ON GENERAL EARTH'S ROTATION THEORY

V.A. BRUMBERG, T.V. IVANOVA
Institute of Applied Astronomy
10, Kutuzov quay, St. Petersburg, 191187, Russia
e-mail: itv@ipa.nw.ru

This paper dealing with the general problem of the rigid–body rotation of the three–axial Earth represents a straightforward extension of (Brumberg and Ivanova, 2007) where the simplified Poisson equations of rotation of the axially symmetrical Earth have been considered. The aim of the present paper is to reduce the equations of the translatory motion of the major planets and the Moon and the equations of the Earth's rotation around its centre of mass to the secular system describing the evolution of the planetary and lunar orbits (independent of the Earth's rotation) and the evolution of the Earth's rotation (depending on the planetary and lunar evolution). In doing so, the techniques of the General Planetary Theory (GPT) (Brumberg, 1995) and the Poisson Series Processor (PSP) (Ivanova, 1995) have been used. The complete equations in terms of the planetary–lunar eccentric and oblique Laplace–type variables $a_i, \bar{a}_i, b_i, \bar{b}_i$ $(i = 1, 2, \ldots, 9)$ and the Earth's rotation parameters $p = (p_i)$, $\bar{p} = (\bar{p}_i)$ $(i = 1, \ldots, 4)$ being the functions of the Euler angles, have the form

$$\dot{X} = i\,\mathcal{N}[PX + R(X,t)], \tag{1}$$

where X and R standing for the vectors of the variables and right–hand members, respectively,

$$X = (a, \bar{a}, b, \bar{b}, p, \bar{p}), \qquad R = (R_1, \ldots, R_6) \tag{2}$$

are vectors with 44 components (a, b and R_i with $i = 1, 2, 3, 4$ for the planets and the Moon are 9–vectors, p and R_5, R_6 for the Earth's rotation are 4–complex–value vectors). $\mathcal{N}$ and P are 44×44 diagonal matrices of the structure

$$\begin{aligned} \mathcal{N} &= \mathrm{diag}(N, N, N, N, n, n, n, n, n, n, n, n), \\ P &= \mathrm{diag}(E_{(9)}, -E_{(9)}, E_{(9)}, -E_{(9)}, E_{(4)}, -E_{(4)}), \end{aligned} \tag{3}$$

$n = -\dfrac{1}{2}\Omega$, Ω being the mean Earth's rotation velocity, N is 9×9 diagonal matrix of the mean motions of the major planets and the Moon, $E_{(9)}$ and $E_{(4)}$ are the unitary matrices of dimension 9×9 and 4×4, respectively. The transformation from X to new variables $Y = (a, \bar{a}, b, \bar{b}, q, \bar{q})$

$$X = Y + \Gamma(Y,t) \tag{4}$$

results in a new system

$$\dot{Y} = i\,\mathcal{N}\big[PY + F(Y,t)\big]. \tag{5}$$

Functions Γ and F are found by iterations as series in powers of Y with quasi–periodic coefficients of t

$$U = R - \mathcal{N}^{-1}\Gamma_Y \mathcal{N} U^*\,, \tag{6}$$

$$\Gamma_t + i\,(\Gamma_Y \mathcal{N} P Y - \mathcal{N} P \Gamma) = i\,\mathcal{N} U^+\,, \tag{7}$$

$$U = U^* + U^+\,, \qquad F = U^*\,. \tag{8}$$

The splitting of U is aimed to ensure the integration of (7) without t–secular terms. Since planetary and lunar secular system was already constructed in GPT, the transformation

$$p_i = q_i + \Gamma_{4+i} \qquad (i = 1, \ldots, 4) \tag{9}$$

transforms the Earth's rotation equations into the secular system

$$\dot{q}_i = i\,n(q_i + F_{4+i}) \tag{10}$$

with

$$F_\kappa = U_\kappa^*(q, \bar{q}, \alpha, \bar{\alpha}, \beta, \bar{\beta}). \tag{11}$$

Here $\alpha, \bar{\alpha}, \beta, \bar{\beta}$ are 9–vectors of slowly changing Laplace–type elements for the major planets and the Moon. Expressions (9) describe the short–period nutation depending on the mean longitudes of the Sun, the Moon and major planets. System (10) is responsible for precession and long–period nutation since it includes lunar evolutionary variables α_9 and β_9 related to the motions of the lunar perigee and node.

The work is now in progress and the final results will be published elsewhere in the future.

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