THE RELATIVISTIC REFERENCE SYSTEMS AS A TOOL TO MODEL EARTH ROTATION

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ABSTRACT. Relativistic modelling of Earth rotation represents one of the most complicated problems of Applied Relativity. The relativistic reference systems of IAU (2000) give a suitable theoretical framework for such a modelling. Recent developments in the post-Newtonian theory of rigid Earth rotation are reported below. We describe the STF approach to compute the post-Newtonian torque, the framework to compute transformations between all relativistic time scales as the relativistic scaling of various parameters of the theory.

1. EARTH ROTATION IN THE RELATIVISTIC CONTEXT

Early attempts to model rotational motion of the Earth in a relativistic framework (see, e.g., Brumberg, 1972) made use of only one relativistic references system to describe both rotational and translational equations of motion. That reference system was usually chosen to be quite similar to the BCRS. This resulted in a mathematically correct, but physically inadequate coordinate picture of rotational motion. For example, from that coordinate picture a prediction of seasonal variations of the LOD with an amplitude of about 75 microseconds has been put forward.

At the end of the 1980s a better reference system for modelling of Earth rotation has been constructed and after a number of modifications and improvements has been adopted as GCRS in the IAU 2000 Resolutions. The GCRS implements the Einstein's equivalence principle and represents a reference system in which the gravitational influence of external matter (the Moon, the Sun, planets, etc.) is reduced to tidal potentials. Thus, for physical phenomena occurring in the vicinity of the Earth the GCRS represents a reference system, the coordinates of which are, in a sense, as close as possible to physically measurable quantities. This substantially simplifies the interpretation of the coordinate description of physical phenomena localized in the vicinity of the Earth. One important application of the GCRS is modelling of Earth rotation. The price to pay when using GCRS is that one should deal not only with one relativistic reference system, but with several reference systems, the most important of which are BCRS and GCRS. This makes it necessary to clearly and carefully distinguish between parameters and quantities defined in the GCRS and those defined in the BCRS.

2. RELATIVISTIC EQUATIONS OF EARTH ROTATION

The model which is used in this investigation was discussed and published by Klioner *et al.* (2001) and recalled in Klioner *et al.* (2008). Let us, however, repeat these equations one again not going into physical details of the model since we will need them in our subsequent discussion. The post-Newtonian equations of motion (omitting numerically negligible terms as explained in Klioner *et al.* (2001)) read

$$\frac{d}{dT}\left(C^{ab}\,\omega^{b}\right)=\sum_{l=1}^{\infty}\frac{1}{l!}\,\varepsilon_{abc}\,M_{bL}\,G_{cL}+L^{a}(\mathbf{C},\omega,\mathbf{\Omega}_{\rm iner}),\tag{1}$$

where $\mathbf{C}=C^{ab}$ is the post-Newtonian tensor of inertia and $\omega=\omega^{a}$ is the angular velocity of the post-Newtonian Tisserand axes (Klioner, 1996), T = TCG, M_L are the multipole moments of the Earth's gravitational field defined in the GCRS, G_L are the multipole moments of the external tidal gravito-electric field in the GCRS. In the simplest situation (a number of mass monopoles) G_L are explicitly given by Eqs. (19)–(23) of Klioner *et al.* (2001).

The additional torque L^a depends on $\mathbf{C}$, ω, as well as on the angular velocity $\mathbf{\Omega}_{\rm iner}$ describing the relativistic precessions (geodetic, Lense-Thirring and Thomas precessions). The definition of $\mathbf{\Omega}_{\rm iner}$ can be found, e.g., in Klioner *et al.* (2001). A detailed discussion of L^a, its structure and consequences will be published elsewhere (Klioner et al. 2009).

The model of rigidly rotating multipoles (Klioner *et al.*, 2001) represents a set of formal mathematical assumptions that make the general mathematical structure Eqs. (1) similar to that of the Newtonian equations of rotation of a rigid body. The assumptions are

$$C^{ab} = P^{ac}\,P^{bd}\,\overline{C}^{cd}, \quad \overline{C}^{cd} = \text{const} \tag{2}$$

$$M_{a_1a_2\dots a_l} = P^{a_1b_1}\,P^{a_2b_2}\dots P^{a_lb_l}\,\overline{M}_{b_1b_2\dots b_l}, \quad \overline{M}_{b_1b_2\dots b_l} = \text{const}, \quad l \geq 2, \tag{3}$$

where the orthogonal matrix $P^{ab}(T)$ is assumed to be related to the angular velocity ω^a used in (1) as

$$\omega^a = \frac{1}{2}\,\varepsilon_{abs}\,P^{db}(T)\,\frac{d}{dT}\,P^{dc}(T). \tag{4}$$

The meaning of these assumptions is that both the tensor of inertia C^{ab} and the multipole moments of the Earth's gravitational field M_L are "rotating rigidly" and that their rigid rotation is described by the same angular velocity ω^a that appears in the post-Newtonian equations of rotational motion. It means that in a reference system obtained from the GCRS by a time-dependent rotation of spatial axes both the tensor of inertia and the multipole moments of the Earth's gravitational field are constant.

No acceptable definition of a physically rigid body exists in General Relativity. The model of rigidly rotating multipoles represent a minimal set of assumptions that allows one to develop the post-Newtonian theory of rotation in the same manner as one usually does within Newtonian theory for rigid bodies. In the model of rigidly rotating multipoles only those properties of Newtonian rigid bodies are saved which are indeed necessary for the theory of rotation. For example, no assumption on local physical properties ("local rigidity") is made. It has not been proved as a theorem, but it is rather probable that no physical body can satisfy assumptions (2)–(4). The assumptions of the model of rigidly rotating multipoles will be relaxed in a later stage of the work when a non-rigid Earth will be discussed. On the other hand, such a model has been always tacitly used in the model of SLR data.

3. POST-NEWTONIAN EQUATIONS OF ROTATIONAL MOTION IN NUMERICAL COMPUTATIONS

Looking at the post-Newtonian equations of motion (1)–(4) one can formulate several problems to be solved before the equations can be used in numerical calculations:

A. How to parametrize the matrix P^{ab}?

B. How to compute M_L from the standard models of the Earth gravity field?

C. How to compute G_L from a solar system ephemeris?

D. How to compute the torque $\varepsilon_{abc}\,M_{bL}\,G_{cL}$ out of M_L and G_L?

E. How to deal with different time scales (TCG, TCB, TT, TDB) appearing in the equations of motion, solar system ephemerides, used models of Earth gravity, etc.?

F. How to treat the relativistic scaling of various parameters when using TDB and/or TT instead of TCB and TCG?

G. How to find relativistically meaningful numerical values for the initial conditions and various parameters?

Question A has been already discussion in Section 3 of Klioner et al. (2008). Questions B–D are considered in Section 4 below. Question E is discussed in Section 5. Question F is the subject of Section 6. An analysis of question G will be published elsewhere.

4. STF MODEL FOR THE TORQUE

The relativistic torque requires computations with STF tensors M_L and G_L. For this project special numerical algorithms for numerical calculations have been developed. The detailed algorithms and their derivation will be published elsewhere. Let us give here only the most important formulas. For each l the component $D_a = \varepsilon_{abc}\, M_{bL-1}\, G_{cL-1}$ of the torque in the right-hand side of Eq. (1) can be computed as ($A_l = 4\, l\, \pi\, l!/(2l+1)!!$, $a^{+}_{lm} = \sqrt{l(l+1) - m(m+1)}$)

$$D_1 = \frac{1}{A_l}\left(\sum_{m=0}^{l-1} a^{+}_{lm}\,\left(-\mathcal{M}^R_{lm}\,\mathcal{G}^I_{l,m+1} + \mathcal{M}^I_{l,m+1}\,\mathcal{G}^R_{lm}\right) + \sum_{m=1}^{l-1} a^{+}_{lm}\,\left(\mathcal{M}^I_{lm}\,\mathcal{G}^R_{l,m+1} - \mathcal{M}^R_{l,m+1}\,\mathcal{G}^I_{lm}\right)\right), \tag{5}$$

$$D_2 = \frac{1}{A_l}\left(\sum_{m=0}^{l-1} a^{+}_{lm}\,\left(-\mathcal{M}^R_{lm}\,\mathcal{G}^R_{l,m+1} + \mathcal{M}^R_{l,m+1}\,\mathcal{G}^R_{lm}\right) + \sum_{m=1}^{l-1} a^{+}_{lm}\,\left(-\mathcal{M}^I_{lm}\,\mathcal{G}^I_{l,m+1} + \mathcal{M}^I_{l,m+1}\,\mathcal{G}^I_{lm}\right)\right), \tag{6}$$

$$D_3 = \frac{2}{A_l}\,\sum_{m=1}^{l} m\,\left(\mathcal{M}^I_{lm}\,\mathcal{G}^R_{lm} - \mathcal{M}^R_{lm}\,\mathcal{G}^I_{lm}\right). \tag{7}$$

The coefficients $\mathcal{G}^R_{lm}$ and $\mathcal{G}^I_{lm}$ characterizing the tidal field can be computed from Eqs. (19)–(23) of Klioner *et al.* (2001) as explicit functions of the parameters of the solar system bodies: their masses, positions, velocities and accelerations. A Fortran code to compute $\mathcal{G}^R_{lm}$ and $\mathcal{G}^I_{lm}$ for $l < 7$ and $0 \le m \le l$ has been generated automatically with a specially written software package for *Mathematica*. It is possible to develop a sort of recursive algorithm to compute $\mathcal{G}^R_{lm}$ and $\mathcal{G}^I_{lm}$ for any l similar to the corresponding algorithms for, e.g., Legendre polynomials.

The coefficients $\mathcal{M}^R_{lm}$ and $\mathcal{M}^I_{lm}$ characterizing the gravitational field of the Earth can be computed as

$$\mathcal{M}^R_{l0} = \frac{l!}{(2l-1)!!}\,\left(\frac{4\pi}{2l+1}\right)^{1/2}\,M_E\,R_E^l\,C_{l0}, \tag{8}$$

$$\mathcal{M}^R_{lm} = (-1)^m\,\frac{1}{2}\,\frac{l!}{(2l-1)!!}\,\left(\frac{4\,\pi}{2l+1}\,\frac{(l+m)!}{(l-m)!}\right)^{1/2}\,M_E\,R_E^l\,C_{lm},\ \ 1 \le m \le l, \tag{9}$$

$$\mathcal{M}^I_{lm} = (-1)^{m+1}\,\frac{1}{2}\,\frac{l!}{(2l-1)!!}\,\left(\frac{4\,\pi}{2l+1}\,\frac{(l+m)!}{(l-m)!}\right)^{1/2}\,M_E\,R_E^l\,S_{lm},\ \ 1 \le m \le l, \tag{10}$$

where M_E is the mass of the Earth, R_E its radius, C_{lm} and S_{lm} are usual harmonics (potential coefficients) of the Earth gravitational field. If only Newtonian terms are considered in the torque this formulation with STF tensors is fully equivalent to the classical formulation with Legendre polynomials (e.g., Bretagnon *et al.*, 1997, 1998). If the relativistic terms are taken in account, the only known way to express the torque is that with STF tensors.

5. TIME TRANSFORMATIONS

An important aspect of relativistic Earth rotation theory is the treatment of different relativistic time scales. The numerical code described in (Klioner *et al.*, 2008) contains a subsystem dealing with the transformations between time scales TCB, TCG, TT and TDB. The transformation between TDB and TT *at the geocenter* (all the transformations in this Section are meant to be "evaluated at the geocenter") are computed along the lines of Section 3 of (Klioner, 2008b). Namely,

$$\mathrm{TT} = \mathrm{TDB} + \Delta\mathrm{TDB}(\mathrm{TDB}), \tag{11}$$

$$\mathrm{TDB} = \mathrm{TT} - \Delta\mathrm{TT}(\mathrm{TT}), \tag{12}$$

$$\mathrm{TCG} = \mathrm{TCB} + \Delta\mathrm{TCB}(\mathrm{TCB}), \tag{13}$$

$$\mathrm{TCB} = \mathrm{TCG} - \Delta\mathrm{TCG}(\mathrm{TCG}), \tag{14}$$

so that

$$\frac{d\Delta\mathrm{TDB}}{d\mathrm{TDB}} = A_{\mathrm{TDB}} + B_{\mathrm{TDB}}\,\frac{d\Delta\mathrm{TCB}}{d\mathrm{TCB}}, \tag{15}$$

$$A_{\mathrm{TDB}} = \frac{L_B - L_G}{1 - L_B}, \tag{16}$$

$$B_{\mathrm{TDB}} = \frac{1 - L_G}{1 - L_B} = A_{\mathrm{TDB}} + 1, \tag{17}$$

$$\frac{d\Delta\mathrm{TT}}{d\mathrm{TT}} = A_{\mathrm{TT}} + B_{\mathrm{TT}}\,\frac{d\Delta\mathrm{TCG}}{d\mathrm{TCG}}, \tag{18}$$

$$A_{\mathrm{TT}} = \frac{L_B - L_G}{1 - L_G}, \tag{19}$$

$$B_{\mathrm{TT}} = \frac{1 - L_B}{1 - L_G} = 1 - A_{\mathrm{TT}}, \tag{20}$$

$$\frac{d\Delta\mathrm{TCB}}{d\mathrm{TCB}} = F(\mathrm{TCB}) = \frac{1}{c^2}\,\alpha(\mathrm{TCB}) + \frac{1}{c^4}\,\beta(\mathrm{TCB}), \tag{21}$$

$$\frac{d\Delta\mathrm{TCG}}{d\mathrm{TCG}} = \frac{F(\mathrm{TCG} - \Delta\mathrm{TCG})}{1 + F(\mathrm{TCG} - \Delta\mathrm{TCG})}, \tag{22}$$

where functions α and β are given by Eqs. (3.3)–(3-4) of (Klioner, 2008b) and Eq. (22) represents a computational improvement of Eq. (3.8) of (Klioner, 2008b). Clearly, the derivatives $\frac{d\Delta\mathrm{TCB}}{d\mathrm{TCB}}$ and $\frac{d\Delta\mathrm{TCG}}{d\mathrm{TCG}}$ must be expressed as functions of TDB and TT, respectively, when used in (15)–(18).

The differential equations for ΔTDB and ΔTT are first integrated numerically for the whole range of the used solar system ephemeris (any ephemeris with DE-like interface can be used with the code). The initial conditions for ΔTDB and ΔTT should be chosen according to the IAU 2006 Resolution defining TDB: for $JD_{\mathrm{TT}} = 2443144.5003725$ one has $JD_{\mathrm{TDB}} = 2443144.5003725 - 6.55 \times 10^5/86400$ and vice verse. The results of the integrations for the pairs ΔTDB and $\frac{d\Delta\mathrm{TDB}}{d\mathrm{TDB}}$, and ΔTT and $\frac{d\Delta\mathrm{TT}}{d\mathrm{TT}}$ are stored with a selected step in the corresponding time variable (TDB for ΔTDB and its derivative, and TT for ΔTT and its derivative). A cubic spline on the equidistant grid is then constructed for each of these 4 quantities. The accuracy of the spline representation is automatically estimated using additional data points computed during the numerical integration. These additional data points lie between the grid points used for the spline and are only used to control the accuracy of the spline. The splines precomputed and validated in this way are stored in files and read in by the main code upon request. These splines are directly used for time transformation during the numerical integrations of Earth rotation. Although this spline representation requires significantly more stored coefficients than, for example, a representation with Chebyshev polynomials with the same accuracy, the spline representation has been chosen because of its extremely high computational efficiency. More sophisticated representations may be implemented in future versions of the code.

6. RELATIVISTIC SCALING AND TIME SCALES

Let us again consider the post-Newtonian equations of rotational motion (1)–(4). Obviously, there are two classes of quantities entering these equations that are defined in the BCRS and GCRS and, therefore, naturally parametrized by TCB and TCG, respectively. It is important to realize that the post-Newtonian equations of motion are only valid if non-scaled time scales TCG and TCB are used. If TT and/or TDB are needed, the equations should be changed correspondingly.

The relevant quantities defined in the GCRS and parametrized by TCG are: (1) the orthogonal matrix P^{ab}, angular velocity ω^a and corresponding angles φ, ψ and ω with which they are parametrized (see, Klioner *et al.*, 2008); (2) the tensor of inertia C^{ab}; (3) the multipole moment of Earth's gravitational field M_L. In principle, (a) G_L and (b) Ω^a_{iner} are also defined in the GCRS and parametrized by TCG, but these quantities are computed using positions $\mathbf{x}_A$, velocities $\mathbf{v}_A$ and accelerations $\mathbf{a}_A$ of solar system bodies. The orbital motion of solar system bodies are modelled in BCRS and parametrized by TCB or TDB. The definition of G_L is conceived in such a way that positions, velocities and accelerations of solar system bodies in BCRS should be taken at the moment of TCB corresponding to the required moment of TCG with spatial location taken at the geocenter (Klioner *et al.*, 2001; Klioner, Voinov, 1993; Soffel *et al.*, 2003). Let us recall that the transformation between TCB and TCG is a 4-dimensional one that involves the spatial location of an event.

In all theoretical works aimed to derive and/or analyze the rotational equations of motion in the GCRS one uses TCG as coordinate time scale parametrizing the equations. Although the natural time variable for the equations of Earth rotation is TCG, in principle, using a corresponding re-parametrization any time scale (including TCG, TT, TCB and TDB) can be used as independent time variable. Thus, simple rescaling of the first and second derivatives of the angles entering the equations of rotational motion should be applied to use TT instead of TCG:

$$\frac{d\theta}{d\mathrm{TCG}} = (1 - L_G)\,\frac{d\theta}{d\mathrm{TT}}, \tag{23}$$

$$\frac{d^2\theta}{d\mathrm{TCG}^2} = (1 - L_G)^2\,\frac{d^2\theta}{d\mathrm{TT}^2}, \tag{24}$$

where θ is any of the angles φ, ψ and ω used in the equations of motion to parametrize the orientation of the Earth. If TDB is used as independent variable the corresponding formulas are more complicated:

$$\frac{d\theta}{d\mathrm{TCG}} = (1 - L_G)\left(\left.\frac{d\mathrm{TT}}{d\mathrm{TDB}}\right|_{\mathbf{x}_E}\right)^{-1}\frac{d\theta}{d\mathrm{TDB}}, \tag{25}$$

$$\frac{d^2\theta}{d\mathrm{TCG}^2} = (1 - L_G)^2\left(\left.\frac{d\mathrm{TT}}{d\mathrm{TDB}}\right|_{\mathbf{x}_E}\right)^{-2}\frac{d^2\theta}{d\mathrm{TDB}^2} - (1 - L_G)^2\left(\left.\frac{d\mathrm{TT}}{d\mathrm{TDB}}\right|_{\mathbf{x}_E}\right)^{-3}\left.\frac{d^2\mathrm{TT}}{d\mathrm{TDB}^2}\right|_{\mathbf{x}_E}\frac{d\theta}{d\mathrm{TDB}}, \tag{26}$$

where the derivatives of TT w.r.t. TDB should be evaluated at the geocenter (i.e., for $\mathbf{x} = \mathbf{x}_E$). These relations must be substituted into the equations of rotation motion to replace the derivatives of the angles φ, ψ and ω w.r.t. TCG as appear e.g., in Eqs. (7)–(9) of (Bretagnon *et al.*, 1998). It is clear that the parametrization with TDB makes the equations more complicated.

The values of the parameters naturally entering the equations of rotational motion must be interpreted as unscaled (TCB-compatible or TCG-compatible) values. If scaled (TT-compatible or TDB-compatible) values are used, the scaling must be explicitly taken into account. The relativistic scaling of parameters read (see e.g. Klioner, 2008a):

$$GM_A^{\mathrm{TT}} = (1 - L_G)\,GM_A^{\mathrm{TCG}}, \quad GM_A^{\mathrm{TCG}} = GM_A^{\mathrm{TCB}}, \quad GM_A^{\mathrm{TDB}} = (1 - L_B)\,GM_A^{\mathrm{TCB}}, \tag{27}$$

$$X^{\mathrm{TT}} = (1 - L_G)\,X^{\mathrm{TCG}}, \quad x^{\mathrm{TDB}} = (1 - L_B)\,x^{\mathrm{TCB}}, \tag{28}$$

$$V^{\mathrm{TT}} = V^{\mathrm{TCG}}, \quad v^{\mathrm{TDB}} = v^{\mathrm{TCB}}, \tag{29}$$

$$A^{\mathrm{TT}} = (1 - L_G)^{-1}\,A^{\mathrm{TCG}}, \quad a^{\mathrm{TDB}} = (1 - L_B)^{-1}\,a^{\mathrm{TCB}}, \tag{30}$$

where GM_A is the mass parameter of a body, x, v, and a are parameters represents spatial coordinates (distances), velocities and accelerations in the BCRS, respectively, while X, V, and A are similar quantities in the GCRS.

Now, considering the source of the numerical values of the parameters used in the equations of Earth rotation we can see the following.

a. The position $\mathbf{x}_A$, velocities $\mathbf{v}_A$, accelerations $\mathbf{a}_A$ and mass parameters GM_A of the massive solar system bodies are taken from standard JPL ephemerides and are TDB-compatible.

b. The radius of the Earth comes together with the potential coefficients C_{lm} and S_{lm} from a model of the Earth's gravity field (e.g., GEMT3 was used in SMART). These values come from SLR and dedicated techniques like GRACE. GCRS with TT-compatible quantities is used to process these data. Therefore, the values of the radius of the Earth is TT-compatible. Obviously, C_{lm} and S_{lm} have the same values when used with any time scale. The mass parameter GM_E of the Earth coming with the Earth gravity models is also TT-compatible.

c. From the definitions of $\mathcal{M}^R_{lm}$ and $\mathcal{M}^I_{lm}$ given above and formulas for G_L given by Eqs. (19)–(23) of Klioner *et al.* (2001), it is easy to see that the TCG-compatible torque $F^a = \sum_{l=1}^{\infty}\frac{1}{l!}\,\varepsilon_{abc}\,M_{bL}\,G_{cL}$ can be computed using TDB-compatible values of mass parameters GM_A^{TDB}, positions $\mathbf{x}_A^{\mathrm{TDB}}$, velocities $\mathbf{v}_A^{\mathrm{TDB}}$ and accelerations $\mathbf{a}_A^{\mathrm{TDB}}$ of all external bodies, TDB-compatible value of the mass parameter of the Earth GM_E^{TDB} and the value of Earth radius formally rescaled from TT to TDB as $R_E^{\mathrm{TDB}} = (1 - L_B)\,(1 - L_G)^{-1}\,R_E^{\mathrm{TT}}$. Denoting the resulting torque by F^a_{TDB}, it can be seen that the TCG-compatible value is $F^a_{\mathrm{TCG}} = (1 - L_B)^{-1}\,F^a_{\mathrm{TDB}}$.

d. The values of the Earth's moments of inertia $\mathcal{A}_i$, $i = 1, 2, 3$ can be represented as $G\mathcal{A}_i = GM_E R_E^2 k_i$, where k_i is a factor characterizing the distribution of the matter inside Earth. Clearly, the factors k_i do not depend on the scaling. Therefore, the moments of inertia can be scaled as

$$\mathcal{A}_i^{\mathrm{TT}} = (1 - L_G)^3 \, \mathcal{A}_i^{\mathrm{TCG}}. \tag{31}$$

The last question is how to interpret the values of the moments of inertia $\mathcal{A}_i = (A, B, C)$ and the initial conditions for the angles φ, ψ and ω and their derivatives given in (Bretagnon *et al.*, 1998). Obviously, the initial angles at J2000 are independent of the scaling. For the other parameters in question it is not possible to clearly claim if the given values are TDB-compatible or TT-compatible. Arguments in favor of both interpretations can be given. The rigorous solution here is only possible when all calculations leading to these quantities are repeated in the framework of General Relativity. In this paper we prefer to interpret the SMART values of $\mathcal{A}_i$, $\dot\varphi$, $\dot\psi$ and $\dot\omega$ as being TT-compatible. Therefore, if TDB is used as independent variable, the values of the derivatives should be changed accordingly. For any of these angles one has

$$\frac{d\theta}{d\mathrm{TDB}} = \left(\left. \frac{d\mathrm{TT}}{d\mathrm{TDB}} \right|_{\mathbf{x}_E} \right) \frac{d\theta}{d\mathrm{TT}} \,. \tag{32}$$

Thus, we have all tools to treat correctly the relativistic scaling of all relevant parameters of the Earth rotation theory as well as relativistic time scales. A numerical integration of Earth rotation over the full range of DE403 shows that the effect of these two factor is relatively small: periodic effects of an amplitude of $0.15 - 0.25$ μas (depending on the angle) and a period of 18 years plus secular trends in φ and ψ (-68.4 and 74.7 μas per century, respectively). Further details will be published elsewhere.

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