LUNAR LASER RANGING AND EARTH ORIENTATION

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ABSTRACT. Lunar Laser Ranging (LLR) is carried out for more than 38 years. Several parameters of the Earth–Moon system can be determined with high accuracy by dedicated data analysis. These comprise, e.g., coordinates of the stations and the reflectors, lunar gravity field, orbit and rotation of the Moon, the secular tidal acceleration but also gravitational physics parameters. Here, we focus on the determination of Earth orientation parameters. Long–term nutation coefficients for the 18.6 years period are determined in the global adjustment and compared with results from studies of other analysis centres and parameters of the MHB2000 model of (Mathews et al., 2002). Furthermore, Earth rotation $\Delta UT0$ and variation of latitude $\Delta\phi$ are determined by the daily decomposition method from post–fit residuals. In our LLR analysis, different EOP series are applied and their effect on the Earth–Moon parameters investigated. The results of this study are presented, too.

1. MODEL AND ANALYSIS

The existing model to analyse LLR data at the Institut für Erdmessung (IfE) is based on Einstein's theory of gravity. It is fully relativistic and complete up to the first post–Newtonian ($1/c^2$) level, e.g. (Müller et al., 2008). The basic observation equation for the station–reflector distance is defined in the Barycentric Celestial Reference Frame (BCRF). The station and reflector coordinates have to be transformed from their respective reference frames (the terrestrial (TRF) or selenocentric (SRF) reference frame) into the inertial frame. Here, the Earth orientation parameters (EOP) are used for the Earth and the libration angles, computed by numerical integration, for the Moon. The Earth–Moon distance is obtained by numerical integration of the corresponding equation of motion, considering Newtonian and relativistic effects.

Based on the LLR model two groups of parameters for the Earth–Moon system (ca 180 in total) are determined by a weighted least–squares adjustment of the observations. The Newtonian parameters are, e.g., initial position, velocity and physical librations of the Moon, coordinates of LLR observatories and retro–reflectors, orbit and mass of the Earth–Moon system, lunar gravity field, long–periodic nutation parameters and the lag angle, indicating the lunar tidal acceleration. These parameters are summarised in the so–called standard solution. The post–fit residuals of the solution can be further investigated for Earth orientation parameters, see below. By extending the standard solution, it is possible to solve for parameters related to general relativity, like temporal variation of the gravitational constant (Müller and Biskupek, 2007) and metric parameters. It is also possible to investigate the strong equivalence principle and preferred–frame effects (Soffel et al., 2008).

Figure 1 shows the annually averaged weighted post–fit residuals of the standard solution with data from Dec. 1969 to Mar. 2008 (16230 normal points). It reflects the precision of the LLR measurement and analysis model, about $20 - 30$ cm up to the middle 80ies. From 1985 on, more stations started to observe the Moon and the residuals decreased. In the last years, only two stations, one with reduced accuracy, tracked the Moon, so that the residuals increased again. For more details see (Müller et al., 2008).

2. EARTH ROTATION FROM LLR DATA BY DAILY DECOMPOSITION

To investigate the effect of different EOP series in the LLR analysis, the series IERS EOP C04 and COMB2006 were used as input for the global standard solution. Then, the post–fit residuals were analysed to determine corrections for Earth rotation $\Delta UT0$ and variation of latitude $\Delta\phi$. In a further step, the $\Delta\phi$ corrections were used to iteratively improve the results of the global standard solution.

Both EOP series were obtained from the combination of "operational" EOP series derived from the

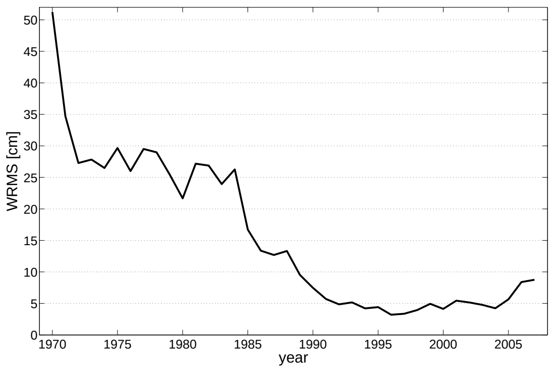

Figure 1: Weighted post–fit residuals (observed minus computed Earth–Moon distance) annually averaged

various space–geodetic techniques VLBI (Very Long Baseline Interferometry), GPS (Global Positioning System), SLR (Satellite Laser Ranging) and LLR, as well as optical observations, see (Gambis, 2004; Gross, 2007). Additionally, in the C04 series DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite) data were included. Further differences between the series on the data side are, that not exactly the same time periods were used and the filter techniques to combine the data were different. Another difference between the series is the treatment of tidal effects. In the C04 series periods from 5 days to 18.6 years are corrected by the model of (Defraigne & Smits, 1999), in the COMB2006 series periods from 5 days to 35 days are corrected using the procedure of (Yoder et al., 1981). As the comparison of the two series (C04 minus COMB2006) shows, the difference in all components (x_P, y_P and $\Delta UT1$) is large in the period between 1970 and 1982. It is between -18 and 34 mas for polar coordinate x_P and between -42 and 15 mas for polar coordinate y_P. For Earth rotation $\Delta UT1$, the difference is between -2.1 and 3.3 ms. From 1982 on, the difference decrease more and more, for polar coordinates it is near zero now. For $\Delta UT1$ it is about -0.005 ms today, because of the different treatment of the tidal effects.

The two EOP series, i.e., the pole coordinates x_P/y_P and Earth rotation $\Delta UT1$, are used in the analysis for the transformation between the celestial and terrestrial systems. In addition, the long–periodic, diurnal and sub–diurnal effects of the ocean are corrected according to the IERS Conventions 2003.

To determine Earth rotation from LLR data, the post–fit residuals are first sorted by station–reflector combinations and merged in daily sets. One set must include the minimum of three station–reflector pairs. Out of 16230 observations, 1179 daily sets for the station OCA in Grasse and 752 daily sets for the station McDonald in Texas were found. These sets are analysed in a second least–squares adjustment (daily decomposition method, see (Dickey et al., 1985) applying the following model:

$$r(t) = r_{\Delta UT0} + r_{\Delta\phi} + r_n, \tag{1}$$

where the post–fit residuals of the first least–squares fit are assumed to be caused by contributions from Earth rotation

$$r_{\Delta UT0} = 2\ \Delta UT0\ r_E\ \cos\phi\ \sin H\ \cos\delta, \tag{2}$$

from variation of latitude

$$r_{\Delta\phi} = 2\ \Delta\phi\ r_E\ (\sin\phi\ \cos\delta\ \cos H - \sin\delta\ \cos\phi) \tag{3}$$

and a part containing other effects r_n, like systematic ranging errors and model errors. $\Delta UT0$ and $\Delta\phi$ enter the respective equation as

$$\Delta\phi = x_P\ \cos\lambda - y_P\ \sin\lambda \tag{4}$$

and

$$\Delta UT0 = \Delta UT1 + \tan\phi\ (x_P\ \sin\lambda + y_P\ \cos\lambda) \tag{5}$$

with the declination δ, the hour angle H of the Moon, the latitude ϕ and the Earth's radius r_E. Figure 2 shows the result for the determination of $\Delta\phi$ from the post–fit residuals for the McDonald station by using the different EOP series. The daily solutions (dots), calculated with eq. (3), are smoothed by a spline filter. The curves of the two calculations show large differences in the 70ies and middle 80ies, the

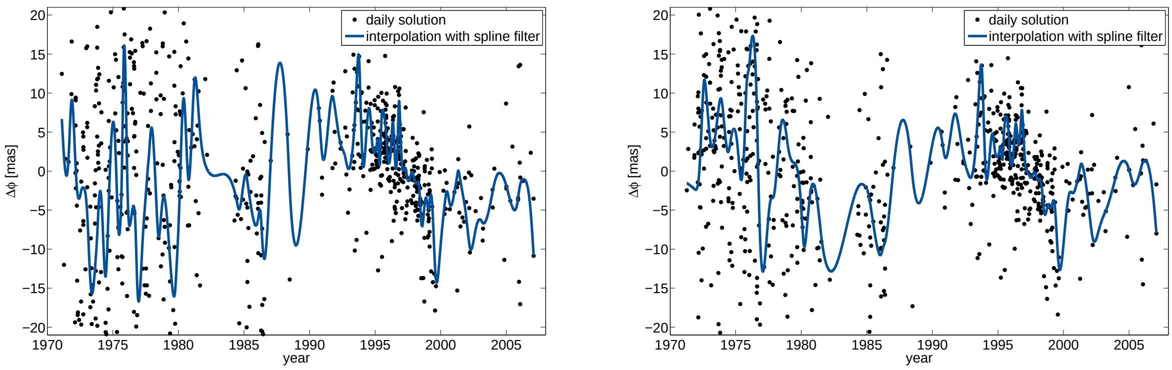

Figure 2: Results of the daily decomposition method for $\Delta\phi$ when using the EOP series C04 (left) and COMB2006 (right) as input in the global adjustment

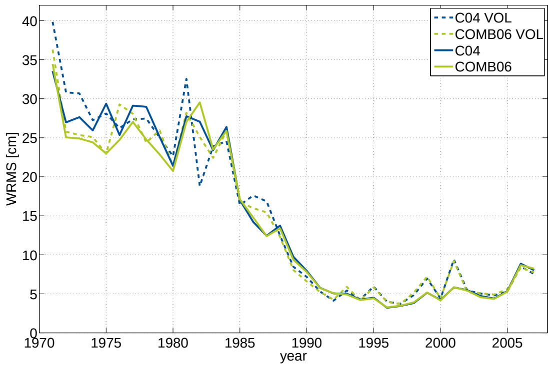

Figure 3: Results of using the $\Delta\phi$ corrections in the global adjustment

period where the two EOP series show large differences, too. From the middle 80ies on, when both EOP series are very similar, also the results are very similar here.

In the next step, these spline–interpolated $\Delta\phi$ values are iteratively used as corrections in the global adjustment, figure 3 shows the corresponding results. Here, two calculations have been made for both EOP series: One as reference without the estimated $\Delta\phi$ values (solid lines) and one with the $\Delta\phi$ correction applied (dotted lines). It can be seen, that again in the time span from the middle 80ies on, where the EOP series are very similar, also the residuals are very similar. It is furthermore obvious, that the residuals can not really be improved by using the $\Delta\phi$ correction. It seems, that the input EOP series are already very accurate. In the time span up to the 80ies, the residuals by using the COMB2006 series are smaller than that one based on the C04. But also here, the results of the global adjustment can not really be improved by applying the determined $\Delta\phi$ correction.

3. NUTATION

A further study was dedicated to the determination of nutation parameters from LLR data. In the IERS Conventions 2003 nutation is described as

$$\Delta\psi = \sum_{i=1}^{N} (A_i + A_i' t)\ \sin(ARG) + (A_i'' + A_i''' t)\ \cos(ARG) \tag{6}$$

$$\Delta\epsilon = \sum_{i=1}^{N} (B_i + B_i' t)\ \cos(ARG) + (B_i'' + B_i''' t)\ \sin(ARG) \tag{7}$$

with $ARG = \sum_j^5 N_j F_j$, N_j : multiplier, F_j : Delaunay parameter. In the global adjustment, the non–time–dependent nutation coefficients A_i, A_i'', B_i and B_i'' of the 18.6 years period were determined and compared with the values of the MHB2000 model (Mathews et al., 2002). Table 1 gives some preliminary

Table 1: Preliminary results for nutation coefficients of the 18.6 years period

	A_i[mas]	A_i" [mas]	B_i [mas]	B_i" [mas]
MHB2000 model	-17206.42	3.33	9205.23	1.54
our investigation	-17203.42	4.15	9204.49	3.81
standard deviation	0.39	0.31	0.17	0.15

results. The values of all coefficients show large differences to the values of the model, the largest in A_i and B_i''. These are also seen in the analysis of other groups (Williams, 2008).

4. CONCLUSIONS

The investigations show, that the determination of variation of latitude $\Delta\phi$ from LLR data is possible. Using these values in the next iteration step of the LLR analysis does, however, not significantly improve the residuals. Obviously, the input EOP series are already of good quality. As further work other filters, apart from the spline filter, will be tested. Also values for length of day LOD will be calculated to compare them with LOD results from VLBI.

The estimated nutation coefficients for the 18.6 years period show large differences to the MHB2000 model, which are not understood yet. Here, further investigation is needed. In a next step, the coefficients for more periods (9 years, 1 year) will be determined and compared to the model.

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