# ABSOLUTE PARALLAXES AND PROPER MOTIONS FROM THE PARSEC PROGRAM

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ABSTRACT. The PARallaxes of Southern Extremely Cool objects (PARSEC) program is designed to measure trigonometric parallaxes of 150 confirmed brown dwarfs in the southern hemisphere with the aim of using distances as fundamental calibrators for the investigation of star formation and evolution in the very low-mass regime. A scientifically useful addition to the primary scope of the project is the derivation of stellar proper motions, by combining observations from the full field of view, linked to the UCAC2 catalogue, with first-epoch data from 2MASS. To date, a proper motion catalogue of about 200,000 objects has been compiled. Tailored reduction techniques allow to attain milliarcsecond accuracy in the derived astrometric parameters, as validated by external comparisons.

## 1. OBSERVATIONAL STRATEGY

PARSEC observations are carried out with the Wide Field Imager on the ESO 2.2 m telescope at La Silla. The detector is a mosaic of 8 CCDs sized 2k x 4k 15$\mu$m pixels, providing a scale of 0.2$''$/pixel and a total field of view of 0.3 square degrees. All images are taken in the $z$ filter (central wavelength 964.8 nm), a suitable compromise between the optimal QE of the system in the $I$ band and the expected brightness of the targets, whose $(I - z)$ is typically larger than 1.5. Exposure times are indicatively 150 s and 300 s for bright ($z < 18$) and faint ($z \geq 18$) objects respectively; during nights with particularly poor seeing ($> 1.5''$), times are adjusted to obtain a highest-pixel signal of $> 100$ counts above the background.

With earliest observations dating back to April 2007, a frequency of 3-4 observing runs per year and an envisaged time span of the program of 4 years, the parallax ellipse is optimally sampled for almost all the targets. The end points of the ellipse's major axis correspond to the most crucial observations, which occur when the so-called parallax factor $F$ reaches its highest numerical value. In fact, the star's apparent displacement in right ascension, due to its annual parallax, is adequately given by $\Delta\alpha cos\delta = F\pi$, where $\pi$ is the trigonometric parallax and $F \equiv (Y cos\alpha - X sin\alpha)$ is a function of the star's right ascension $\alpha$ and the Sun's geocentric equatorial coordinates $(X, Y) = (cos\lambda_\odot, sin\lambda_\odot cos\epsilon)$, $\epsilon$ being the inclination between the equatorial and ecliptic planes. It is clear that, having fixed the measurement error on the stellar displacement, a larger value of $F$ would result in a smaller error on the estimated parallax. It can be shown that $|F|$ is maximum when the geocentric angle between the Sun and the star is 90°. When planning our observations, we also require that the target be near the meridian (+/- 20 min) in order to minimize differential refraction effects; these two conditions are simultaneously best met at evening/morning twilight, when stars crossing the meridian are approximately perpendicular to the direction of the setting/rising sun. Therefore, twilight hours bear the most relevance to our observing program.

Targets are picked from the nightly schedule according to a priority flag which reflects the observing history of each object and which is updated at every run. Finally, after an initial acquisition, the pointing is refined to move the target always in the same $(x, y)$ position, which falls in the top third of CCD#7. For all the subsequent parallax reductions we only use data from this portion of the detector: this is sufficiently large that we have enough reference objects for a transformation to a common system and sufficiently small to assume that a variation in astrometric distortion over the observational campaign would be smaller than the errors of a linear transformation.

Once the raw CCD data are pre-reduced using the prescriptions illustrated in Andrei et al. (2010), the stellar density profiles are fitted with a bi-dimensional Gaussian model giving an estimation of the objects' *centroiders* $(x(t), y(t))$ at the time of observation, which are our basic astrometric measurements.

## 2. THE DERIVATION OF PARALLAXES

The target's relative parallax and its relative-to-absolute correction are derived following the methods discussed in TOPP (Smart et al. 2003, 2007). These have been extensively tested and successfully used to produce the first PARSEC results (Andrei et al. 2010), i.e., preliminary parallaxes for 10 brown dwarfs, 2 of which within 10 parsecs, with a median *rms* error of 4.2 milliarcseconds.

In this contribution we focus on some aspects concerning the problem of rank deficiency intrinsic to the task of parallax determination with small-field astrometry. Let's consider $n$ frames taken at different observing times $t_\nu; \nu = 1, ..., n$ and $m$ stars observed in each of those frames. For any star $\mu$; $\mu = 1, .., m$, assuming that the change in position is only due to parallax and proper motion effects, its longitudinal *standard coordinate* on the tangential plane, identified by the intersection of the telescope optical axis with the celestial sphere, can be modelled as:

$$\xi_\mu(t_\nu) \equiv \xi_{\nu\mu} = \xi_{0\mu} + \Delta t_\nu \mu_\nu + F_\nu \pi_\mu \tag{1}$$

where $\xi_0$ is the tangential position at a chosen reference epoch $t_0$ and $\Delta t_\nu = t_\nu - t_0$; $\mu, \pi$ are the star's *proper motion* in right ascension and its *parallax*, and $F$ is the parallax factor defined in the previous section.[1] On the other hand, the $\xi_\mu(t_\nu)$ are functions of the true values of the measured coordinates $x_{\nu\mu}$, which we assume to be well represented by the linear transformation:

$$\xi_{\nu\mu} = x_{\nu\mu} + A_\nu x_\mu + B_\nu y_\mu + C_\nu \tag{2}$$

where the $A_\nu, B_\nu, C_\nu$ are the so-called *plate parameters*. Equations 1 and 2 can be trivially combined to obtain the *observation equation* for star $\mu$ on frame $\nu$. We introduce now vector $\mathbf{l^T} = (\mathbf{l_1^T}, ..., \mathbf{l_n^T})$ where $\mathbf{l_\nu^T} = (x_{\nu 1}, ...x_{\nu m})$ is the vector of measurements of all the stars on frame $\nu$. Moreover, let's $\mathbf{p^T} = (A_1, B_1, C_1, ..., A_n, B_n, C_n, \xi_1, \mu_1, \pi_1, ..., \xi_m, \mu_m, \pi_m)$ be the vector of instrumental and astrometric unknowns. With this formalism, the final system of equations can be expressed in matricial form as:

$$\mathbf{X} \cdot \mathbf{p} = -\mathbf{l} \tag{3}$$

The explicit form of $\mathbf{X}$ is given by:

$$\mathbf{X} = \begin{pmatrix} \mathbf{M_1} & & & & -\mathbf{I} & -\Delta t_1 \mathbf{I} & -F_1 \mathbf{I} \\ & \mathbf{M_2} & & & -\mathbf{I} & -\Delta t_2 \mathbf{I} & -F_2 \mathbf{I} \\ & & \ddots & & \vdots & \vdots & \vdots \\ & & & \mathbf{M_n} & -\mathbf{I} & -\Delta t_n \mathbf{I} & -F_n \mathbf{I} \end{pmatrix}$$

where $\mathbf{I}$ is the identity matrix, $\Delta t_i$ and $F_i$ are defined as in equation 2, and the matrices

$$\mathbf{M_i} = \begin{pmatrix} x_{i1} & y_{i1} & 1 \\ x_{i2} & y_{i2} & 1 \\ \vdots & \vdots & \vdots \\ x_{im} & y_{im} & 1 \end{pmatrix}$$

differ from each other inasmuch as the measurements of stellar positions in a given field of view change in value from one frame to another. The number of columns of matrix $\mathbf{X}$ is $3m + 3n$, but its rank is only $3(m + n) - 3$, meaning that some a-priori known position, proper motion and parallax must be used in order to solve the system of equations 3. If we assume that the plate parameters are small ($<< 1$), which is generally the case provided that the CCD axes are properly aligned and the scale factor well known, the matrices $\mathbf{M_i}$ will be nearly identical, and can therefore be approximated by a unique matrix $\mathbf{M}$ by choosing, e.g., $\mathbf{M} = \mathbf{M_1}$. With this substitution, the dimension of the null space of $\mathbf{X}$ becomes equal to 9 (Eichhorn 1988). If no approximations are made, only 3 *singular values* of $\mathbf{X}$ are exactly equal to zero, and the *rank deficiency* of the problem decreases mathematically to 3; however, 6 of the remaining

[1] Analogous formulae hold for the $\zeta$ component in latitude, which is neglected in this treatment. We note that the parallax factor in declination is sensibly smaller than the one in right ascension, bearing therefore much less weight into the estimation process

singular values are *nearly* zero; therefore, in a numerical sense, the singularity of the problem is still equal to 9. The physical meaning of such indeterminacy relies in the impossibility of distinguishing, solely based on the measurements, whether an observed translation, rotation, or scale factor are accountable to instrumental effects as opposed to some global astrometric behaviour of the stellar field.

To solve equation 3 in the least-square sense, one must determine a particular solution out of the infinite solutions of the associated normal system. Two different approaches are considered: the first one is to add to matrix $\mathbf{X}$ nine linearly independent constraint equations in order to eliminate the rank deficiency; the second one consists in finding a *minimum norm* solution using, e.g., the singular value decomposition method, which allows to construct an orthonormal set of vectors spanning the *range* of $\mathbf{X}$.

In the first approach, a suitable choice of constraints, as suggested by Eichhorn (1988) and subsequently analysed by Rapaport (1998), is represented by the set of equations

$$\mathbf{M}^T\mathbf{P}_s = \mathbf{C}$$

with $\mathbf{M}^T$ defined above;

$$\mathbf{P}_s = \begin{pmatrix} \xi_{01} & \mu_1 & \pi_1 \\ \xi_{02} & \mu_2 & \pi_2 \\ \vdots & \vdots & \vdots \\ \xi_{0m} & \mu_m & \pi_m \end{pmatrix}$$

being the unknown astrometric parameters of the *reference* stars, and $\mathbf{C}$ an arbitrary 3x3 matrix. While for the constraint equations involving stellar positions, the elements of $\mathbf{C}$ can be calculated using *reference* stars' catalogue values, this is not the case for the ones concerning parallaxes and proper motions, which are not usually available. A suitable choice is to put all the other elements of $\mathbf{C}$ equal to zero, which means fixing at zero the barycenter of parallaxes and proper motions of the reference stars, plus adding an orthogonalization condition with respect to the plate measurements.

Both the described approaches should be equivalent, as in principle one can fully recover one solution from the other by noticing that the so-called *general solution*, i.e., the one generating the complete set of solutions of system 3, is given by the sum of a *particular* solution and a linear combination of the orthonormal vectors spanning the null space of $\mathbf{X}$. However, as already stated, the rank deficiency of the problem is not exactly 9, and solutions obtained with different set of constraints are not exactly equivalent. The method adopted in TOPP is an iterative one, naturally converging to a minimum-norm-type solution without the necessity of adding extra conditions. We are currently investigating the extent to which the choice of different constraints can influence the final determination of the relative-to-absolute parallax correction.

## 3. THE DERIVATION OF PROPER MOTIONS

The raw data from the entire field of view of the ESO Wide Field Imager have been used to compute proper motions of anonymous objects down to the magnitude limit of an average CCD exposure, i.e., $I \simeq 19$. To this end, independently for each CCD and each observing time, we have determined an astrometric reduction relative to the Second US Naval Observatory CCD Astrograph Catalog (UCAC2, Zacharias et al. 2004). With an average number of reference stars per CCD equal to 20, polynomial fits of degree 2 or 3, depending on the actual number of UCAC2 stars available, were used to transform the $(x, y)$ measurements onto equatorial coordinates $(\alpha, \delta)$. Then, each object was matched to the Two Micron All Sky Survey (2MASS, Skrutskie et al. 2006) point source catalogue. We employed a nearest-neighbor match, which should be sufficient to avoid mismatches even in the presence of high proper motions, given that the epoch difference between 2MASS and PARSEC observations is small and that our targets are clear of the Galactic disk. However, we are developing a more robust matching algorithm making use of GSC2.3 positions at different epochs.

In Andrei et al. (2010), results for 197,500 sources are analysed by means internal and external comparisons showing that our proper motions are well behaved, with a median *rms* error of 5 mas/year, both for right ascension and declination. Figure 1 is a plot of the reduced proper motion $H(K) = K + 5log(\mu_{tot}) + 5$, as a function of the $z - K$ color for all objects in the PARSEC proper motion catalogue. The $z$ magnitudes come from a zero-point correction to the instrumental magnitudes of the first observations while $K$ are 2MASS magnitudes. It can be seen that brown dwarf objects, which are marked with diamonds, are well segregated in this diagram.

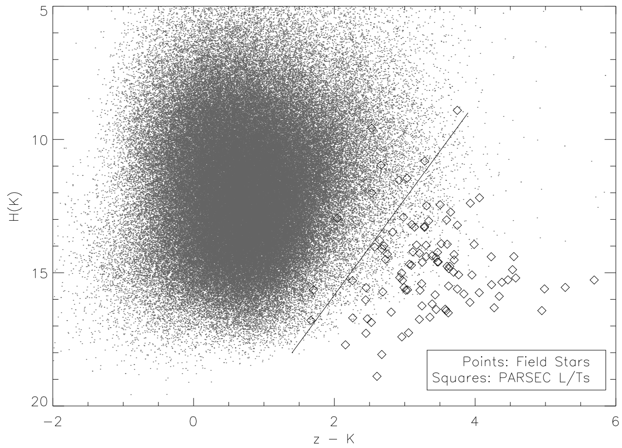


Figure 1: A reduced proper motion diagram ($H(k)$ versus $z - K$) of the 197,500 objects in the PARSEC proper motion catalogue. The region of possible brown dwarf candidates is delimited by the solid line

## 4. CONCLUSIONS

The PARSEC program has been successfully using ESO 2.2m Wide Field Camera observations to derive parallaxes of selected brown dwarfs, as well as proper motions of field stars at the milliarcsecond level accuracy, with direct astrophysical applications. Different methods for overcoming the rank deficiency inherent to the parallax derivation are being investigated. The PARSEC proper motion catalog, which to-date counts $\simeq 200,000$ objects, can be exploited to identify new brown dwarf candidates via the reduced proper motion diagram, for spectroscopic follow-up.

## 5. REFERENCES

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