

AN ANALYTICAL THEORY OF THE ROTATION OF VENUS

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ABSTRACT. Although a full and detailed discussion of the effects of the Venus atmosphere and Venus interior on its rotation has already been done by several authors (Correia & Laskar, 2001; Yoder, 1995), a precise analytical determination of its precession and nutation coefficients has been poorly investigated. In that aim, we use the same theoretical framework as Kinoshita (1977) for the Earth, in order to construct the tables of nutation of Venus, together with its precession constant. A comparative study with the Earth is done, which shows that two fundamental characteristics of Venus lead to very specific rotational properties: its very long sidereal retrograde rotation (243 d) and its very small flattening.

1. INTRODUCTION

One of the important results of Venus' radar ranging observations was the discovery of its retrograde rotation, with a period of -243.16 days. Various efforts were made to explain this particular rotational mode. Soon after the discovery, Goldreich & Peale (1970) explained that thermally driven atmospheric tidal torque and dissipation of energy at the boundary between a rigid mantle and a differentially rotating liquid core are possible mechanics for maintaining the retrograde spin. Lago & Cazenave (1979) have studied the past evolution of the rotation of Venus using the hypothesis that only solar tidal torques and core-mantle coupling have been active since formation. They have found quite conceivable that Venus had originally a rotation quite similar to the other planets and had evolved in 4.5×10^9 years from a rapid and direct rotation to the slow retrograde one. Yoder(1995) has made a full account of the various internal mechanisms acting on Venus' obliquity, as core friction, CMB (core-mantle boundary) ellipticity, and resonant excitations. More recently Correia & Laskar (2001) showed that the atmospheric torque on Venus is large enough for a synchronous rotation to become unstable. Then two configurations appear, one prograde and the other retrograde. From numerical simulations they investigated four possible scenarios of change of the obliquity, one leading to the present state. Although these various studies concern the evolution of Venus' rotation at very long time scales, few attempts were made to modelize accurately the rotation for a short time span. An analytical study of the rotation of rigid Venus was carried out by Habibullin (1995) with rather uncommon parametrization, showing that Venus rotates almost uniformly with libration harmonics being negligible. In the following we propose an alternative construction of a rigid Venus rotation contradicting in some extent these last results.

2. MODELING VENUS' ROTATION

In Table 1 we present the various orbital, rotational and physical characteristics of Venus together with the corresponding values for the Earth. Some elements tend to make the amplitudes of the precession constant and the coefficients of nutation larger for Venus than for the Earth : these are the smaller heliocentric distance as well as the very long period of rotation. On the opposite the comparative small value of the dynamical ellipticity should reduce these amplitudes. The fact that the Venus eccentricity is significantly smaller than the Earth's should also lead to a smaller number of coefficients for a given level of amplitude. The moments of inertia A , B , and C have been taken from Habibullin (1995).

In our study, Venus is considered as a rigid body. Therefore in order to construct the analytical theory of its rotation, we use the same analytical framework as Kinoshita (1977) when computing the coefficients of nutation of the Earth for a rigid model. Thus we use the Andoyer variables (Andoyer, 1923) allowing a full description of the rotation, by considering three different and close axis : the axis of figure, the axis of angular momentum and the axis of rotation. The precession-nutation angles in longitude and obliquity are respectively given by h and I with : $H = G \cos I$.

Table 1: Results of cross-identification between A–H and HB.

	Earth	Venus
Semi-major axis	1.000000 U.A	0.723332 U.A
Eccentricity	0.0167	0.0068
Density	5.51 g/cm^3	5.25 g/cm^3
Equatorial radius	6378 km	6051 km
Mass	$5.97 \times 10^{24} kg$	$4.87 \times 10^{24} kg$
Period of revolution	365.25 d	224.70 d
Inclination	0	$3^\circ.39$
Obliquity	$23^\circ.43$	$2^\circ.63$
Period of rotation	0.997 d	-243.02 d
Ascending node	$174^\circ.87$	$76^\circ.67$
Triaxiality $\frac{A-B}{4C}$	-5.34×10^{-6}	-2.67×10^{-6}
Dyn. Flattening : $\frac{2C-A-B}{2C}$	3.27×10^{-3}	1.7×10^{-5}
$\frac{B-A}{2*(2*C-A-B)}$	-0.0016	-0.15

Then the Hamiltonian for the Venus' rotation becomes : $K = F_0 + U + E$, where F_0 stands for the Hamiltonian of the free motion of rotation, U is the perturbing potential and E is a part of the Hamiltonian corresponding to a change of reference plane, from Venus' orbital plane at the fundamental epoch J2000.0 (the equivalent of the ecliptic of J2000.0 for the Earth) to the orbital plane at the date t . At the first order U can be divided into two parts, one involving the Venus' dynamical flattening (term depending on $2C - A - B$) and the other one Venus' triaxiality (term depending on $A - B$): $U = U_1 + U_2$ with :

$$U_1 = \frac{k^2 M'}{r^3} \left[\frac{2C - A - B}{2} P_2(\sin \delta) \right] \quad U_2 = \frac{k^2 M'}{r^3} \left[\frac{A - B}{4} P_2^2(\sin \delta) \cos 2\alpha \right] \quad (1)$$

Where M' is the mass of the Sun, r its distance from Venus geocenter and α et δ its planetocentric coordinates (respectively the longitude et latitude of the Sun with respect to a meridian origin. The P_n^m are the classical Legendre functions.

The basic canonical equations are (Kinoshita, 1977) :

$$\frac{d}{dt}(L, G, H) = -\frac{\partial K}{\partial(l, g, h)} \quad \frac{d}{dt}(l, g, h) = \frac{\partial K}{\partial(L, G, H)} \quad (2)$$

Where K is obtained by integrating directly U . Following the same methodology as Kinoshita (1977) we have computed the precession constant as well as the nutation coefficients of Venus. Complete results will be published soon (Cottureau & Souchay, in prep.) The scaling factor k for these computations is given by $k_{Venus} = 3(n^2/\omega) \times (2C - A - B/2C)$. Although the dynamical flattening ($2C - A - B/2C$) at the nominator is much smaller than for the Earth, this is compensated by the fact that the angular rotational velocity ω at the denominator is itself much smaller. Thus we have calculated that the precession constant is $\dot{\psi} = 6133.7''/cy$ by taking the same values of A , B , and C as Habibullin (1995). It is roughly 3.5 times larger than the solar part of the precession constant of the Earth. The largest nutation coefficients has a $3''$ amplitude and a period (112.35 d) half the period of revolution of Venus. Complete computations are on the way (Cottureau & Souchay, in prep.)

3. REFERENCES

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