# MODELLING IRREGULARITIES OF THE EARTH'S ROTATION

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ABSTRACT. The methods of celestial mechanics can be used to construct a mathematical model for the perturbed rotational motions of the deformable Earth that can adequately describe the astrometric measurements of the International Earth Rotation Service. This model describes the gravitational and tidal influences of the Sun and Moon. Fine resonant interactions of long period zonal tides (annual, semi-annual, monthly and biweekly) with the diurnal and semidiurnal tides are revealed. These interactions can be reliably confirmed via a spectral analysis of the IERS data. Emphasis is placed on the variations of the day duration on short time intervals with periods of one year and less (interaannual fluctuations).

To study the axial rotation of the deformable Earth, we will use the classical dynamical EulerLiouville equations with a varying inertia tensor:

$$J\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times J\boldsymbol{\omega} = \mathbf{M}, \quad \boldsymbol{\omega} = (p, q, r)^T, \quad J = J^* + \delta J, \quad J^* = const,$$
$$J^* = diag(A^*, B^*, C^*), \quad \delta J = \delta J(t), \quad \|\delta J\| \ll \|J^*\|, \tag{1}$$
$$\mathbf{M} = \mathbf{M}_K + \mathbf{M}^S + \mathbf{M}^L.$$

Here, $\boldsymbol{\omega}$ is the angular-velocity vector in a coordinate system (reference frame) fixed to the Earth. The axes of this frame approximately coincide with the main central inertia axes $J^*$ of the "frozen" Earths figure allowing for the equatorial bulge. The reference frame chosen agrees qualitatively and quantitatively with the ITRF coordinates. The small variations in the inertia tensor $\delta J$ can contain harmonics due to regular perturbations exerted by the solar and lunar gravitational diurnal tides, and probably some other harmonics as well (annual, semiannual, monthly, biweekly, etc). Additional perturbations result from differentiating the vector of the kinetic moment of the deformable Earth. These terms are included in the vector $\mathbf{M}_K$, which has a very complex structure and is, in turn, additively included in $\mathbf{M}$. Note that the equations for the components $p$ and $q$ were examined during analyses of the Earths polar oscillations.

Let us rewrite the third equation of (1) for the Earths axial rotation component $r(t)$:

$$C^*\dot{r} + (B^* - A^*)pq + (J_{qr}p - J_{pr}q)r = M_r^S + M_r^L. \tag{2}$$

Here, $J_{pr}$ and $J_{qr}$ are small nondiagonal elements of the inertia tensor, and $M_r^{S,L}$ are the gravitational tidal solar and lunar perturbing moments, respectively [1]. For example, $M_r^S$ includes:

$$M_r^S = 3\omega_0^2[(B^* + \delta B - (A^* + \delta A))\gamma_p\gamma_q + \delta J_{pq}(\gamma_p^2 - \gamma_q^2) +$$
$$+\delta J_{qr}\gamma_p\gamma_r - \delta J_{pr}\gamma_q\gamma_r], \tag{3}$$
$$\gamma_p = \sin\theta\sin\varphi, \quad \gamma_q = \sin\theta\cos\varphi, \quad \gamma_r = \cos\theta.$$

where $\omega_0$ is the frequency of the orbital motion; $\gamma_p$, $\gamma_q$, and $\gamma_r$ the direction cosines of the radius-vector in the fixed frame; $\psi$, $\theta$, and $\varphi$ the Euler angles; and $A^*$, $B^*$, and $C^*$ the effective main central moments of inertia, allowing for deformations of the "frozen" Earth, which can be calculated accurately. The coefficients $\delta A$, $\delta B$, $\delta J_{pq}$, $\delta J_{qr}$, and $\delta J_{pr}$ are due to the tidal diurnal and semidiurnal gravitational influences of the Moon and Sun, which cannot be measured directly. These coefficients can be indirectly estimated using measurements of the characteristics of the process themselves. Averaging over the fast variable $\varphi$ ($\varphi$ is the angle of the proper rotation) yields the simple expression for $M_r^S$

$$M_r^S = 3\omega_0^2[\chi_{1r}^S \sin^2\theta + \chi_{2r}^S \sin\theta\cos\theta]. \tag{4}$$

The quantities $\chi_{1r}^S$ and $\chi_{2r}^S$ in (4) are due to the semidiurnal and diurnal tides, respectively, and these result from the $\varphi$-averaging of the coefficients of $\sin^2\theta$ and $\sin\theta\cos\theta$ in the solar gravitational force moment components:

$$\chi_{1r}^S = \frac{1}{2}\left\langle \frac{\delta B - \delta A}{C^*}\sin 2\varphi \right\rangle_\varphi - \left\langle \frac{\delta J_{pq}}{C^*}\cos 2\varphi \right\rangle_\varphi, \tag{5}$$

$$\chi_{1r}^S = \frac{1}{2}\left\langle \frac{\delta J_{qr}}{C^*}\sin \varphi \right\rangle_\varphi - \left\langle \frac{\delta J_{pr}}{C^*}\cos \varphi \right\rangle_\varphi.$$

Integrating (2), we obtain the *l.o.d.* fluctuations:

$$\begin{aligned} l.o.d.(\tau) = c &+ a_c^S\cos(2\pi\tau) + a_s^S\sin(2\pi\tau) + b_c^S\cos(4\pi\tau) + b_s^S\sin(4\pi\tau) + \\ &+ a_c^L\cos(2\pi\nu_m\tau) + a_s^L\sin(2\pi\nu_m\tau) + b_c^L\cos(2\pi\nu_f\tau) + b_s^L\sin(2\pi\nu_f\tau). \end{aligned} \tag{6}$$

Here, $\nu_m$ and $\nu_f$ are the frequencies of the monthly and biweekly oscillations due to the lunar perturbation, while the unknown quantities $c$, $a_{c,s}^{S,L}$, and $b_{c,s}^{S,L}$ must be determined from the IERS measurements via least-squares fitting. These coefficients are uniquely related to the unknowns contained in (2). The parameter $\tau$ in (6) and below is measured in standard years.

With account for the spectral analysis of the IERS data, the model parameters are identified by means of the least squares method. A statistically reliable interpolation of the measured data over a time interval of one year (and longer) is obtained, and the nonuniformities of the Earths rotation (that is, the variations of the day duration and the UT1-UTC correction) are predicted on an interval of 4 to 10 months. A comparison of the real and theoretical trajectories for the irregularity of tidal oscillations of the rotational angular velocity indicates agreement between the constructed model and the IERS observations.

The theoretical model constructed demonstrates a good agreement with the IERS forecast, according to the accuracy of the approximation.

## REFERENCES

Akulenko L.D., Markov Yu.G.,Perepiolkin V.V. Rykhlova L.V., 2008, “Intrayear Irregularities of the Earth’s Rotation”, Astronomical Journal, vol. 52, No. 7, pp. 590–597.