TOWARDS A LONG-TERM PARAMETRIZATION OF PRECESSION

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ABSTRACT. The IAU 2000/2006 precession-nutation model is designed to provide the coordinates X, Y of the Celestial Intermediate Pole (CIP) with respect to the Geocentric Celestial Reference System (GCRS), with microarcsecond precision for several centuries around the central epoch J2000. Its precessional part is expressed in terms of polynomial developments of the time elapsed from this epoch. However, when extrapolated to more distant epochs (comparable to the basic 26000-yr period of CIP motion around the ecliptic pole), it starts to diverge rapidly from reality. The aim of this paper is to estimate the accuracy of the present model as function of the length of the interval, and to propose new developments for X, Y, based on long-periodic functions of time. The goal is to obtain accuracy that approaches the present IAU developments for epochs close to J2000, and a better fit to reality for longer intervals.

1. INTRODUCTION

The position of the Celestial Intermediate Pole (CIP) in the Geocentric Celestial Reference System (GCRS) at any given date includes the motion due to precession-nutation together with a frame bias (of about 23 mas) between the GCRS and the J2000 equatorial system. Expressions for predicting the CIP directions, based on the IAU 2000A precession-nutation, can be found in Capitaine et al. (2003a), in the IERS Conventions (2003) and in the IAU SOFA software (Wallace 1998). Expressions based on the IAU 2006 precession have been provided by Capitaine et al. (2003b) and Wallace and Capitaine (2006). The developments of the CIP's GCRS X, Y coordinates are given as polynomials of t which originate mainly from precession, plus a series of Fourier and Poisson terms representing the contribution from nutation. These developments, which ensure a microarcsecond accuracy valid over an interval of several centuries, aim to meet the requirements of high-accuracy applications. Outside this interval the errors quickly grow with time. In reality, precession represents a complicated and very long-periodic process, with periods equal to hundreds of centuries; this can be demonstrated by numerical integration of the respective equations of motion of the Earth in the solar system and its rotation (see below).

It appears necessary to develop expressions that would allow more realistic long-term behavior, comparable to that of the Euler angle approaches. The purpose of this paper is to provide a development for these quantities for use in the long term (covering several precession cycles) and to evaluate their accuracy through numerical comparison with precession-nutation ephemerides based on other precession formulations. A first such attempt was made by one of us (Vondrák 2007), where long-term developments of the precession of the ecliptic and equator were derived separately. This long term study will be mainly based on precession only, the nutation part being the short-periodic (i.e., with periods shorter than several tens of years) component of the motion.

2. NUMERICAL INTEGRATION AND X, Y POLE COORDINATES

In order to obtain the long-term behavior of Earth's orientation in space we use the numerical integration of both Earth's motion in solar system (precession of the ecliptic) and its rotational motion (general precession and obliquity of the ecliptic).

A. Precession of the ecliptic (angles Π_A, π_A):
The basis for this is the numerical integration of the motion of the solar system, using the integrator package Mercury 6 (Chambers, 1999), in the interval 2000cy with step equal to 1cy.

B. General precession, obliquity (angles p_A, ε_A):
The basis is the numerical integration of general precession and obliquity LA93 (Laskar et al. 1993) available for the interval 1My with step equal to 10cy. Additional corrections are applied to account for the secular change of dynamical ellipticity ($\dot{J}_2$) and the secular change of obliquity.

The relations of these four angles to other parameters describing precession are shown in Fig. 1. To calculate the precession part of X, Y we obtain first the auxiliary angles α, β, γ from the triangle $\Upsilon\Upsilon_o$N:

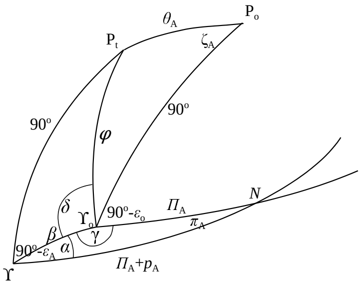

Figure 1: Precession quantities

$$
\begin{aligned}
\cos\beta &= \cos\Pi_A \cos(\Pi_A + p_A) + \sin\Pi_A \sin(\Pi_A + p_A)\cos\pi_A \\
\sin\beta\cos\alpha &= \cos\Pi_A \sin(\Pi_A + p_A) - \sin\Pi_A \cos(\Pi_A + p_A)\cos\pi_A \\
\sin\beta\sin\alpha &= sin\Pi_A \sin\pi_A \\
\sin\beta\sin\gamma &= \sin(\Pi_A + p_A)\sin\pi_A \\
\sin\beta\cos\gamma &= \sin\Pi_A \cos(\Pi_A + p_A) - \cos\Pi_A \sin(\Pi_A + p_A)\cos\pi_A,
\end{aligned}
\qquad (1)
$$

then the angles φ, δ by solving the triangle $\Upsilon\Upsilon_o$P$_t$

$$
\begin{aligned}
\cos\varphi &= \sin\beta\sin(\varepsilon_A + \alpha) \\
\sin\varphi\cos\delta &= -\cos\beta\sin(\varepsilon_A + \alpha) \\
\sin\varphi\sin\delta &= \cos(\varepsilon_A + \alpha)
\end{aligned}
\qquad (2)
$$

and finally, from triangle Υ_oP$_t$P$_o$ and accounting for small celestial pole and equinox offsets, we get

$$
\begin{aligned}
X &= \sin\theta_A\cos\zeta_A + 0.0146'' \sin\theta_A \sin\zeta_A - 0.016617'' \\
Y &= -\sin\theta_A\sin\zeta_A + 0.0146'' \sin\theta_A \cos\zeta_A - 0.006951'',
\end{aligned}
\qquad (3)
$$

where $\sin\theta_A\cos\zeta_A = \cos\varphi$, $\sin\theta_A\sin\zeta_A = \sin\varphi\cos(\gamma+\delta-\varepsilon_o)$. We used these formulas to calculate X, Y in the interval $\pm$2000cy with 1cy step.

Next we compared these long-term integrated values with different models, in the interval of $\pm$200 centuries (about 1.5 precession cycles) around J2000 (see Fig. 2). The first three models – by Lieske et al. (1977), designated Lieske; by Simon et al. (1994), designated Simon; and by Capitaine et al. (2003b), designated Capitaine I – are such that the values X, Y are calculated from polynomial expressions for the angles θ_A, ζ_A. In the last model (Capitaine II) the values X, Y are directly calculated from their polynomial expressions provided in (Capitaine et al. 2003b). It is evident that the models are not graphically distinguishable in the interval $\pm$50cy around J2000, but they start to differ significantly outside the interval $\pm$100cy. Generally speaking, a better fit is obtained if X, Y values are calculated from the polynomial expressions for θ_A, ζ_A, using Eqs. (3) rather than from the direct development of X, Y.

3. ESTIMATION OF PERIODIC TERMS OF THE MODEL

To develop a long-term precession formula, valid over several precession cycles, the expressions for X, Y must contain long-periodic terms. Therefore, we made a spectral analysis of integrated coordinates X, Y, based on a least-squares approach, to find hidden periodicity. The most pronounced ones were then compared with the periods found by Laskar (1993, 2004) from much longer interval. We identified our periods with Laskar's whenever possible and made a least-squares estimation of 14 dominant sine/cosine amplitudes plus a cubic parabola (to account for missing longer periods).

In order to obtain a model as close to P03 as possible near the epoch J2000, we used numerical integration outside the interval $\pm$10cy, and P03 values inside this interval, with much higher weights.

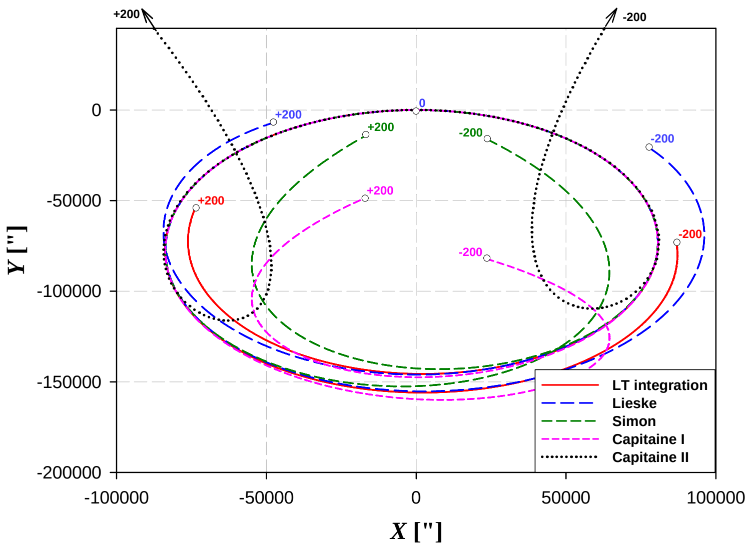

Figure 2: Different models of precession X, Y in the interval ±200cy around J2000

Small additional corrections were then applied to the constant, linear and quadratic terms to keep the derivatives up to the 2nd order identical with P03 model. The results (in arcseconds) are given as

$$\begin{aligned} X =& 5452.121068 + 0.4936640T - 0.00037051T^2 - 188 \times 10^{-9}T^3 + \\ &+ \sum_{i=1}^{14} (C_{xi} \cos 2\pi T/P_i + S_{xi} \sin 2\pi T/P_i) \\ Y =& -73748.904862 - 0.7300392T - 0.00018363T^2 + 212 \times 10^{-9}T^3 + \\ &+ \sum_{i=1}^{14} (C_{yi} \cos 2\pi T/P_i + S_{yi} \sin 2\pi T/P_i)\,. \end{aligned} \quad (4)$$

The periodic terms of Eqs. (4), where T counts in centuries from J2000.0, are given in Table 1 below.

Table 1: Periodic terms in long-term expressions for X, Y

term	P[cy]	C_x	S_x	C_y	S_y
p	256.75	-890.392958	81486.055678	74993.013701	1624.771025
σ_3	708.15	-8442.032827	786.472556	623.634003	7772.231028
$p - g_2 + g_5$	274.20	2645.487483	1175.748879	1183.101287	-2262.613844
$p + g_2 - g_5$	241.45	2799.283269	-1163.092273	-1010.239126	-2564.893806
s_1	2309.00	-165.543689	-3021.082069	-2654.217193	217.164493
s_6	492.20	872.202300	639.204007	699.627348	-846.622884
$p + s_4$	396.10	45.589521	129.102969	152.109075	-1394.137691
$p + s_1$	288.90	-523.682245	-419.460618	-926.684032	379.173891
$p - s_1$	231.10	-827.551724	529.877488	444.781911	757.410362
	1610.00	-539.346941	-60.246349	-151.914565	462.551085
	620.00	-193.676517	524.751903	557.485310	239.959374
$2p + s_3$	157.87	-403.471752	-13.830660	-26.992841	374.350053
	220.30	180.209107	-196.580144	-147.305110	-172.499874
	1200.00	-9.210712	-52.971311	12.498143	-28.484470

The differences of both integrated values and model defined by Eqs. (4) in the interval ±20cy are depicted in Fig. 3. A similar comparison in much longer interval (±2000cy) shows that the integration and our new model differ only very slightly (not more than several arcminutes), while their difference from P03 grows extremely rapidly.

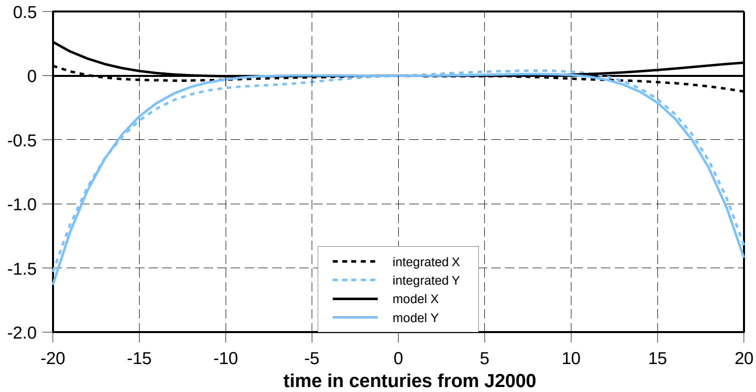

Figure 3: Differences of integrated and modeled values $X, Y\,[''']$ from P03

4. DISCUSSION

Most of the precession models used so far, being expressed as polynomials of time, are valid, with a high accuracy, only for a few centuries around J2000). Their errors grow rapidly outside this interval – more than 10° 200 centuries from J2000. Generally speaking, models based on polynomials for the classical precession angles give better results than those obtained from the time polynomials for the GCRS CIP coordinates X, Y.

We demonstrate the possibility of constructing a model of precession for predicting the CIP direction in the GCRS that yields results comparable to P03 in a short-time interval (a few centuries) around J2000, and follows the periodical character of precession in a long-term sense (hundreds of millennia), with only very slowly decreasing accuracy (several arcminutes) ±0.2My away from J2000.

5. REFERENCES

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