REMARKS ABOUT RELATIVISTIC DEEP SPACE NAVIGATION

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ABSTRACT. The principal objective of spacecraft navigation is to determine the present and future trajectory of a probe. This is done by measuring the spacecraft's radial velocity and/or ranging to further improve the predicted spacecraft trajectory. Many intricate relativistic effects are involved all the way through the data analysis: Einstein-Infeld-Hoffmann equations in the predicted equations of motion, Shapiro delay in the calculation of precision light times during the tracking and finally relativistic time scales when we confront the global modelling with raw data of observation. We present here some remarks concerning that modelling by studying the order of magnitude of neglected terms.

1. FORMULATION OF DEEP SPACE NAVIGATION

One of the first task is to produce a spacecraft ephemeris usually done by a numerical integration. It gives a file of spacecraft positions and velocities as functions of the satellite ephemeris time. The equations of motion of a spacecraft are based on the well-known Einstein-Infeld-Hoffmann equations. In addition, one needs to solve the relativistic light propagation equation in order to get the solution for the total light time travel. In the solar system, this equation is given in a barycentric reference frame as follow

$$c\,(t_2 - t_1) = |\boldsymbol{x}_2 - \boldsymbol{x}_1| + \sum_A \frac{2\,\mu_A}{c^2}\,\ln\frac{|\boldsymbol{r}_{1A}| + |\boldsymbol{r}_{2A}| + |\boldsymbol{r}_{21}|}{|\boldsymbol{r}_{2A}| + |\boldsymbol{r}_{1A}| - |\boldsymbol{r}_{21}|} + \mathcal{O}(c^{-4}), \tag{1}$$

where μ_A is the gravitational constant and index 1 and 2 correspond to an emission and reception of a signal, respectively. The last interesting observable is the Doppler shift, $\delta\nu/\nu$, which can be calculated as follows:

$$\frac{\delta\nu}{\nu} = \frac{d\,\rho}{d\,t}\,, \tag{2}$$

where ρ is the two-ways precision light time (so taking into account Eq. (1), see Eq. (7)) and dt is here related to a terrestrial time scale. Because the old IAU definition of time coordinate in the barycentric frame, the only difference between TDB (dynamical barycentric time) and TT (terrestrial time) are due to periodic terms. Indeed, the difference between the International Atomic Time, TAI, and the satellite Ephemeris Time, T_{eph}, generally used (Moyer, 2003) is given by:

$$\frac{dT_{eph}}{dTAI} = 1 - \frac{U}{c^2} - \frac{1}{2}\frac{v^2}{c^2} + L\,, \quad \text{with } L = \tfrac{1}{c^2}\Big\langle U + \tfrac{v^2}{2}\Big\rangle, \tag{3}$$

$<>$ denoting the long-term average value of the quantity contained within them, U and v being the external gravitational potential of the Earth and its orbital velocity, respectively. To maintain unchanged equations of motion, light time and Doppler shift, the spatial coordinates in the barycentric frame have to be rescaled by the factor L (order of magnitude $\approx 10^{-8}$) to keep the speed of light unchanged. As μ_A/c^2 have also the dimension of length, they have to be rescaled by the factor L. Then, we have three scaling laws as follow:

$$t = (1+L)\tilde{t}, \quad \boldsymbol{x} = (1+L)\tilde{\boldsymbol{x}}, \quad \mu_A = (1+L)\tilde{\mu}_A\,. \tag{4}$$

2. TIME SCALE UP TO THE ORDER $1/c^4$

It is obvious that from Eq. (3), terms of order $1/c^4$ are neglected. The correct time dilation equation between the IAU conventional barycentric coordinate time TCB and the geocentric coordinate time TCG is given as follows:

$$\frac{dTCG}{dTCB} = 1 + \frac{1}{c^2}\mathcal{A}(TCB) + \frac{1}{c^4}\mathcal{B}(TCB) + \mathcal{O}\left(\frac{1}{c^6}\right). \tag{5}$$

It immediately produces another scaling factor $\bar{L}$ because we can expect a supplementary linear drift due to these terms. To quantify this possible additional linear drift, we can compute Eq. (5) with a planetary ephemerides. Hervé Manche, from the INPOP group, provided us the difference between Eqs (3) and (5). The additional linear drift $\bar{L}$ is something about 10^{-17}. What are the consequences on the equations of motion, the light time (1)? On the scaling of mass, time and position, we thus obtain:

$$\tilde{t} = \left(1 + \frac{\bar{L}}{1+L+\bar{L}}\right)\bar{t} = (1+\bar{L})\,\bar{t} + \mathcal{O}(L\bar{L}), \tag{6}$$

$\tilde{\mu}_A \approx (1+\bar{L})\,\bar{\mu}_A$ and $\tilde{\boldsymbol{x}} \approx (1+\bar{L})\,\bar{\boldsymbol{x}}$, the Taylor expansion being truncated at $\mathcal{O}(L\bar{L})$ because $L\bar{L} \approx 10^{-25}$ which is really negligible. We see immediately that these equations will not be affected by this factor. However we can suspect a possible impact on the calculation of the Doppler shift (2). Indeed it needs the evaluation of the two-ways precision light times which are given by (Moyer, 2003):

$$\begin{aligned}\rho =& (t_3 - t_2) + (t_2 - t_1) - (T_{eph} - TAI)_{t_3} + (T_{eph} - TAI)_{t_1} - (TAI - UTC)_{t_3} + (TAI - UTC)_{t_1} \\ &-(UTC - ST)_{t_3} + (UTC - ST)_{t_1} + ...,\end{aligned} \tag{7}$$

where indexes 1 and 3 correspond to the emission and reception of a signal on Earth, respectively, and index 2 refers to the spacecraft. Here $UTC - ST$ represents a time scale transformation from the local time ST at tracking station to UTC. In practice, we see that the relation $T_{eph} - TAI$ is required; so we have two additional terms $\Delta(T_{eph} - TAI)_{t_3}$ and $\Delta(T_{eph} - TAI)_{t_1}$ because of the modification due to $\bar{L}$. It leads to a drift in the time-tagging. However the Doppler shift is calculated from $d\rho/dt$, which means that we finally have only a constant offset of order 10^{-17} in the Doppler data, fully negligible.

3. MODIFY THE SHAPIRO DELAY?

It is well known that Eq. (1) is obtained from the gravitational field of motionless mass-monopole bodies. We can imagine to take into account some gravitomagnetic effects due to their motion. In that case, the spacetime metric has a g_{0i} contribution as follow

$$g_{0i}(c\,t, \boldsymbol{x}) \propto \frac{1}{c^3}\sum_A \frac{\mu_A}{|\boldsymbol{x} - \boldsymbol{x}_A|} v_A^i, \tag{8}$$

where v_A^i is the velocity of body A. In a simple situation, we can suppose that the bodies are moving with a constant velocity. The solution of this problem is known but usually expressed by means of retarded potentials, not so convenient to use in practice. However, we can derive quite easily an order of magnitude. One of the main gravitomagnetic contribution will be a modification of the usual Shapiro delay by a factor $\boldsymbol{v}_A.(\boldsymbol{x}_2 - \boldsymbol{x}_1)/|\boldsymbol{x}_2 - \boldsymbol{x}_1|$. For the Earth, we have $GM_E|\boldsymbol{v}_E|/c^4 \approx 10^{-15}$. Maybe it will be interesting in the future to study in details the impact of these extra-terms on the precision two-ways light time, and consequently on the Doppler shift.

4. REFERENCES

Moyer, T. D., 2003, Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation, (John Wiley & Sons: Hoboken)