

RECENT ADVANCES IN THEORETICAL MODELING AND OBSERVATION OF EARTH ROTATION AT DAILY AND SUBDAILY PERIODS

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ABSTRACT. This paper gives an overview of the recent advances in theoretical modeling and observation of Earth rotation at daily and subdaily periods. The dominant part of this group, with the size up to 1 mas, is due to the gravitationally forced ocean tides. There is also a small variation (about 0.03 mas) due to the direct influence of the tidal gravitation on the triaxial structure of the Earth. The remaining part (up to 0.1 mas) is a geophysical effect driven by the daily cycle in solar heating giving rise to high frequency variations of the atmospheric and oceanic angular momenta (AAM, OAM). The observational evidence of diurnal and semidiurnal signals in polar motion and UT1 concerns mostly the purely harmonic ocean tide effects. The variations of geophysical origin have been detected in the high resolution AAM and OAM data, though there are still significant differences between results from various models. Recent developments of technologies used to monitor Earth rotation, the space geodetic techniques - VLBI, GNSS, SLR, but also the ring laser gyroscope, rises a question about observability of diurnal and subdiurnal components of Earth rotation. Of particular importance is the possibility of monitoring high frequency geophysical signals which are irregular to some extent therefore cannot be described by a simple harmonic model.

1. INTRODUCTION

Polar motion and UT1 contain physical signals within the diurnal and subdiurnal frequency bands. A common feature, besides the high frequency, is their small size: the total peak-to-peak size is only up to about 1 milliarcsecond (mas) corresponding to 3 cm at the Earth surface. Such variations could not be observed by the methods of optical astrometry and early space geodetic measurements, because the observations were not sufficiently accurate and their sampling interval was significantly longer than 1 day. Hence all earlier predictions were purely theoretical based on the knowledge about the shape and internal constitution of the Earth. All observational evidence of diurnal and subdiurnal variations in Earth rotation has been gathered during the last two decades. The high resolution observations of Earth orientation are still under development. For instance, one purpose of the VLBI 2010 system is continuous measurement of the Earth Orientation Parameters (EOP). Another example which will be discussed in a more detailed way in Sec. 3 is determination of high frequency polar motion by the ring laser. An important and independent method of modeling diurnal/subdiurnal variations in Earth orientation is by the high resolution estimation of the total angular momentum of geophysical fluids, primarily the atmosphere and the oceans.

Despite the small size, the diurnal and subdiurnal signals in Earth rotation are important for understanding the high frequency global dynamics of the solid Earth and the overlying fluid layers. The research concerning such signals is also important for validation of the high resolution determinations of Earth rotation parameters and of the procedures applied for data reduction. A possible benefit from such research could be empirical verification of the equations of Earth rotation at high frequencies.

In the next section we will give a brief systematic review of the known diurnal and subdiurnal signals in Earth rotation. For each effect we will describe the physical mechanism, the method of mathematical modeling and the observational evidence. A separate part, Sec. 3, will be devoted to the observations of diurnal polar motion by the ring laser, and their relation to standard determinations of the EOP.

2. DIURNAL/SUBDIURNAL COMPONENTS OF POLAR MOTION AND UT1

In this section we will briefly discuss all known perturbations of Earth rotation with daily and subdaily periods. The perturbations are divided into several groups based on their physical cause. For each group we describe the way of modeling and the observational evidence gathered so far. We also report about own research and estimation efforts.

Astronomical variations. The underlying physical mechanism are the lunisolar and planetary torques exerted upon the rotating Earth. Such torques are regular and predictable. A standard mathematical model consists of a sum of polynomial and (quasi)harmonic terms. The main corresponding effect in Earth rotation is the astronomical precession-nutation which is due to the influence of tidal gravitation on those features of the Earth's mass distribution which are expressed by the zonal terms of geopotential. This component of perturbation is expressed by the conventional a priori model, currently the IAU 2000 precession-nutation model (IERS, 2004). All possible imperfections of the conventional model as well as geophysical signals such as the free core nutation (FCN) contribute to the celestial pole offsets $\delta X, \delta Y$ which are determined by very long baseline interferometry (VLBI). In case of other space geodetic techniques, such as global navigation satellite system (GNSS), satellite laser ranging (SLR), all unmodeled effects can contribute to polar motion as a retrograde diurnal variation.

A minor part of the astronomical variation, called “subdiurnal nutation” or libration, is due to the influence of the tidal gravitation on the non-zonal terms of geopotential. In case of polar motion it contains both the long periodic terms and the prograde diurnal variation with total amplitude up to 50 microarcseconds (μas). The corresponding theoretical model is a part of the IERS Conventions 2003 (IERS, 2004; Table 5.1). In case of UT1, the perturbation is semidiurnal with total amplitude up to 75 μas . The model has been developed in several works (Wünsch, 1991; Chao et al., 1991; Brzeziński and Capitaine, 2002) but is not included in the IERS Conventions.

Ocean tide contributions. The underlying physical mechanism is the influence of the gravitationally-forced ocean tides with diurnal and semidiurnal periods, upon the rotating Earth via the angular momentum exchange. This effect is also regular and predictable. The model of the Ocean Tidal Angular Momentum (OTAM) consists of a sum of harmonic terms with tidal frequencies. The frequencies are well known from the tidal developments, while the amplitudes and phases can be derived from the ocean tide models and/or from the observations of Earth rotation. The ocean tides contribute to all three components of Earth rotation, including precession-nutation, polar motion and UT1. In case of polar motion and UT1 this is dominant effect in the diurnal and semidiurnal frequency bands. The current OTAM model recommended by IERS Conventions 2003 is based on hydrodynamic ocean tide model constrained by TOPEX/Poseidon altimetry data (Chao et al., 1996). The equatorial retrograde diurnal component of the OTAM which contribute to nutation has been accounted for in the IAU 2000 precession-nutation model. In case of polar motion the ocean tide influence consists of semidiurnal and prograde diurnal variation with amplitudes up to 330 μas and 150 μas , respectively (IERS 2004; Table 8.2). In case of UT1, the ocean tide influence appears as semidiurnal and diurnal variations with amplitudes up to 260 μas and 240 μas , respectively (IERS 2004; Table 8.3).

Parameters of the model of the ocean tide effect in polar motion and UT1 have been estimated from the space geodetic observations: VLBI solutions – (Herring and Dong, 1994; Gipson, 1996); SLR solution – (Watkins & Eanes, 1994) with later updates by R. Eanes; GPS solution – (Rothacher et al., 2001). Our recent analysis (Bolotin and Brzeziński, 2006) using VLBI data spanning 1984.0–2006.2 revealed significant corrections to the conventional model, the largest in each group being:

- polar motion, prograde diurnal – $30 \pm 5 \mu\text{as}$ (K_1);
- polar motion, retrograde semidiurnal – $16 \pm 4 \mu\text{as}$ (M_2);
- polar motion, prograde semidiurnal – $14 \pm 4 \mu\text{as}$ (S_2);
- UT1, diurnal – $34 \pm 5 \mu\text{as}$ (K_1);
- UT1, semidiurnal – $20 \pm 4 \mu\text{as}$ (M_2).

Here in parentheses are given standard codes of the tidal constituents. Our recommendation is that the model of the OTAM should be recomputed using the recent estimates from space geodesy and the available data sets from the satellite altimetry.

Diurnal atmospheric tides. The influence of diurnal atmospheric tides on Earth rotation was discussed in details by Brzeziński (2008). Here we will recall some basic facts. The diurnal cycle in solar heating give rise to variations in AAM with main components S_1, S_2 of periods 24 and 12 hours, and their side lobes due to seasonal modulations. These harmonic components are superimposed on the background variation of stochastic character. Similar effect can be seen in the nontidal OAM due to the ocean response

to the atmospheric forcing. All diurnal and semidiurnal harmonics of AAM and OAM expressing the atmospheric thermal tides are added to the harmonics with the same frequencies but produced by the gravitational ocean tides. With exception of S_1 , in all cases the ocean tide contributions are significantly larger than the corresponding thermal effects expressed by AAM and OAM.

The diurnal and semidiurnal variations of AAM and OAM excite small perturbations in all three components of Earth rotation, including precession-nutation, polar motion and UT1. So far, only the S_1 contributions to Earth rotation could be detected in both the space-geodetic observations and geophysical models:

precession-nutation – prograde annual term, amplitude about $100 \mu\text{as}$ from VLBI data and roughly the same magnitude from AAM and OAM;

polar motion – prograde 24-hour harmonic, amplitude between 30 and $40 \mu\text{as}$ from space geodesy and below $10 \mu\text{as}$ from AAM and OAM;

UT1 – 24-hour harmonic, amplitude between 25 and $40 \mu\text{as}$ from space geodesy and below $20 \mu\text{as}$ from AAM and OAM.

There were also several attempts to estimate the S_3 harmonic of thermal origin from the continuous observation campaigns (Haas and Wünsch, 2006; Salstein et al., 2008), but they have been not conclusive so far. Finally we note that the thermal contribution to the S_2 component in polar motion and UT1 has been included in the conventional model of the ocean tide effect. In case of S_1 , only the contribution to prograde annual nutation has been modeled as a so-called Sun-synchronous empirical correction. The contributions of S_1 to polar motion and UT1 are still not included in the conventional models, though significantly exceed the level of detectability. In addition, the stochastic component of excitation by the atmospheric diurnal tides needs regular monitoring in the time domain.

Atmospheric normal modes. An important feature of the global-scale dynamics of the atmosphere is the presence of the so-called normal modes. Three of them could be detected as pseudoharmonic oscillations in the equatorial components of AAM, hence contributing to polar motion. Their standard codes and central periods are the following:

ψ_3^1 Rossby-Haurwitz wave (toroidal mode), retrograde period about 10 days;

ψ_1^1 mixed Rossby-gravity wave (the Yanai wave), retrograde period about 1.2 days;

ξ_2^1 gravity wave (spheroidal mode), prograde period about 0.6 days.

Brzeziński *et al.* (2002) investigated these three atmospheric normal modes using the NCEP-NCAR reanalysis AAM data with 6-hourly resolution. They estimated the mean amplitudes of the corresponding polar motion to be: $\psi_3^1 - 0.54 \text{ mas}$ (pressure term) and 0.15 mas (wind term); $\psi_1^1 - 30 \mu\text{as}$ (wind term) and $2 \mu\text{as}$ (pressure term); and $\xi_2^1 - 2 \mu\text{as}$ (wind term) and $2 \mu\text{as}$ (pressure term). (Here by “mean amplitude” we understand square root of the total variance of oscillation). Brzeziński and Ponte (2005) extended the estimation by taking account the ocean response to the atmospheric forcing, expressed by the barotropic OAM model with hourly resolution. They derived the following combined influence of the atmosphere-ocean system: $\psi_3^1 - 0.32 \text{ mas}$ (matter term) and 0.27 mas (motion term); $\psi_1^1 - 45 \mu\text{as}$ (motion term) and $15 \mu\text{as}$ (matter term); and $\xi_2^1 - 5 \mu\text{as}$ (motion term) and $2 \mu\text{as}$ (matter term).

Among the two normal modes which are in the diurnal/subdiurnal frequency band, only the ψ_1^1 is significant. However, due to its broadband nature, it can only be expressed as time series of Earth rotation parameters with subdiurnal resolution.

3. DIURNAL POLAR MOTION OBSERVED BY RING LASER

Ring laser gyroscope is a promising emerging technology for directly and continuously measuring changes in Earth rotation; see (Schreiber et al., 2004) for details. A single instrument is capable to determine the polar motion of the instantaneous rotation axis, in contrast to the space geodetic techniques which report the terrestrial and celestial motions of the conventional Celestial Intermediate Pole (CIP); see (IERS, 2004) for details. However, due to the instrumental drift, the measurements of ring laser are not stable over periods longer than a few days. Therefore only the diurnal and subdiurnal variations in polar motion can be estimated. Among several instruments which have been developed so far, the most accurate for monitoring high frequency polar motion is the G ring laser in Wettzell (Germany). As shown in Fig. 1, its measurements are quite well correlated with prediction from the theoretical model of the forced diurnal polar motion (Brzeziński, 1986).

Here we will address the question about the possible benefits from the use of the ring laser data for monitoring variations of Earth rotation. We start from recalling some definitions and elementary kinematical relationships.

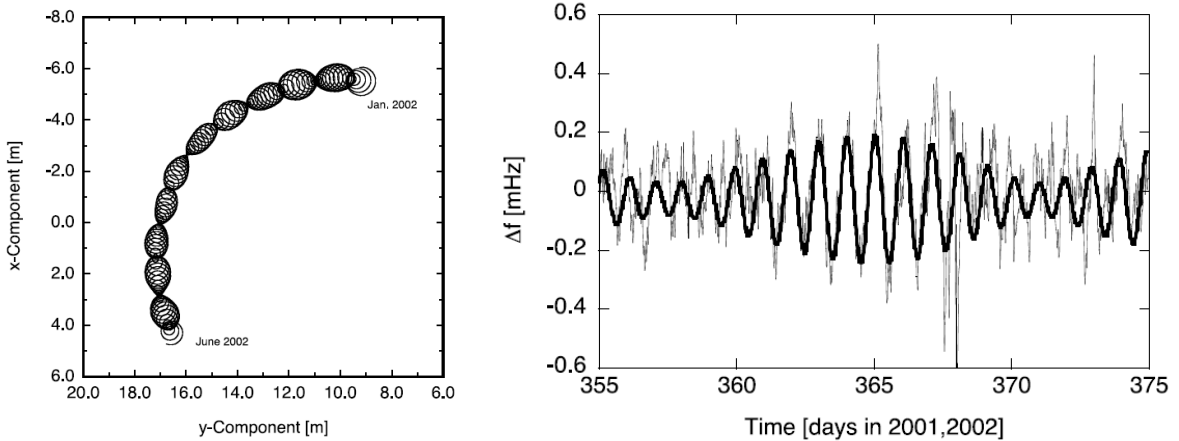


Figure 1: Model of the forced diurnal polar motion of the IRP (Brzeziński, 1986) – left, and its comparison to the measurements of the G ring laser in Wettzell, Germany – right; from (Schreiber et al., 2004).

Let $\vec{\omega} = \Omega [m_1, m_2, 1+m_3]^T$ be the rotation vector of the Earth, where $\Omega = 7\,292\,115 \times 10^{-11}$ rad/s denotes the average angular velocity of rotation, and m_ℓ , $\ell = 1, 2, 3$, are unitless parameters describing small departures from the uniform rotation (from observations $|m_\ell| < 10^{-6}$). The instantaneous rotation pole (IRP) which is identified with the unit vector $\vec{\omega}/|\vec{\omega}| \approx [m_1, m_2, 1]^T$, is usually expressed by the complex parameter $m = m_1 + im_2$, with $i = \sqrt{-1}$. The time variations of m are described as polar motion of the IRP.

The space geodetic techniques express time variations of Earth rotation by the so-called Earth Orientation Parameters (EOP) $\{x_p, y_p, \delta X, \delta Y, \delta UT1\}$. The first two of EOP's, x_p, y_p , describe the terrestrial orientation of the axis defining the CIP, the next two parameters $\delta X, \delta Y$, describe small difference between the observed celestial direction of this axis and the direction predicted by the conventional precession-nutation model, and finally $\delta UT1$ expresses changes in the rotation angle around the CIP axis; see (IERS, 2004) for definition. The complex parameter $p = x_p - iy_p$ is used to describe polar motion of the CIP, while $P = \delta X + i\delta Y$ describe the nutation. Each perturbation in the direction of the CIP axis is treated either as polar motion, or as nutation, depending on frequency. If the space-referred period of perturbation is longer than 2 days, then the perturbation is treated as nutation P , for other periods it is considered as polar motion p . The corresponding perturbations in polar motion of the IRP can be expressed by the following first-order relationships (Brzeziński and Capitaine, 1993)

$$m = p - i \frac{\dot{p}}{\Omega}; \quad m = i \frac{\dot{P}}{\Omega} e^{-i\theta}, \quad (1)$$

where $\theta = \Omega t + \theta_o$ denotes the sidereal rotation angle with $\theta_o \approx \text{const.}$

Assuming harmonic form of perturbation $p = p(\sigma)e^{i\sigma t}$, $P = P(\sigma')e^{i\sigma' t}$, where $p(\sigma)$ and $P(\sigma')$ are the complex amplitudes, the relationships (3) take the following form

$$m = p(\sigma) \left(1 + \frac{\sigma}{\Omega}\right) e^{i\sigma t}; \quad m = \frac{\sigma'}{\Omega} P(\sigma') e^{i[\sigma' t - (\theta + \pi)]} \approx \frac{\sigma'}{\Omega} P(\sigma') e^{i[(-\Omega + \sigma')t - (\theta_o + \pi)]}. \quad (2)$$

According to the definition of the CIP (IERS, 2004), $|\sigma'| \leq 0.5\Omega$ and from eq.(2) the frequency of the corresponding perturbation of m , $\bar{\sigma} = -\Omega + \sigma'$, is nearly diurnal retrograde: $\bar{\sigma} \in \langle -1.5\Omega, -0.5\Omega \rangle$. The perturbations of the CIP having the Earth-referred frequencies outside the diurnal retrograde band, $\sigma \in (-\infty, -1.5\Omega) \cup (-0.5\Omega, \infty)$, are treated as polar motion.

Having regular determinations of m by the ring laser, we could easily combine them with standard observations of p and P by space geodesy, using the linear relationships (3). One obvious benefit would be the interpolation of EOP's with subdiurnal sampling. Below, we will make a quantitative discussion taking into account various known equatorial components of Earth rotation, and using the estimates derived by Brzeziński (2004). The low frequency polar motion will be disregarded because it can not be observed by the ring laser.

Precession-nutation of the CIP. The corresponding terrestrial perturbation of the IRP, expressed by m , is nearly diurnal retrograde. The total size is below 28 mas (Brzeziński, 1986) which is much smaller than the observed precession-nutation of the CIP. This is the main signal in polar motion m which can be derived from the ring laser observation; see Fig. 1. From eq.(2) one can deduce that the total difference between the precession-nutation models IAU 2000 and IAU 1980, which amounts to about 10 mas, corresponds to less than 25 μ as in m . This is by more than one order of magnitude below the current accuracy of m from the ring laser data which is at the level of 1 mas (Paulo Mendes Cerveira, private communication). The geophysical signal in P , which is primarily the free core nutation, has the amplitude below 0.4 mas. The corresponding signal in m is below 1 μ as, hence completely negligible.

Prograde diurnal polar motion. We have $\sigma \approx \Omega$ and from eq.(2) the corresponding relationship between m and p is $m \approx 2p$. When taking into account the size of p , one can see that the amplitudes of m will be below 0.3 mas.

Semidiurnal polar motion. For the retrograde component $\sigma \approx -2\Omega$ we derive $m \approx -p$. The maximum amplitude of m is the same as for p , that is up to about 0.3 mas. For the prograde component $\sigma \approx +2\Omega$ the relationship is $m \approx 3p$ and the amplitude of the ocean tide influence can reach the level of 1 mas.

One should bear in mind that after removing the a priori harmonic model, the size of diurnal and semidiurnal variations in polar motion will be probably by one order of magnitude smaller than assumed in the above analysis, that is below the level of 0.1 mas. The analysis shows that the observations of the ring laser are potentially useful for continuous monitoring of the prograde diurnal and retrograde/prograde semidiurnal signals in polar motion. However, as these signals are extremely small, the uncertainty of the estimated position of the IRP should be better than 0.1 mas.

4. CONCLUDING REMARKS

We presented in this work a systematic review of the perturbations in Earth rotation with daily and subdaily periods. Below, we summarize the most important conclusions and recommendations concerning each group of effects considered here.

- Astronomical variations: Model of the semidiurnal variation in UT1 due to the influence of tidal gravitation upon the triaxiality of the Earth should be included in the IERS Conventions.
- Ocean tide contributions: Observations of space geodesy revealed significant differences with the conventional model. The model recommended by the IERS Conventions 2003 should be recomputed using the available data sets from satellite altimetry and from space geodesy.
- Diurnal atmospheric tides and high frequency normal modes: The 24-hour S_1 component of thermal origin should be added to the conventional a priori model of diurnal variations in polar motion and UT1. Also the side lobes of S_1 and S_2 should be accounted for in the model. A search for the S_3 thermal component should be continued using the high frequency geophysical models and geodetic observations of Earth rotation. Regular subdiurnal observations of Earth rotation are needed to express the influences of the stochastic component of S_1 and of the ψ_1^1 mode.
- Polar motion observations of ring laser: The observations of ring laser are much less accurate than VLBI in determining those components of rotation which are currently expressed as precession-nutation of the CIP, but are potentially useful for continuous monitoring of the prograde diurnal and retrograde/prograde semidiurnal signals in polar motion. However, for significant results the uncertainty of the estimated position of the IRP should be better than 0.1 mas. The observations of ring laser can be combined with space geodetic polar motion data using the first-order kinematical relationships between the motions of the CIP and IRP.

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