

# INVESTIGATION OF THE SHORT PERIODIC TERMS OF THE RIGID AND NON-RIGID EARTH ROTATION SERIES

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**ABSTRACT.** The aims of this research are the investigation of the short periodic terms in a long time new high-precision series of the rigid (S9000A) and non-rigid Earth rotation (SN9000A). Choice of the optimal spectral analysis scheme for this investigation is carried out. The previous version of the rigid Earth rotation series S9000 has been updated the new version the high-precision rigid Earth rotation series S9000A, which contained the short periodic terms (diurnal and sub-diurnal). The high-precision non-rigid Earth rotation series SN9000A containing the short periodic terms (diurnal and sub-diurnal), which is expressed as a function of the three Euler angles with respect to the fixed ecliptic plane and equinox J2000.0 and is dynamically adequate to the ephemerides DE404/LE404 over 2000 years, has been constructed.

## 1. INTRODUCTION

In the previous investigations were constructed the high-precision rigid S9000 and non-rigid Earth rotation series SN9000, which are expressed as functions of Euler angles and are dynamically adequate to the ephemeris DE404/LE404 over 2000 years. The main purpose of this research is the construction a new high-precision rigid S9000A and non-rigid Earth rotation series SN9000A, which will include the short periodic terms (diurnal and sub-diurnal).

The method of this investigation is following:

- 1.To update the previous version of the high-precision numerical solution of the rigid Earth rotation (V.V. Pashkevich, G.I. Eroshkin and A.Brzezinski, 2004), (V.V. Pashkevich and G.I. Eroshkin, Proceedings of “Journées 2004”) by using the integration step of 1 day and the interpolation step of 0.1 day. All calculations have been carried out with the quadruple precision (Real 16) on PC Intel®Core™2 Duo E6600 (2.4GHz). In this investigation is considered only the Kinematical (relativistic) case.

- 2.The initial conditions have been calculated from the model of the rigid Earth rotation SMART97 (P.Bretagnon, G.Francou, P.Rocher, J.L.Simon, 1998). The discrepancies between the numerical solution of the rigid Earth rotation and the semi-analytical solution SMART97 were expressed in Euler angles over 2000 years with 0.1 day spacing.

- 3.Investigation of the discrepancies, which contained the diurnal and sub-diurnal periodic terms, has been carried out by the least squares (LSQ) and by the refined spectral analysis (SA) algorithms (V.V. Pashkevich and G.I. Eroshkin, 2005), (V.V. Pashkevich and G.I. Eroshkin, Proc. “Journées 2005”).

- 4.The high-precision rigid Earth rotation series S9000 (V.V. Pashkevich and G.I. Eroshkin, 2005 ) has been update by including the short periodic terms (diurnal and sub-diurnal).

- 5.The new high-precision non-rigid Earth rotation series (SN9000A) has been constructed. This series is expressed in the function of the three Euler angles and containing the short periodic terms. For it is applied the method of P. Bretagnon et al. (1999) and the transfer function of P.M. Mathews et al. (2002).

## 2. INVESTIGATION RIGID AND NON-RIGID EARTH ROTATION SERIES

In order to choice of the optimal spectral analysis scheme for the investigation of the short periodic terms will consider two algorithms (Figure 1) and (Figure 2). The ALGORITHM - 1 (Figure 1) (V.V. Pashkevich and G.I. Eroshkin, Proc. “Journées 2005”) have only one iteration, which investigated the discrepancies D between numerical solution and semi-analytical solution of the rigid Earth rotation for all periodical terms. This procedure is accomplished in the following manner. After the removal of the

secular trend from discrepancies and the compute the new precessional and GMST polynomials by scheme of V.V.Pashkevich and G.I. Eroshkin, (2005), the spectra of the discrepancies are constructed by using the set of nutation terms of SMART97 solutions. The amplitudes of the spectra of the discrepancies are computed by the LSQ method for each harmonic of SMART97. The spectra of the discrepancies are bounded by the periods from 0.3284255 days to 1000 years for the time interval of 2000 years. The determination of the coefficients of the harmonics is accomplished successively starting from the maximum term in the power spectra by the LSQ method:

- a) The amplitude of the largest harmonic is determined by the LSQ method.
- b) If the absolute value of the amplitude  $Am$  of a harmonic exceeds the absolute value  $\sigma$  of its mean square error then this harmonic is removed from the discrepancies. The new harmonic is determined as a sum of the calculated periodic term and the corresponding harmonic of SMART97.
- c) The steps a) and b) are performed up to the end of the spectra.

The new series S9000A1 are constructed in the result of summing the determined periodic terms and the corresponding harmonics of SMART97.

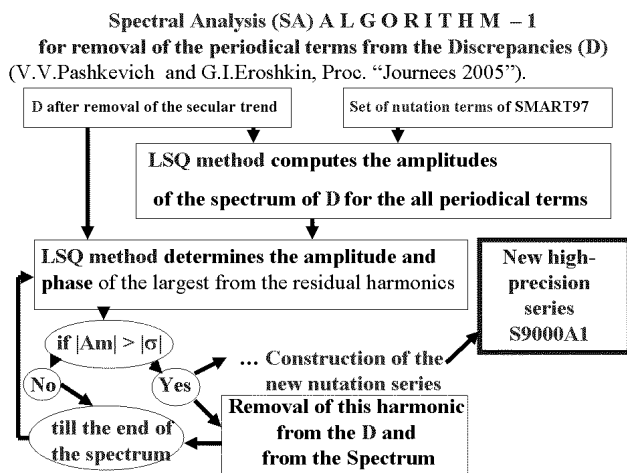


Figure 1: Algorithm 1

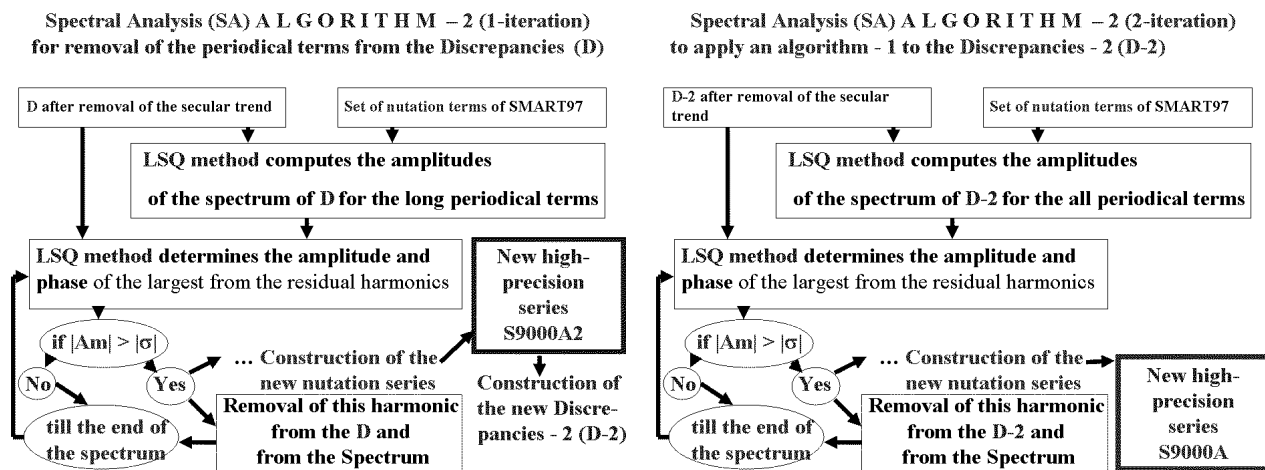


Figure 2: Algorithm 2

The ALGORITHM - 2 (Figure 2) have two iterations. The first iteration investigated the discrepancies D for only the long periodical terms. (This procedure is similar procedure of the ALGORITHM - 1. Only the spectra of the discrepancies are bounded by the periods from 1.0003 days to 1000 years for the time interval of 2000 years.) The second iteration is the ALGORITHM - 1, which investigated the

new construction (after the first iteration) discrepancies D-2 between new numerical solution and new semi-analytical solution of the rigid Earth rotation for all periodical terms. The residuary discrepancies (Figure 3) between numerical solution and semi-analytical solution of the rigid Earth rotation for case of the ALGORITHM - 2 are better than for case of the ALGORITHM - 1. So, in this investigation the ALGORITHM - 2 is better than the ALGORITHM - 1. The investigation of the discrepancies, which contained the diurnal and sub-diurnal periodic terms, has been carried out by the least squares (LSQ) and by the new spectral analysis (SA) ALGORITHM - 2. As the result of this research is the new high-precision rigid Earth rotation series S9000A, which including the short periodic terms (diurnal and sub-diurnal).

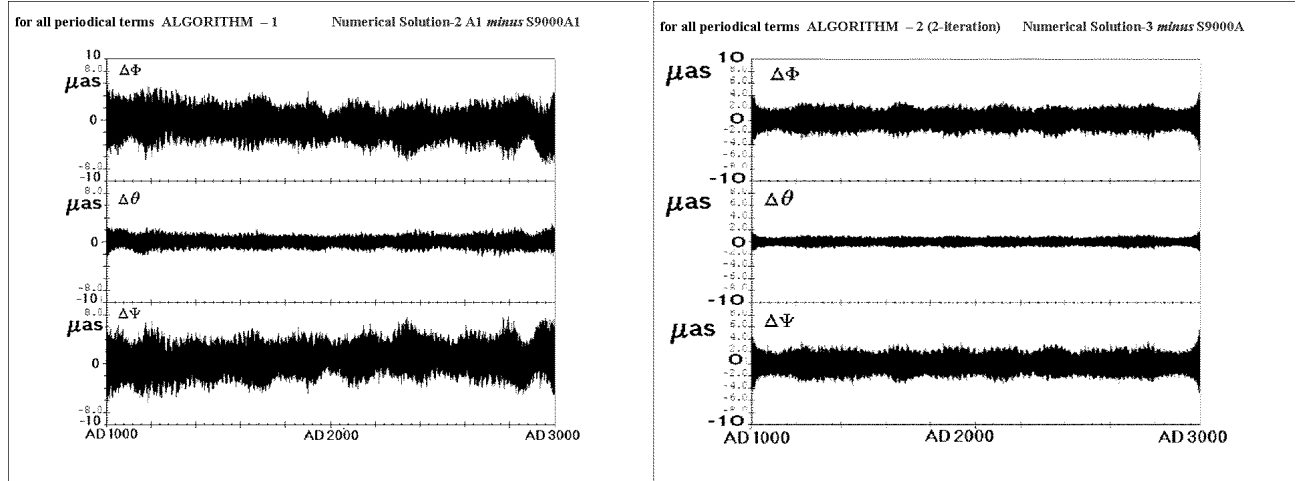


Figure 3: The residuary discrepancies between numerical solution and semi-analytical solution of the rigid Earth rotation after formal removal of the secular trends in the proper rotation angle

For the construction of the non-rigid Earth rotation series (SN9000A) in this investigation applied the method of P. Bretagnon et al. (1999) and the transfer function (MHB) and geophysical model of P.M. Mathews et al. (2002). The comparison of different Earth rotation solutions for selected diurnal periodical terms in the Euler angles are demonstrated Table 1. Others the diurnal and semidiurnal periodic terms have very small difference or have not difference.

Period (days), Argument	Solution rigid Earth rotation	$\psi(\sin)$ $\mu\text{as}$	$\psi(\cos)$ $\mu\text{as}$	$\theta(\sin)$ $\mu\text{as}$	$\theta(\cos)$ $\mu\text{as}$	$\phi(\sin)$ $\mu\text{as}$	$\phi(\cos)$ $\mu\text{as}$
1.03474 $3\lambda_3 + 3D - 2F - \phi$	SMART97	-0.0133	0.0016	-0.0010	-0.0085	-0.0129	0.0016
	S9000A	-0.0806	-0.0612	0.0073	-0.0065	-0.0754	-0.0564
0.99758 $\lambda_3 + D - l - \phi$	SMART97	-19.8544	2.4906	-0.9877	-7.8729	-18.2334	2.2873
	S9000A	-19.8521	2.4941	-0.9891	-7.8720	-18.2314	2.2908
Period (days), Argument	Solution non-rigid Earth rotation	$\psi(\sin)$ $\mu\text{as}$	$\psi(\cos)$ $\mu\text{as}$	$\theta(\sin)$ $\mu\text{as}$	$\theta(\cos)$ $\mu\text{as}$	$\phi(\sin)$ $\mu\text{as}$	$\phi(\cos)$ $\mu\text{as}$
1.03474 $3\lambda_3 + 3D - 2F - \phi$	SMN	-0.0225	-0.0100	-0.0044	-0.0006	-0.0213	-0.0090
	SN9000A	-0.0898	-0.0728	0.0039	0.0014	-0.0838	-0.0670
0.99758 $\lambda_3 + D - l - \phi$	SMN	-19.8316	-1.0929	0.4389	-7.8522	-18.2125	-1.0005
	SN9000A	-19.8293	-1.0894	0.4375	-7.8513	-18.2105	-0.9974

Table 1: The Comparison of different Earth rotation solutions for selected diurnal periodical terms in the Euler angles. Here  $\lambda_3 + D - F = \Omega + 180^\circ$ ;  $\lambda_3$  is the mean longitude of the Earth; D is the difference between the mean longitudes of the Moon and the Sun;  $\Omega$  is the mean longitude of the ascending node of the lunar orbit; F is the mean argument of the Moon's latitude; l is the mean anomaly of the Moon; SMN=SMART97+MHB; SN9000A=S9000A+MHB; MHB- Mathews et al., 2002.

### 3. CONCLUSIONS

1. The optimal spectral analysis scheme for this investigation (with respect to the accuracy) has been determined. It is algorithm-2, which used two iterations. The first iteration investigated the discrepancies for only the long periodical harmonics, while the second iteration investigated the discrepancies for all harmonics.

2. The new version the high-precision rigid Earth rotation series S9000A has been constructed. It contains the short periodic terms (diurnal and sub-diurnal).

3. The high-precision non-rigid Earth rotation series SN9000A (containing the short periodic terms) has been constructed. This series is expressed in the function of the three Euler angles with respect to the fixed ecliptic plane and equinox J2000.0 and is dynamically adequate to the ephemerides DE404/LE404 over 2000 years.

*Acknowledgements.* The author highly appreciates for very useful discussions with Prof. A. Brzeziński. The investigation was carried out at the Central (Pulkovo) Astronomical Observatory of the Russian Academy of Sciences and at the Space Research Centre of Polish Academy of Science, under a financial support of the Cooperation between Russian and Polish Academies of Sciences, Theme No 31.

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