NOMENCLATURE AND NUMERICAL STANDARDS FOR IAU MODELS AND IERS CONVENTIONS FOR EARTH ROTATION

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ABSTRACT. Important resolutions regarding Earth rotation have been adopted by IAU and IUGG since 2000, which include new concepts, a new nomenclature and new models for the quantities necessary for expressing the celestial orientation of the Earth. This presentation summarizes the consequences of the joint IAU 2000/2006 resolutions on the IAU models and the IERS Conventions that implement those models. It then looks at the changes in the status and numerical values for the constants associated with the IAU models for precession, nutation and the angle for Earth rotation, with respect to those of the previous IAU System of astronomical constants and previous lists of Current Best Estimates.

1. INTRODUCTION

The high accuracy observations of Earth rotation require a coordinated use of IAU-approved formulations, numerical standards and software that implement the IAU models. Several IAU resolutions on reference systems that are related to Earth's rotation have been passed in 2000 and 2006 including new nomenclature, models and standards. The IERS Conventions provide a practical and fully detailed implementation of these models and standards, expressed in terms of the new nomenclature, while the authoritative algorithms for implementing the IAU resolutions are provided by SOFA (Standards Of Fundamental Astronomy).

The aim of this paper is to review the changes resulting from the joint IAU 2000/2006 resolutions in the status and the numerical values for the constants associated with the new IAU models for precession, nutation and the angle for Earth rotation with respect to those of the current (1976) IAU System of astronomical constants and previous lists of Current Best estimates. The role of SOFA will be considered in the following paper of these Proceedings (cf. Wallace 2009, this Volume).

2. THE IAU 2000/2006 RESOLUTIONS FOR EARTH ROTATION

The IAU 2000 Resolutions have refined the definition of the astronomical reference systems and transformations between them and adopted the IAU 2000 precession-nutation. Those resolutions came into force on 1 January 2003 and were endorsed by IUGG in July 2003. The IAU 2006 Resolutions have adopted a new precession model and have addressed definition, terminology or orientation issues relative to reference systems and time scales. These have been endorsed by IUGG in 2007 with additional refinements for the definition of terrestrial reference systems.

The major points on Earth's rotation of the IAU 2000/2006 and IUGG 2003/2007 resolutions are:

- The systems of space-time coordinates for the solar system and the Earth within the framework of General Relativity have been specified (i) by IAU 2000 Resolution B1.3 (i.e. the Barycentric and Geocentric Celestial Reference Systems, BCRS and GCRS, respectively) and (ii) by IUGG 2007 Resolution 2 for the terrestrial systems (i.e. the Geocentric and Terrestrial Reference Systems, GTRS and ITRS, respectively).

- A high precision model for precession and nutation has been adopted in two stages. The first stage (IAU 2000 Resolution B1.6) was the adoption of the IAU 2000 precession-nutation (Mathews et al. 2002), which has been implemented in the IERS Conventions 2003. The second stage (IAU 2006 Resolution B1) was the adoption of the P03 Precession (Capitaine et al. 2003; Hilton et al. 2006) (i.e. the IAU 2006 precession) as a replacement for the precession part of the IAU 2000A precession-nutation, beginning on 1 January 2009.

- The definition of the Earth's Pole has been refined (IAU 2000 Resolution B1.7), so that the Celestial Intermediate Pole, CIP, is the intermediate pole, separating nutation from polar motion by a convention in the frequency domain. The GCRS direction of the CIP has been recommended as a replacement for the classical precession-nutation angles (IAU 2000 Resolution B1.8).

- The definition of the Earth's rotation around the CIP axis has been provided by the Earth Rotation Angle (ERA) between the "non-rotating origins" (Guinot, 1979) on the CIP equator (IAU 2000 Resolution B1.8). The diurnal rotation is expressed through a conventional linear transformation of UT1.

- The names of the pole and origins at date t have been harmonized to "intermediate" by IAU 2006 Resolution B2, i.e. celestial and terrestrial "intermediate" origins, CIO and TIO (instead of CEO and TEO, respectively, as originally named in the IAU 2000 resolutions).

3. NEW PARAMETERS AND NOMENCLATURE RELATED TO EARTH ROTATION

According to the new concepts adopted by IAU 2000 Resolution B1.8, the GCRS coordinates X, Y of the CIP unit vector and the Earth rotation angle, ERA, replace the classical precession and nutation angles and Greenwich sidereal time, GST, respectively (see more details in Capitaine et al. (2003)). Two recommendations from the IAU Working Group on "Nomenclature for Fundamental Astronomy" (NFA) (Capitaine et al. 2008) have been endorsed by IAU 2006 Resolution B2 while the detailed IAU 2006 nomenclature for fundamental astronomy are provided in the IAU 2006 NFA Glossary that is available at `http://syrte.obspm.fr/iauWGnfa`. A few examples of that Glossary are given in the following:

Celestial Intermediate Origin (CIO): origin for right ascension on the intermediate equator in the celestial intermediate reference system. It is the non-rotating origin in the GCRS that is recommended by the IAU 2000 Resolution B 1.8, where it was designated the Celestial Ephemeris Origin. The CIO was originally set close to the GCRS meridian and throughout 1900-2100 stays within 0.1 arcseconds of this alignment.

Celestial Intermediate Pole (CIP): geocentric equatorial pole defined by IAU 2000 Resolution B1.7 as being the intermediate pole, in the transformation from the GCRS to the ITRS, separating nutation from polar motion. It replaced the CEP on 1 January 2003. Its GCRS position results from (i) the part of precession-nutation with periods greater than 2 days, and (ii) the retrograde diurnal part of polar motion (including the free core nutation, FCN) and (iii) the frame bias. Its ITRS position results from (i) the part of polar motion which is outside the retrograde diurnal band in the ITRS and (ii) the motion in the ITRS corresponding to nutations with periods less than 2 days. The motion of the CIP is realized by the IAU precession-nutation plus time-dependent corrections provided by the IERS.

Earth Rotation Angle (ERA): angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO), positively in the retrograde direction. It is related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1 (see IAU 2000 Resolution B1.8). Its time derivative is the Earth's angular velocity. Previously, it has been referred to as the stellar angle.

4. IAU MODELS FOR EARTH ROTATION AND THEIR IERS IMPLEMENTATION

The precession-nutation model is composed of a nutation part and a precession part. In addition are frame bias values between the J2000 mean pole and equinox and the Geocentric Celestial Reference System (GCRS).

The IAU 2000 semi-analytical series for nutation is composed of 1365 lunisolar and planetary terms with "in-phase" and "out-of-phase" components with amplitudes from $17''.2$ to $0.1\,\mu$as and periods between 3 d and 101 cy. This series is based on the REN2000 solution (Souchay et al. 1999) for the nutation of a rigid Earth model, transformed to nutation of a non-rigid Earth model with the MHB2000 "transfer function" (Mathews et al. 2002) expressed as function of seven Basic Earth Parameters (BEP) fitted to VLBI data. The IAU 2000 precession part consists only of corrections to the precession rates of the IAU 1976 precession; this is why IAU 2000 Resolution B1.6 recommended the development of new expressions for precession consistent with dynamical theories and with IAU 2000A nutation.

The IAU 2006 precession (Capitaine et al. 2003) provides improved polynomial expressions up to the 5th degree in time, t, both for the precession of the ecliptic and the precession of the equator, the latter being consistent with dynamical theory while matching the IAU 2000A precession rate for continuity reasons. The precession of the equator was derived from the dynamical equations expressing the motion of the mean pole about the ecliptic pole with the value $\epsilon_0 = 84381''.406$ (Chapront et al. 2002) for the mean J2000 obliquity of the ecliptic. This includes various contributions from the non-rigidity of the Earth as well as corrections for the perturbing effects in the observed quantities.

The coordinate transformation from the ITRS to the GCRS at the date t of the observation that is considered in Chapter 5 of the IERS Conventions (2003) is written as:

$$[\text{GCRS}] = Q(t)R(t)W(t)\ [\text{ITRS}],$$

where $Q(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the GCRS motion of the CIP, the ERA and the ITRS motion of the CIP, respectively.

The IERS implementation of the ITRS-to-GCRS coordinate transformation compliant with the latest IAU resolutions should be based (i) for $Q(t)$: on the IAU 2006/2000 precession-nutation model plus IERS estimated celestial pole offsets, (ii) for $R(t)$: on the IAU 2000 expression for ERA(UT1) and the IERS estimated UT1, and (iii) for $W(t)$: on the IERS estimated pole coordinates, plus a modeled part of polar motion (cf. IERS Conventions 2003). The IAU 2000/2006 implementation of the precession-nutation model has been provided by Capitaine & Wallace (2006) and Wallace & Capitaine (2006).

5. NUMERICAL STANDARDS FOR EARTH ROTATION QUANTITIES

Numerical standards for Earth rotation in previous systems

In the first system of fundamental astronomical constants (adopted by the "Conférence internationale des étoiles fondamentales", Paris, 1896), the constants associated with precession-nutation, were (1) the "constant of precession", p, for the yearly amount of precession of the equinox, (2) the "constant of nutation", N_0, for the amplitude of the 18.6-yr nutation in obliquity, and (3) the obliquity of the ecliptic, ϵ. In the IAU 1976 System of astronomical constants, the precession-nutation constants have been unchanged, though they have been called "general precession in longitude at standard epoch 2000", p, "obliquity of the ecliptic at standard epoch 2000", ϵ, and "constant of nutation at standard epoch 2000", N; their numerical values have been updated to the best estimates at that time. In the Numerical Standards of the IERS Conventions (2003), the constants, ϵ_0 and ψ_1 associated with precession corresponds to a change of status for precession, i.e. replacing the classical constant of precession that was related to the general precession at J2000 with the precession rate ψ_1 of the equator at J2000 (with the IAU 2000 numerical value). There is no longer a constant associated with nutation, which corresponds to a change of status for nutation. This is due to the fact that, since the IAU 1980 nutation model, the nutation amplitudes are no longer related to the amplitude of the 18.6-yr term (as it was the case in the past). There is an additional IERS constant for Earth's rotation, i.e. ω, for the "nominal mean angular velocity of the Earth", the numerical value of which is the International Association of Geodesy (IAG) value.

Numerical standards for the IAU 2000/2006 model for Earth rotation

(i) Precession: The IAU 2006 precession of the equator is based on the expressions for the precession quantities ψ_A, and ω_A, which have been directly derived from the precession equations; it also includes expressions for a number of derived precession quantities, such as the GCRS CIP's X, Y. The numerical standards associated with the IAU 2006 precession are those for the coefficients of the polynomial expressions for the precession quantities. Those coefficients are dependent on (1) the J2000 precession rates of the equator, (2) the J2000 obliquity of the ecliptic, (3) the precession of the ecliptic and (4) the J_2 rate.

One option for representing the IAU 2006 precession with a very few numerical standards would be to consider only the IAU 2006 values for the J2000 precession rates X_1, Y_1, of the equator; this would correspond to retaining the same number of constants for precession in the list of numerical standards for fundamental astronomy, but with a change of status. Another option would be to consider the IAU 2006 values for the most relevant physical parameters associated with precession, such as the dynamical flattening, H_d and the J_2 rate (i.e. $H_d = 3.27379448\ \ 10^{-3}$ and $dJ_2/dt = -3.0 \times 10^{-11}$/yr), plus the J2000 obliquity of the ecliptic ($\epsilon_0 = 84381''.406$). These few constants could only provide a first order approximation for precession and it should be clear that the complete IAU 2006 precession expressions must be used for high accuracy needs.

(ii) Nutation: there is a large number of numerical standards associated with the IAU 2000 nutation. The fundamental values are the numerical values for the nutation amplitudes, which are directly related to the numerical values for (1) the REN2000 amplitudes and (2) the MHB2000 Basic Earth parameters, BEP (e.g. the dynamical ellipticities, e, of the whole Earth, and e_f of its fluid core, respectively).

Hence, the numerical standards associated with nutation could be either the IAU 2000 nutation amplitudes, or the REN2000 nutation amplitudes plus the MHB2000 Basic Earth parameters, which in both cases, would correspond to a very large number of values. A few numerical values out of this series can only provide a very poor approximation of nutation.

(iii) Earth's rotation: the conventional relationship defining UT1 (which is intended to be a representation of the hour angle of the "fictitious mean" Sun) from the ERA is:

$ERA(\mathrm{UT1}) = 2\pi \times [0.7790572732640 + 1.00273781191135448\ (\text{Julian UT1 date} - 2451545.0)]$,
where 0.7790572732640, 1.00273781191135448 rev/day are"defining constants".

The "nominal" mean angular velocity of the Earth, $\omega = 7.292\ 115 \times 10^{-5}$ rad/s (cf. IERS Conventions 2003), has been chosen to have the number of significant digits limited to those for which the value can be considered a constant; that value is "significantly" different from the actual value for the true Earth's angular velocity, which is known to have relative variations of the order of 10^{-7}.

Therefore, the numerical standards associated with the Earth's rotation angle and rate, and compliant with the IAU 2000/2006 resolutions, consist of values for two conventional constants for the relationship that defines UT1 from ERA and one conventional constant for the nominal mean angular velocity of the Earth. The observed variations in the Earth's angular velocity must refer to that nominal value.

The IAU 2000/2006 Resolutions have modified the status and numerical values of the constants associated with Earth rotation. A few constants for representing precession and nutation are no longer appropriate. The IAU precession-nutation model is necessary to ensure a precision compliant with that of the current observations. For continuity reasons, two numerical values can be provided for precession, but with making clear that those values are only a part of the constants of the IAU 2006 precession that in fact includes expressions as functions of time of various quantities. Nutation cannot be represented by a few constants, except for low precision needs; it must be represented by the IAU 2000 nutation model. A very few numerical values for conventional constants provide a model for the Earth's rotation angle and rate, which must be completed with their observed variations.

6. REFERENCES

Capitaine, N., Wallace, P.T., Chapront, J., 2003, "Expressions for IAU 2000 precession quantities", A&A 412, 567–586.

Capitaine, N., Wallace, P.T., 2006, "High precision methods for locating the celestial intermediate pole and origin", A&A 450, 855–872.

Capitaine, N., Andrei, A., Calabretta, M. Dehant, V., Fukushima, T., Guinot, B., Hohenkerk, C., Kaplan, G., Klioner, S., Kovalevsky, J., Kumkova, I., Ma, C., McCarthy, D., Seidelmann, K., and Wallace, P., 2007, "Proposed terminology in fundamental astronomy based on IAU 2000 resolutions", in *Transactions of the IAU* XXVIB, van der Hucht, K.A. (ed), 14, pp. 474–475.

Chapront, J., Chapront-Touzé, M. and Francou, G., 2002, "A new determination of lunar orbital parameters, precession constant and tidal acceleration from LLR measurements", A&A 387, 700–709.

Guinot, B., 1979, "Basic Problems in the Kinematics of the Rotation of the Earth", in *Time and the Earth's Rotation*, McCarthy, D.D. and Pilkington, J.D. (eds.), D. Reidel Publishing Company, pp. 7–18.

Hilton, J., Capitaine, N., Chapront, J., et al., 2006, "Report of the International Astronomical Union Division I Working Group on Precession and the Ecliptic", Celest. Mech. Dyn. Astr. 94, 3, 351.

IAU 2006, *Transactions of the IAU* XXVIB; van der Hucht, K.A. (ed).

IERS Conventions (2003), *IERS Technical Note 32*, D.D. McCarthy and G. Petit (eds), Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004.

IUGG 2007, IUGG Resolutions, http://www.iugg.org/resolutions/perugia07.pdf .

Mathews, P.M., Herring, T.A., Buffet, B.A., 2002, "Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth's interior", J. Geophys. Res. , 107(B4), doi: 10.1029/2001JB000390.

Souchay, J., Loysel, B., Kinoshita, H., and Folgueira, M., 1999, "Corrections and new developments in rigid Earth nutation theory: III. Final tables REN-2000 including crossed-nutation and spin-orbit coupling effects", A&AS 135, 111.

Wallace P.T., 2009, "Recent SOFA Developments", This Volume.

Wallace P.T., Capitaine, N., 2006, "Precession-nutation procedures consistent with IAU 2006 resolutions", A&A 459, 3, 981.