THE KINEMATICAL ANALYSIS OF PROPER MOTIONS AND RADIAL VELOCITIES OF STARS BY MEANS OF THE VECTOR SPHERICAL HARMONICS

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ABSTRACT. The paper describes the application of the 3-D vector spherical harmonics (henceforth VSH) to the investigation of stellar kinematics. The VSH technique is suitable for present and future catalogues which contain all three components of velocity vector: proper motions and radial velocities. In general, the VSH allows to detect all the systematic components in the stellar velocity field and does not depend on any model. If some physical model is used, the VSH not only determines the parameters of the model, but detects the systematic components which are beyond the model. The application of the VSH to the Hipparcos data complimented with radial velocities discovers the systematic components which are beyond the linear Ogorodnikov-milne model.

1. OUTLINE OF THE METHOD

It is widely known that in the linear approximation the stellar velocity field may be described with the Ogorodnikov-milne model

$$\vec{V} = \vec{V}_0 + M^+ \vec{r} + M^- \vec{r}, \tag{1}$$

where $\vec{V}_0$ — taken with opposite sign velocity of the Sun with respect to given centroid of stars. This velocity is defined by components U, V, W in the directions of the principal galactic axes x, y, z; M^+ — the diverging matrix with the dilation coefficients M^+_{11}, M^+_{22}, M^+_{33}, and M^+_{12}, M^+_{13} M^+_{23} standing for shears in the galactic planes (x, y), (x, z), (y, z), M^- — the rotation matrix with the components ω_1, ω_2 ω_3 of the angular rotation vector in the proper motions about axes x, y, z;

Let $\mathbf{e}_l, \mathbf{e}_b, \mathbf{e}_r$ be the unit vectors in longitude, latitude and radial velocity directions. Using notation $K_{nkp}(l, b)$ for spherical harmonics, introduce Vectorial Spherical Functions as follows:

$$\mathbf{V}_{nkp}(l, b) = K_{nkp}(l, b)\mathbf{e}_r. \tag{2}$$

$$\mathbf{T}_{nkp} = \frac{1}{\sqrt{n(n+1)}} \left(\frac{\partial K_{nkp}(l, b)}{\partial b} \mathbf{e}_l - \frac{1}{\cos b} \frac{\partial K_{nkp'}(l, b)}{\partial l} \mathbf{e}_b \right), \tag{3}$$

$$\mathbf{S}_{nkp} = \frac{1}{\sqrt{n(n+1)}} \left(\frac{1}{\cos b} \frac{\partial K_{nkp}(l, b)}{\partial l} \mathbf{e}_l + \frac{\partial K_{nkp}(l, b)}{\partial b} \mathbf{e}_b \right). \tag{4}$$

We have shown that with $M^*_{11} = M^+_{11} - M^+_{22}$; $x = M^+_{33} - \frac{1}{2}(M^+_{11} + M^+_{22})$; $y = \frac{1}{3}(M^+_{11} + M^+_{22} + M^+_{33})$; r standing for distance in $[ps]$ and proper motions expressed in $[km\ s^{-1}\ kps^{-1}]$ the equations of condition may be written in terms of VSF as follows:

$$
\begin{aligned}
\mu_l \cos b\, \mathbf{e}_l + \mu_b\, \mathbf{e}_b = & -U/r\, \frac{\mathbf{S}_{111}(l,b)}{\rho_{11}} - V/r\, \frac{\mathbf{S}_{110}(l,b)}{\rho_{11}} - W/r\, \frac{\mathbf{S}_{101}(l,b)}{\rho_{10}} + \\
& + \omega_1\, \frac{\mathbf{T}_{111}(l,b)}{\rho_{11}} + \omega_2\, \frac{\mathbf{T}_{110}(l,b)}{\rho_{11}} + \omega_3\, \frac{\mathbf{T}_{101}(l,b)}{\rho_{10}} + \\
& + \frac{M_{13}^+}{3}\, \frac{\mathbf{S}_{211}(l,b)}{\rho_{21}} + \frac{M_{23}^+}{3}\, \frac{\mathbf{S}_{210}(l,b)}{\rho_{21}} + \frac{M_{12}^+}{6}\, \frac{\mathbf{S}_{220}(l,b)}{\rho_{22}} + \\
& + \frac{M_{11}^*}{12}\, \frac{\mathbf{S}_{221}(l,b)}{\rho_{22}} + \frac{x}{3}\, \frac{\mathbf{S}_{201}(l,b)}{\rho_{20}},
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
V_r/r = & -U/r\, \frac{K_{111}(l,b)}{R_{11}} - V/r\, \frac{K_{110}(l,b)}{R_{11}} - W/r\, \frac{K_{101}(l,b)}{R_{10}} + \\
& + M_{13}^+\, \frac{2K_{211}(l,b)}{3R_{21}} + M_{23}^+\, \frac{2K_{210}(l,b)}{3R_{21}} + M_{12}^+\, \frac{K_{220}(l,b)}{3R_{22}} + \\
& + M_{11}^*\, \frac{K_{221}(l,b)}{6R_{22}} + x\, \frac{2K_{201}(l,b)}{3R_{20}} + y\, \frac{K_{001}(l,b)}{R_{00}},
\end{aligned} \tag{6}
$$

where

$$
R_{nk} = \sqrt{\frac{2n+1}{4\pi}} \begin{cases} \sqrt{\frac{2(n-k)!}{(n+k)!}}, & k > 0; \\ 1, & k = 0, \end{cases} \tag{7}
$$

$$
\rho_{nk} = \frac{R_{nk}}{\sqrt{n(n+1)}}\ . \tag{8}
$$

This apparently new result gives possibility to use decomposition of the velocity field on VSF for two goals: determination of the Ogorodnikov-Milne parameters and searching the systematics beyond the model.

Application of this method to the proper motions and parallaxes from HIPPARCOS catalogue complimented with known radial velocity gave evidence that beside the classical components the real velocity field does contain extramodel systematic components partial explanation of which may be given in frames of generalized kinematical model of the second order. The further details of the FSF method will be published elsewhere.

Acknowledgements. The authors appreciate the support of this work by the grant 1323.2008.2 of the President of Russian Federation and by the grant 2.1.1.5077 of the Ministry of Education and Sciences.

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