SOLVING THE ROTATIONAL EARTH'S EQUATIONS IN RECTANGULAR COORDINATES FOR A NON-RIGID EARTH

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ABSTRACT. The Descartes prize 2003 was awarded by the European Union to the IAU/IUGG Working Group on 'Nutation for a nonrigid Earth'. One of the Descartes-nutation projects following this prize is named "Advances in the integration of the equations of the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions". The first paper devoted to this project has been published by Capitaine at al. (2006) (it will be denoted $XYRE06$ in the following) and the second one (denoted $XYRE08$ in the following) is about to be submitted by Capitaine and Folgueira (2008). This poster contains a brief report of Sub-project entitled *"Solving the rotational Earth's equations in rectangular coordinates for a non-rigid Earth"* which is currently under development as an extension to a non-rigid Earth model of the two previous works ($XYRE06$ and $XYRE08$) considered for the rigid Earth.

1. BACKGROUND

The aim of $XYRE06$ and $XYRE08$ consisted in obtaining the coordinates of the celestial intermediate pole CIP, (X,Y), in the geocentric celestial reference system ($GCRS$), from the direct integration of the differential equations in the terms of these coordinates.

1. The rigorous rotational equations (see Capitaine at al. 2006) were developed for a rigid Earth in terms of $(X, Y)_R$ and were then simplified for an axially symmetric Earth ($A = B$) as follows:

$$\begin{aligned} -\ddot{Y} + \frac{C\Omega}{A}\dot{X} &= \frac{L}{A} + F'' \\ \ddot{X} + \frac{C\Omega}{A}\dot{Y} &= \frac{M}{A} + G'', \end{aligned} \tag{1}$$

   where F'' and G'' are functions, at the 2nd order of X, Y and their 1st and 2nd time derivatives, L and M are the components of the torque in the Celestial intermediate reference system (denoted $CIRS'$) linked to the CIP and the point Σ (see Capitaine et al. 2006) and A, C are the principal moments of inertia of the rigid Earth.

2. The most complete semi-analytical developments of the solutions for the Moon (ELP2000), the Sun and the planets (VSOP97), were used to to compute the semi-analytical development of the external torque.

3. The equations were integrated using the method of variations of parameters and a semi-analytical integration with successive approximations to obtain the X, Y solution.

2. RESEARCH DESIGN OF THE PROJECT

The main goal of this work is to use the same method as in $XYRE08$, but with a non-rigid Earth model in order to obtain the solution for the $GCRS$ coordinates of the CIP (X,Y) for a non-rigid Earth.

1. We will first consider a deformable Earth for which the dynamical equations have been provided by Capitaine at al. (2006) in terms of $(X, Y)_{DE}$:

$$\begin{aligned} -\ddot{Y} + \Gamma\dot{X} &= \tfrac{L}{A} + d\psi_1^L + d\psi_1^{TD} + F'' \\ \ddot{X} + \Gamma\dot{Y} &= \tfrac{M}{A} + d\psi_2^L + d\psi_2^{TD} + G'' , \end{aligned} \tag{2}$$

   with :

$$\Gamma = (C\Omega/A)/\left(1 + \tfrac{k_1}{k_S}\tfrac{C-A}{A}\right), \tag{3}$$

   where, $\psi = \psi_1 + i\psi_2$ is the excitation function, $d\psi_L$ is the contribution due to the secular variation of the dynamical ellipticity of the Earth, $d\psi_{TD}$ is the contribution due to the tidal deformation of the Earth and L and M are components of the external torque which could be obtained as in the case for rigid Earth. The differential equations (2) will be solved semi-analytically by the method of variation of parameters in a similar way as for a rigid Earth, but taking into account (i) the new frequency for a deformable Earth, $\Gamma = f(\sigma, k_2, k_S)$, and (ii) the additional contributions $d\psi_L$ and $d\psi_{TD}$.

2. Then, the computed solutions $(X, Y)_{DE}$ will be checked by comparison with the solutions obtained in a classical way :

$$(\Delta\psi,\ \Delta\varepsilon)_{\text{REN2000}} \xrightarrow{\text{Transfer Function (DE)}} (\Delta\psi,\ \Delta\varepsilon)_{DE} \xrightarrow{\text{(Capitaine at al. 2003)}} (X, Y)^1_{DE}.$$

3. The model will be improved step by step taking into account different non-rigid effects (elasticity, fluid core, anelasticity, etc.). Figure (1) displays the values taken by the so-called *transfer function* (see Mathews et al. 2002) for tidal frequencies in the diurnal band for increasingly complex Earth models. Anelasticity has weak effects, making it non distinguishable on the plot from elasticity.

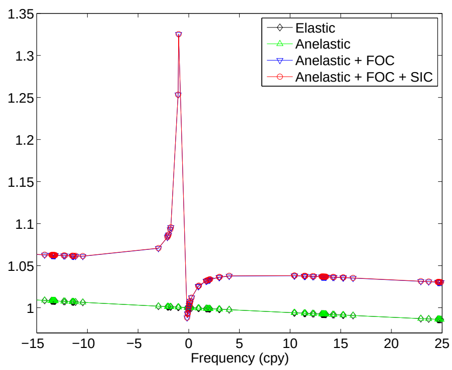

Figure 1: Values of the transfer function for various Earth models where *FOC* and *SIC* are free core nutation and solid inner core, respectively.

Acknowledgements. This research was made possible thanks to the Descartes-nutation project and two research stays V. Stefka and M. Folgueira at Paris Observatory (SYRTE Department).

3. REFERENCES

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