RELATIVISTIC ASPECTS IN ASTRONOMICAL STANDARDS AND THE IERS CONVENTIONS

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ABSTRACT. In the last years, the definition of reference systems and their application to astronomy and geodesy has been passed into Resolutions of the scientific unions, following work of several working groups and of the community at large. However some work remains to be done to disseminate the information and to resolve a few residual questions. We review some topics that are still debated in the ad-hoc working groups and commissions. The IAU framework for relativity has also been introduced in the IERS Conventions, but some work remains to be done to obtain full consistency and conformity throughout the document, as will be reviewed. Finally, several updates under realization or under consideration for the IERS Conventions are presented.

1. THE RELATIVISTIC FRAMEWORK

In order to describe observations in astronomy and geodesy, one has to choose the proper relativistic reference systems best suited to the problem at hand. A barycentric celestial reference system (BCRS) should be used for all experiments not confined to the vicinity of the Earth, while a geocentric celestial reference system (GCRS) is physically adequate to describe processes occurring in the vicinity of the Earth. These systems have been defined in a series of Resolutions passed by scientific Unions, mostly the International Astronomical Union (IAU), in the past 20 years, see a more complete description of the work until year 2000 in (Soffel et al., 2003).

1.1 1991 and the following years

The reference systems were first defined by the IAU Resolution A4 (1991) which contains nine recommendations, the first four of which are summarized below.

In the first recommendation, the metric tensor for space-time coordinate systems $(t, \mathbf{x})$ centered at the barycenter of an ensemble of masses is recommended in the form

$$\begin{aligned} g_{00} &= -1 + 2U(t,\mathbf{x})/c^2 + \mathcal{O}(c^{-4}), \\ g_{0i} &= \mathcal{O}(c^{-3}), \\ g_{ij} &= \delta_{ij}\left(1 + 2U(t,\mathbf{x})/c^2\right) + \mathcal{O}(c^{-4}), \end{aligned} \tag{1}$$

where c is the speed of light in vacuum (c = 299792458 m/s) and U is the Newtonian gravitational potential (here a sum of the gravitational potentials of the ensemble of masses, and of a external potential generated by bodies external to the ensemble, the latter potential vanishing at the origin). The recommended form of the metric tensor can be used, not only to describe the barycentric reference system of the whole solar system, but also to define the geocentric reference system centered in the center of mass of the Earth with U, now depending upon geocentric coordinates.

In the second recommendation, the origin and orientation of the spatial coordinate grids for the barycentric and geocentric reference systems are defined.

The third recommendation defines TCB (Barycentric Coordinate Time) and TCG (Geocentric Coordinate Time) as the time coordinates of the BCRS and GCRS, respectively, and, in the fourth recommendation, another time coordinate named TT (Terrestrial Time), is defined for the GCRS as

$$\mathrm{TT} = \mathrm{TCG} - \mathrm{L_G} \times (\mathrm{JD_{TCG}} - T_0) \times 86400, \tag{2}$$

where $\mathrm{JD_{TCG}}$ is the TCG Julian date, $T_0 = 2443144.5003725$ and where $L_G = U_G/c^2$ with U_G being the gravity potential on the geoid.

Note that the International Union for Geodesy and Geophysics (IUGG), in its Resolution 2 (1991), endorsed the IAU Recommendations and explicitly based its definition of Terrestrial Reference Systems on the IAU relativistic framework.

In 1997 the IAU has supplemented the framework by one more recommendation stating that no scaling of spatial axes should be applied in any reference system, even if scaled time coordinate like TT is used for convenience of an analysis (this is in relation with e.g. discussions on the VLBI model, see Section 3.3).

1.2 2000 and the recent years

In the years following the adoption of the IAU'1991 Resolution, it became obvious that this set of recommendations was not sufficient, especially with respect to planned astrometric missions with μas-accuracies and with respect to the expected improvement of atomic clocks and the planned space missions involving such clocks and improved time transfer techniques. For that reason the IAU WG "Relativity for astrometry and celestial mechanics" together with the BIPM-IAU Joint Committee for relativity suggested an extended set of Resolutions that was finally adopted at the IAU General Assembly in Manchester in the year 2000 as Resolutions B1.3 to B1.5 and B1.9.

Resolution B1.3 concerns the definition of Barycentric Celestial Reference System (BCRS) and Geocentric Celestial Reference System (GCRS). The Resolution recommends to write the metric tensor of the BCRS in the form

$$\begin{aligned} g_{00} &= -1 + 2w/c^2 - 2w^2/c^4 + \mathcal{O}(c^{-5}), \\ g_{0i} &= -4/c^3 w^i + \mathcal{O}(c^{-5}), \\ g_{ij} &= \delta_{ij} \left(1 + 2w/c^2\right) + \mathcal{O}(c^{-4}) \, . \end{aligned} \tag{3}$$

where w is a scalar potential and w^i a vector potential. This extends the form of the metric tensor given in (1), so that its accuracy is now sufficient for all applications foreseen in the next years, including those involving accurate space clocks. For the GCRS, Resolution B1.3 also adds that the spatial coordinates are kinematically non-rotating with respect to the barycentric ones.

Resolution B1.4 provides the form of the expansion of the post-Newtonian potential of the Earth to be used with the metric of Resolution B1.3.

Resolution B1.5 applies the formalism of Resolutions B1.3 and B1.4 to the problems of time transformations and realization of coordinate times in the solar system. Resolution B1.5 is based upon a mass monopole spin dipole model. It provides an uncertainty not larger than 5×10^{-18} in rate and, for quasi-periodic terms, not larger than 5×10^{-18} in rate amplitude and 0.2 ps in phase amplitude, for locations farther than a few solar radii from the Sun. The same uncertainty also applies to the transformation between TCB and TCG for locations within 50 000 km of the Earth.

Some shortcomings appeared in the definition of TT (2) when considering accuracies below 10^{-17}: the uncertainty in the determination of U_g is limited, the surface of the geoid is difficult to realize so that it is difficult to determine the potential difference between the geoid and the location of a clock, and in addition the geoid varies with time. Therefore it was decided to dissociate the definition of TT from the geoid while maintaining continuity with the previous definition. The constant L_g was turned into a defining constant with its value fixed to $6.969290134 \times 10^{-10}$ in Resolution B1.9, which therefore removes the limitations mentioned above when realizing TT from clocks onboard terrestrial satellites.

Finally in 2006 it was decided to redefine the coordinate time TDB, which had been introduced by the IAU in 1976 as a dynamical time scale for barycentric ephemerides. As it had not been unambiguously defined, multiple realizations of TDB were possible. Because such realizations are still widely used for barycentric ephemerides, IAU Resolution B3 (2006) was passed to define TDB as the following linear transformation of TCB:

$$\mathrm{TDB} = \mathrm{TCB} - \mathrm{L_B} \times (\mathrm{JD_{TCB}} - T_0) \times 86400 + TDB_0, \tag{4}$$

where $\mathrm{JD_{TCB}}$ is the TCB Julian date and where $L_B = 1.550519768 \times 10^{-8}$ and $TDB_0 = -6.55 \times 10^{-5} s$ are defining constants. Figure 1 shows graphically the relationships between the time scales following the IAU Resolutions of 1991, 2000 and 2006.

2. RELATIVISTIC ASPECTS IN ASTRONOMICAL STANDARDS

The main task of the IAU Working Group on Numerical Standards for Fundamental Astronomy (Luzum et al., 2008) is to update the IAU Current Best Estimates, eventually defining a new official system of constants. In doing so, it has to apply the relativistic framework defined above, and to disseminate the information on the consequences.

Without duplicating work of the Working group, we here remind the principal items where the application of the relativistic framework has direct impact on the astronomical standards. One concerns the definitions of the constants related to the rates between time coordinates L_C, L_G and L_B. In the present situation (since 2006) the last two are *defining* constants, where $\mathrm{d}TT/\mathrm{d}TCG = 1 - L_G$ and $\mathrm{d}TDB/\mathrm{d}TCB = 1 - L_B$, while L_C is *measured* as $1 - L_C =< \mathrm{d}TCG/\mathrm{d}TCB >$. Thus the relation $1 - L_B = (1 - L_G).(1 - L_C)$, which was used to define L_B, is no more (strictly) valid. Equivalently this means that TT and TDB dont have (strictly) the same rate at the geocenter.

Also of importance to this field is the discussion on the nomenclature (see section 3.1) and the choices to be made on the statute of the Sun's gravitational constant $GM_\odot$ and (correspondingly) that of the Gaussian constant K and the astronomical unit.

3. RELATIVISTIC ASPECTS IN THE IERS CONVENTIONS

In the current work to update the IERS Conventions (2003), relativistic aspects cover three topics. The first is to review the nomenclature throughout the document. The second concerns chapter 10 (models for space-time coordinates and equations of motion) where, in a recent update (see `http://tai.bipm.org/iers/convupdt/convupdt.html`) the transformation from proper time to coordinate time in the vicinity of the Earth is treated and numerical examples are provided for the different terms in the relativistic expression for the acceleration of an Earth satellite. The third concerns chapter 11 (models for signal propagation) and covers models for VLBI and (radio and laser) ranging techniques.

3.1. Nomenclature

Nomenclature issues in the IERS Conventions can be loosely classified in three categories, although several issues are interconnected. The first type concerns the designation of coordinate quantities, the second type relates to the transformation between celestial and terrestrial reference systems, the third type relates to the definition and realization of terrestrial reference systems.

In the first category, we must mention the wording used to designate coordinate quantities (e.g. space coordinates, gravitational constants GM, etc...). In the Conventions (2003) three types of wording are used:

- One wording uses the word "unit", like in "[so-called] TDB unit" (for cases when a scaled coordinate time is used, here TDB), or in "TCB (SI) units" TCB (SI) units (for cases when an unscaled coordinate time is used).
- One uses the word "scale", like in "ITRF ... uses the TT scale".
- One uses a full set of words in a sentence, like in " ... coordinates consistent with TDB".

It is one goal of the IAU Commission "Relativity in fundamental astronomy" (RIFA) to propose a conventional wording (Klioner, 2008).

The second category mostly concerns Chapter 5 (transformation between celestial and terrestrial systems) and is in the process of being reviewed following the work of the IAU Division I Working Group "Nomenclature for Fundamental Astronomy" (NFA), see `http://syrte.obspm.fr/iauWGnfa/NFA_Glossary.html` and (Capitaine, 2008).

Finally the nomenclature to name terrestrial reference systems and frames is to be set consistently throughout the Conventions, mostly in Chapter 4 (Terrestrial Reference System) and Chapter 5 (Transformation between celestial and terrestrial frames). Most of the terms have been considered in the Glossary of the NFA working group. Note also that the IUGG Resolution 2 (2007) was passed to provide updated definitions and terminology concerning Terrestrial Reference Systems. Nevertheless, an IAG Inter-commission Working Group (WG 1.3) has been formed to finalize a complete recommended nomenclature in the field of Geodetic Reference Systems.

3.2. Models for space-time coordinates and the equations of motion

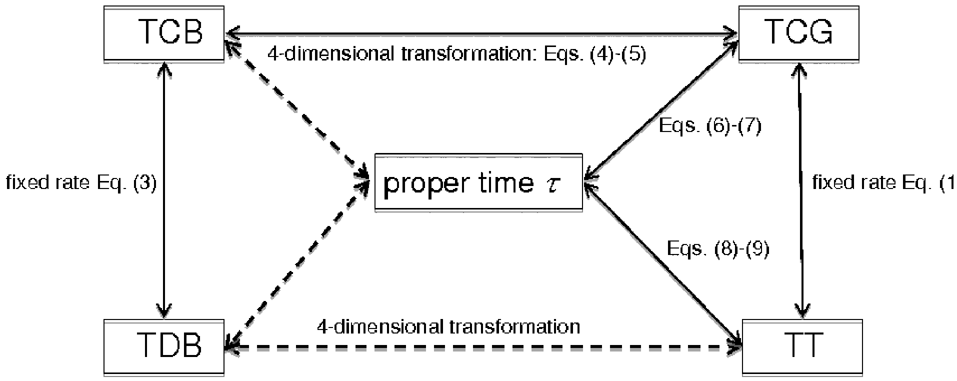

Figure 1: Various relativistic time scales and their relations. Each of the coordinate time scales TCB, TCG, TT and TDB can be related to the proper time τ of an observer, provided that the trajectory of the observer in the BCRS and/or GCRS is known. Transformations shown as dashed lines are not explicitly described in this document.

Chapter 10 (Models for space-time coordinates and the equations of motion) has been updated recently (October 2008) and the presentation of coordinate time scales now accounts for all IAU Resolutions (see section 1). The relationship between all time scales used in this context is shown on Figure 1, as taken from the IERS Conventions.In addition, a new section covers the transformation between proper time and coordinate time in the vicinity of the Earth (typically up to geosynchronous orbit or slightly above). Evaluating the contributions of the higher order terms in the metric (3) applied to the geocentric reference system GCRS, it is found that the IAU'1991 metric (1) is sufficient for time and frequency applications in the GCRS in the light of present clock accuracies.

When considering TT as coordinate time, the proper time of a clock A located at the GCRS coordinate position $\mathbf{x}_A(t)$, and moving with the coordinate velocity $\mathbf{v}_A$, is

$$\frac{d\tau_A}{dTT} = 1 + L_G - 1/c^2 \left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A) + V(X_{\mathrm{A}}) - V(X_{\mathrm{E}}) - x_A^i \partial_i V(X_{\mathrm{E}})\right] \tag{5}$$

Here, U_{E} denotes the Newtonian potential of the Earth at the position $\mathbf{x}_A$ of the clock in the geocentric frame, and V is the sum of the Newtonian potentials of the other bodies (mainly the Sun and the Moon) computed at a location X in barycentric coordinates, either at the position X_{E} of the Earth center of mass, or at the clock location X_{A}. The last three terms are tidal terms and their contribution will be limited to below 1×10^{-16} in frequency and 1 ps in time amplitude up the GPS orbit, so they may be skipped depending on the uncertainty required. Nevertheless, some care needs to be taken when evaluating the Earth's potential U_{E} at the location of the clock as the uncertainty in U_{E} should be consistent with the uncertainty expected on (5). Analytical formulas may be specified e.g. for GPS (Kouba, 2004), however a numerical integration of equation (5) using the proper development for the potential is always worth using. This is specially the case for low Earth orbit satellites (see *e.g.* Larson et al., 2007), where analytical expressions may be significantly in error or even completely misleading.

3.3. Models for signal propagation

The chapter 11 of the Conventions (2003) "Models for signal propagation" describes the relativistic model for VLBI time delay and for Laser ranging.

While no change is expected in the VLBI model, except possible changes linked to the nomenclature, is is worth reminding the past history of this model since its introduction in 1990. In 1990, the so-called "Consensus model" was adopted at a USNO workshop. This was in a "pre-1991" era so that, although relativity was carefully accounted for, the currently agreed notations did not exist at that time. The model appeared in IERS Standards (1992), but was modified in the IERS Conventions (1996), erroneously intending to comply with IAU/IUGG Resolutions, stating that "as the time argument is now based on TAI ...., distance estimates from these conventions will now be consistent 'in principle' with physical distances". However the change would have produced coordinates which would have differed from the usual "TT-compatible" coordinates by a scale change of 1.4×10^{-9}. Furthermore the stated goal is not achievable as no coordinate quantity can be consistent with a physical (proper) quantity over the extension of the Earth. The change was never implemented and the model was eventually restored in its original form in the IERS Conventions (2003) with additional explanations: Indeed the consensus model can provide either "TT-compatible" space coordinates when used with raw VLBI (TT) delays (as is the usual case in VLBI analysis), or it could provide "TCG-compatible" space coordinates if used with delays transformed to TCG. These issues are examined whenever the scale of the terrestrial reference frame is discussed, however it should be stressed that no ambiguity exists in the present model of the Conventions.

The section on laser ranging is to be expanded to cover all ranging techniques by electromagnetic signals in the vicinity of the Earth (up to the Moon). As it has been shown (Klioner, 2007) that post-post Newtonian terms are not required in view of the present uncertainty, no significant model change is expected.

4. CONCLUSIONS

The relativistic framework specified by IAU Resolutions in 1991 and 2000, and supplemented by additional Recommendations, is now complete and adapted to the current and planned applications in astrometry and space geodesy. Work remains to be done to apply it in all fields and, in some cases, a conventionally adopted nomenclature is still missing. This work is under way in IAU working groups (NSFA) and commissions (RIFA) and in the IERS Conventions center and should be concluded in the near future.

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