IMPROVING THE ASTEROID PERTURBATIONS MODELING IN PLANETARY EPHEMERIDES

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ABSTRACT. The objective of this study is to investigate the advantages of modeling the large numbers of weakly perturbing asteroids present in the main belt by a solid ring. With a simple model we perform various Monte Carlo experiments and show that a ring is indeed able to represent a global effect. We estimate at 200 the number of asteroids that need to be eliminated before the global effect becomes dominant and calculate with the Monte Carlo method an independent estimate of the ring's mass.

1. INTRODUCTION: Asteroids are considered as the major obstacle for achieving a satisfactory long-term prediction accuracy in ephemerides. There are hundreds of asteroids whose perturbations on Mars are above the precision limit of available observations (∼1 m for modern ranging data) and there are many more asteroids whose perturbations though individually weak add up to a significant global effect. Krasinsky et al. 2002 proposed to model the effect of a large number of weakly perturbing asteroids by a solid ring, the purpose of this work is to gain further insight into the advantages of such a model in a planetary ephemeris.

2. METHODOLOGY: We chose 24635 asteroids from the ASTORB database for an initial model of the main-belt, this selection was based on the asteroids' semi-major axes (required to be bellow 3.5 AU) and absolute magnitudes (required brighter than 14). A simple model allows us to assign each asteroid with a set of "reasonable" random masses. The model is based on a simplified version of the Statistical Asteroid Model (Tedesco et al. 2005); SIMPS data, taxonomies and dynamical family memberships are used to assign to each asteroid a mean albedo ρ and an albedo uncertainty $\Delta\rho$, in the most common case where no data at all is available, the asteroid is randomly assigned one of four albedo classes and a corresponding ρ and $\Delta\rho$ (see Tedesco et al. 2005 for details). At this point we are able to calculate a diameter for every asteroid: we pick randomly an albedo from the $[\rho - \Delta\rho, \rho + \Delta\rho]$ interval and use the absolute magnitude to obtain a diameter. In order to obtain a mass, we assign a density interval to each of the albedo classes : $[0.5, 2.5]$ for low albedo (class C), $[1.6, 3.8]$ for moderate albedo (class S) and $[1, 5]$ for intermediate and high albedos (classes E and M). By randomly picking diameters together with corresponding albedo class densities, we obtain for each asteroid a reasonable distribution of masses.

In order to test the ability of a solid ring to represent a large number of weakly perturbing asteroids we compare perturbations on the Earth-Mars distance exerted by a ring to perturbations arising from all of the 24635 selected asteroids but the N most perturbing ones. By performing this comparison for different values of N we can estimate the ability of the ring to model a global effect and we can also estimate a limit from which this global effect is dominant. We devised a scheme that allows fast testing of many possible mass configurations of the 24635 asteroids. For each of the selected asteroids, the Solar system is integrated between 1969 and 2010 with and without the asteroid. These integrations are made with the INPOP integrator (Fienga et al. 2008). We can thus obtain for each asteroid the perturbation on the Earth-Mars distance $\Delta D = D_{\text{with}} - D_{\text{without}}$, this perturbation is proportional to the asteroid's mass and it also gives an estimation of $\partial D/\partial M$ $(= \Delta D/M)$. The proportionality relation can be verified empirically by comparing different ΔD for a whole range of the asteroid's mass values. Our scheme is based on the expansion to the first order in asteroid masses of a perturbation generated by all 24635 asteroids: $D(M_1, ..., M_{24635}) - D(0, ..., 0) = \Delta D_1 + ... + \Delta D_{24635}$. The expansion allows to calculate the global perturbation for any given set of asteroid masses by simply adding perturbations calculated once and applying corresponding mass proportionality relations. As it was the case with the proportionality relation, the validity of the expansion can be verified empirically.

3. RESULTS: We denote by ΔD_{ring} the perturbation on the Earth-Mars distance caused by a solid ring centered at the Sun. The perturbation is proportional to the mass of the ring, the dependency of the perturbation on the ring's radius is not linear, but for a radius between 2.5 AU and 3.5 AU it can be properly compensated by a variation of the ring's mass. We denote by $\Delta D_{\text{glob}}(N)$ the perturbation from all the asteroids but the N most perturbing ones (in terms of amplitude). The ability of the ring to model a global effect on the Earth-Mars distance is estimated by comparing ΔD_{ring} to $\Delta D_{\text{glob}}(N)$, in order to account for the possibility to adjust the ring's mass, we normalize ΔD_{ring} and $\Delta D_{\text{glob}}(N)$. Figure 1 shows the evolution of $R(N) = |\frac{\Delta D_{\text{ring}}}{|\Delta D_{\text{ring}}|} - \frac{\Delta D_{\text{glob}}(N)}{|\Delta D_{\text{glob}}(N)|}|$ for a particular set of asteroid masses as well as an average over 100 different sets. We observe two drops in $R(N)$, one for $N \sim 20$ and another one for $N \sim 125$, this shows that from $N \sim 200$ a ring represents indeed well the remaining asteroids, though the similarity is not perfect because $R(N)$ doesn't reach 0.

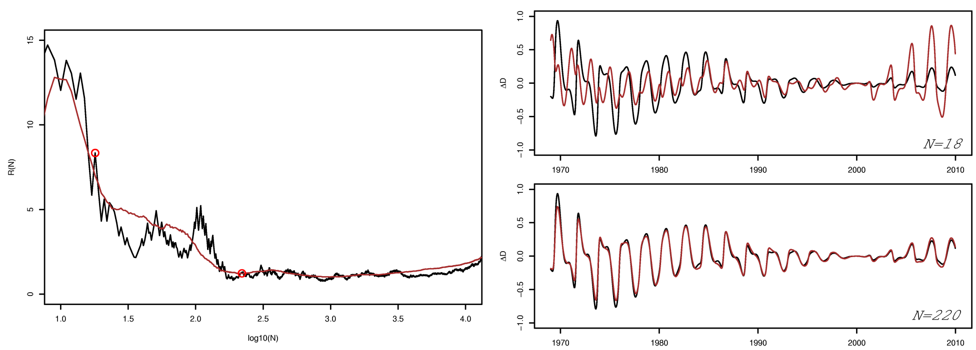

Figure 1: Evolution of $R(N)$, the brown curve is an average over 100 different sets of masses. On the right side are shown the normalized perturbations ΔD_{ring} and ΔD_{glob} for two particular values of N.

The random masses sets allow us to use the Monte Carlo method to estimate an average mass of a ring representing perturbations of all but the 300 most perturbing asteroids: fitting the ring's mass so as to minimize $|\Delta D_{\text{ring}} - \Delta D_{\text{glob}}(N = 300)|$ for 1000 different sets of asteroid masses leads to the estimate $M_{\text{ring}} = 0.6 \pm 0.2 \times 10^{-10} M_{\odot}$ for a ring with radius of 2.8 AU (the estimate becomes $M_{\text{ring}} = 1 \pm 0.3 \times 10^{-10} M_{\odot}$ for a radius of 3.14 AU). The corresponding perturbation $\Delta D_{\text{glob}}(N = 300)$ reaches on average 250 m whereas the residuals after fitting the ring reach 40 m (on the 1969-2010 interval). We emphasize that these estimations are dependent only on the model for assigning masses described in the methodology part, they are obtained independently from a fit to observations of a ring mass as an ephemeris parameter. One should note that the obtained estimation of M_{ring} is close to the value of $0.34 \pm 0.15 \times 10^{-10} M_{\odot}$ fitted for a ring at 2.8 AU in INPOP06 (Fienga et al. 2008).

Besides allowing for an independent estimation of the ring's mass, the Monte Carlo method can also be used to find the most probable list of the top 300 perturbing asteroids in terms of amplitude of the perturbation on the Earth-Mars distance on the 1969-2010 interval. Such a list is quite similar to the list of the 300 asteroids taken individually into account in INPOP06, a notable difference being the presence of 60 Echo, 516 Amstherstia and 585 Bilkis, which have large estimated amplitudes (the perturbation of 60 Echo reaches 170 m) and are absent from INPOP06.

4. CONCLUSION: We showed through a simple Monte Carlo study the ability of a ring to model a global perturbation caused by large numbers of weakly perturbing asteroids. We were also able to give an independent estimate of the ring's mass.

5. REFERENCES

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