 2000 days time span (Cottereau et al., 2010).