

# LUNAR NUMERICAL THEORY EPM2008 FROM ANALYSIS OF LLR DATA

E.I. YAGUDINA  
IAA, RAS, St-Petersburg, Russia  
10 Kutuzov quay, St-Petersburg, 191187, Russia  
e-mail: eiya@ipa.nw.ru

**ABSTRACT.** The analysis of Lunar Laser Ranging (LLR) data from 1970 till 2008 has been carried out to improve the lunar part of the luni-solar ephemerides EPM2008 under developing. The dynamical model of the lunar rotation takes into account the effects of elasticity of the lunar body and the tidal dissipation in the Moon. The orbital parameters of the Moon, parameters of the physical libration and coordinates of the retro-reflectors have been estimated. The new ephemeris are compared with DE403, DE405 and DE421 versions of the JPL ephemerides. All the calculations have been carried out in the framework of the universal software package ERA developed for scientific research in the ephemeris and dynamical astronomy.

## 1. INTRODUCTION

The very precise now days LLR observations are the basis for construction of the modern lunar ephemeris. The high accuracy of the modern LLR data demands dynamical theories of a adequate precision. Such theories are developed and supported by JPL, USA (DE403, DE405, DE421); by IMCCE (INPOP06), Observatoire de Paris and is under developing in IAA RAS (Russia Institute of Applied Astronomy)-EPM-Ephemerides Planets and Moon. The application of EPM to planets is very known and are in use. The preliminary version of lunar numerical theory has been presented (for example, in Krasinsky, 2002), where analysis of 14612 LLR observations of time interval 1970-2001 was fulfilled, the selenodynamical parameters have been obtained. At present analysis of 16320 LLR observations (time interval is 1970-2008) has been included in processing for improving ephemeris of the Moon and obtaining some selenodynamical parameters. To estimate the precision of our EPM2008 ephemeris the result has been compared with three versions of DE ephemerides.

## 2. OBSERVATIONS

In the present analysis 16320 LLR observations have been included in the processing . They have been carried out mainly at McDonald (Texas), where at different epochs three different sites were activated as McDonald, MLRS1 and MLRS2; Cerga station (France) and a set of observations of two years duration made at Haleakala Observatory (Hawaii). Number of observations at each site is shown in Table 1. In LLR

Station	Time interval	Number of LLR observations
McDonald	1970 Mar - 1985 Jun	3439
MLRS1	1985 Jan - 1988 Jan	275
MLRS2	1988 Aug - 2008 Jan	2933
HALEAKALA	1989 Nov - 1990 Aug	694
CERGA	1985 Jan - 2008 Jan	8979
TOTAL	1970 Mar- 2008 Jan	16320

Table 1: Distribution of LLR observations

analysis a number of parameters appear to be strongly correlated and may be only estimated because four reflectors could be observed: 1-Apollo 11, 2-Apollo 14, 3-Apollo 15, 4-Lunahod2. The number

of ranging to Apollo 11, Apollo 14, Apollo 15, Lunahod2 are 1585, 1557, 12724 and 452, respectively. Unfortunately such disparity of the observations distribution deteriorates the reliability of the estimates of a number of selenodynamical parameters. Before 1998 the observations are obtained by request from observatories, later on they have been retrieved from FTP server [ccdisa.gsfc.nasa.gov/pub/slr](http://ccdisa.gsfc.nasa.gov/pub/slr), partly from [oca.eu/gemini/donnees/las1une](http://oca.eu/gemini/donnees/las1une), some of them have been obtained by private correspondence.

### 3. MODEL, EPHEMERIDES AND PARAMETERS

In this paper only a brief summary of the used model is described. The precise dynamical model of the Moon motion has been constructed under Krasinsky by simultaneous numerical integration the equations of orbital and rotational motions of the Moon, major planets, five biggest asteroids (the integration includes reduced equations of 295 asteroids, it is important for the major planets but not for the Moon). Potential of the Moon is calculated up to 4-th order of the zonal index, that of the Earth includes the 2th order harmonics  $C_{20}$  and  $C_{22}$ . Tidal perturbations in the lunar orbital motion caused by tidal dissipation on the Earth's body is computed by the model with a constant lag, the effects of elasticity of the lunar body has been taken into account. The integration was carry out by Everhart's method with the automatically operated choice of the step of integration. Partial of ranging respectively to dynamical parameters of the orbital and rotational model of the Moon are computed mostly by integration of variational equations; in a few cases they have been obtained by integration of the rigorous system of equations with slightly varied values of the parameters under study. The set of parameters includes the lunar initial coordinates and velocities, libration angles and their velocities, Stocks coefficients of the selenopotential, the angle of tide delay, the coordinates of reflectors, observational stations etc. The LLR data set has been also processed with a help of DE403, DE405 and DE421 lunar ephemerides making use of the partials obtained with EPM (these partials are not distributed along with DE ephemerides). However nominal values of many of the estimated parameters in DE ephemerides are not known; that is why only corrections to such parameters could be determined. Then we might implement the improved values of dynamical parameters only to EPM, but not to DE ephemerides. In Table 2 the list of 65 parameters have been improved , all of them being then fed back to EPM by iteration. As lunar rangings

N	Parameters estimated
1-6	Lunar orbital state vector for the epoch JD 2446000.5
7-12	Euler's angles and their time derivatives for the same epoch
13	Lag of the Moon's body tides
14-16	Lunar Love numbers $k_1, h_2, l_2$
17	Lag of the Earth's body tides
18-29	Harmonics of lunar potential from $c_{20}$ to $s_{33}$
30-38	Coordinates of reflectors A11, A14, L2
39	Coordinate X for Appollo 15
40-54	Coordinates of 5 stations
55-56	Corrections to orientation of the Earth's equator $\epsilon, \phi$
57-58	Secular trends $\dot{\epsilon}, \dot{\phi}$
59-60	Secular trends in sidereal angles of the Earth and Moon
61	Undimentional lunar moment of inertia $g = C/mR^2$
62-64	Moon's core-mantle factors $v_1, v_2, v_3$
65	Moon's core-mantle coupling factor $k$

Table 2: List of estimated parameters

are invariant relatively to the rotation of the Earth-Moon system as a whole, all set of parameters of orientation of this system cannot be determined simultaneously. Due to this reason two coordinates of the most often observable reflector Apollo15 have been fixed (longitude and latitude). Values of these two parameters were obtained from a simplified solution made as the first step, in which lunar libration have not been improved. LLR observations are sensitive to the Earth's gravitational constant  $Gm_E$ . The investigation shows that the observable effect cannot be reliably separated from corrections to X

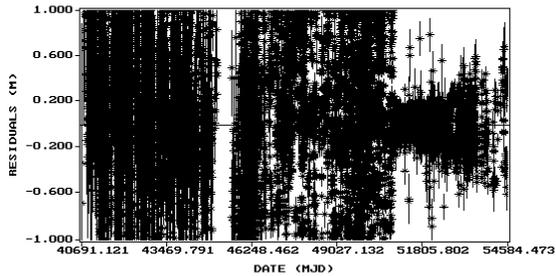


Figure 1: DE403

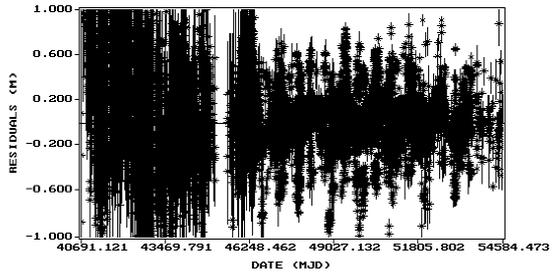


Figure 2: DE405

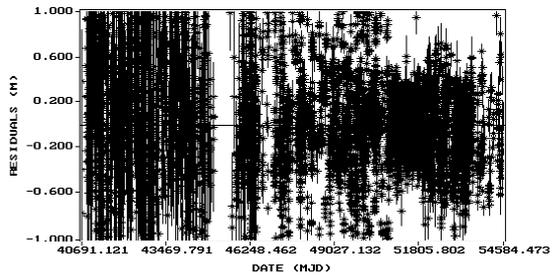


Figure 3: DE421

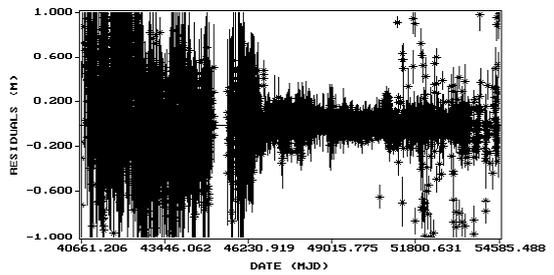


Figure 4: EPM2008

coordinate of the reflectors. Thus the value  $Gm_E$  has not been included to the list of parameters.

#### 4. DISCUSSION OF RESULTS

Using the derivatives from EPM all calculations have been made by help of three versions of DE ephemerides: DE403, DE405, De421. The pre-fit, post-fit residuals and number of included LLR observations in the processing of DE ephemerides along with EPM one can see in Table 3.

ephemeris	pre-fit cm	post-fit cm	Number of obs.
DE403	23.66	5.24	16105
DE405	23.20	5.10	16102
DE421	22.96	5.06	16087
EPM2008	6.32	6.32	16115

Table 3: Internal precision of DE ephemerides compared with EPM

Because the EPM model has been implemented by the obtained corrections the post-fit residuals practically coincide with difference  $O - C$  computed with the improved model equal 0.422ns (6.32 cm). For now days version of EPM post-fit and pre-fit residuals is coincide. For DE ephemerides a similar work could not be carried out in the full scale and only estimated of coordinated of reflectors and Love numbers  $h_2, l_2$  of the Moon have been incorporated. The plots of Fig1, Fig2, Fig3, Fig4 shows us the residuals. In tables 4 corrections to initial coordinates and velocities of the Moon for all DE versions are presented. The plots of Fig1, Fig2, Fig3, Fig4 shows us the residuals. In table 5 the corrections to parameters of Moon's libration are shown. One can see that the corrections to the parameters of libration in all versions of DE ephemerides are big enough but they cannot be feed in ephemeris because the values of these parameters are not known (they are not distributed with the ephemerides).

#### 5. CONCLUSION

1. The investigation has proved a necessity to refine the model of dissipative effect of the lunar

Corrections to coordinates of the Moon	DE403 m	DE405 m	DE421 m
$dX$	$-0.346\pm 0.094$	$2.625\pm 0.092$	$0.021\pm 0.091$
$dY$	$-0.320\pm 0.055$	$-1.516\pm 0.054$	$-0.232\pm 0.053$
$dZ$	$0.988\pm 0.088$	$0.509\pm 0.086$	$0.342\pm 0.085$
Corrections to velocities of the Moon	DE403 m/sec	DE405 m/sec	DE421 m/sec
$dV_x$	$-0.043\pm 0.009$	$0.247\pm 0.008$	$-0.004\pm 0.008$
$dV_y$	$0.105\pm 0.022$	$0.560\pm 0.027$	$0.024\pm 0.022$
$dV_z$	$0.006\pm 0.009$	$0.228\pm 0.008$	$0.001\pm 0.008$

Table 4: The corrections to the DE ephemerides parameters

Corrections to Euler's angles	DE403	DE405	DE421
$d\phi$	$0.137\pm 0.030$	$-0.439\pm 0.029$	$0.922\pm 0.028$
$d\theta$	$-0.299\pm 0.010$	$-0.169\pm 0.009$	$-0.546\pm 0.009$
$d\psi$	$-1.502\pm 0.022$	$0.756\pm 0.022$	$-5.659\pm 0.022$
Corrections time derivatives of Euler's angles	DE403 arc/year	DE405 arcs/year	DE421 arcs/year
$d\dot{\phi}$	$-0.182\pm 0.005$	$-0.103\pm 0.005$	$-0.327\pm 0.005$
$d\dot{\theta}$	$-0.007\pm 0.002$	$0.031\pm 0.002$	$-0.081\pm 0.002$
$d\dot{\psi}$	$0.167\pm 0.005$	$0.095\pm 0.005$	$0.301\pm 0.005$

Table 5: The corrections to the DE ephemerides parameters

rotation for EPM ephemeris by integration the equations of orbital and rotation motions with retarded argument. We used the simplified model of dissipation tides of the Moon. This is the main source of errors the present model.

2. The outward accuracy of DE ephemerides is about 22-24 cm. The inward accuracy of DE ephemerides cannot be used by independent researcher because it is not possible to feed back the correction values.

The study has been carried out by the software package ERA for Ephemeris Astronomy.

## 6. REFERENCES

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