WAVELET ANALYSIS OF TEC MEASUREMENTS OBTAINED USING DUAL FREQUENCY SPACE AND SATELLITE TECHNIQUES

A. KRANKOWSKI[1], T. HOBIGER[2], H. SCHUH[2], W. KOSEK[3] and W. POPIŃSKI[4]
[1]Institute of Geodesy, University of Warmia and Mazury in Olsztyn, Poland
[2]Institute of Geodesy and Geophysics, Vienna University of Technology, Austria
[3]Space Research Centre, Polish Academy of Sciences, Warsaw, Poland
[4]Central Statistical Office, Warsaw, Poland
e-mail: kand@uwm.edu.pl

ABSTRACT. An extensive database of Total Electron Content (TEC) measurements has become available from both ground- and space-based observations. Global Positioning System (GPS) and Very Long Baseline Interferometry (VLBI) observations collected at the IGS (International GNSS Service) and the IVS (International VLBI Service for Geodesy and Astrometry) stations over Europe were used to obtain TEC data during the time interval 1995 till 2003. In this paper, the wavelet analysis is used to determine the wavelet time-frequency spectra of TEC data above one European collocation station - Wettzell. The GPS and VLBI TEC time series of quiet and disturbed ionospheric conditions, utilized in this study, cover one solar cycle. A very good agreement between semidiurnal, diurnal, semiannual, and annual oscillations of TEC estimated using GPS and VLBI observations was obtained. The diurnal and annual oscillations are the most energetic and clearly visible ones especially during increasing and maximum solar activity.

1. INTRODUCTION

The wavelet analysis is a mathematical technique which is very useful for numerical analysis and manipulation of multidimensional discrete data sets. Originally applied in geophysics, analyzing the Earth orientation parameters and atmospheric angular momentum, the wavelet transforms were better and broadly formalized thanks to mathematicians', physicists', and engineers' efforts (Morlet 1983, Popiński et al. 2002). Therefore, the use of wavelet techniques in data analysis has grown, since it represents a synthesis of old techniques associated with robust mathematical results and efficient computational algorithms under the interest of a broad community (Daubechies et al. 1992). In a rapidly developing field, overview papers are particularly useful, and several good ones concerning wavelets are already available (Daubechies et al. 1992, Chui 1992). In atmospheric science applications, the main characteristic of the wavelet technique is the introduction of the time-frequency decomposition. A well known example of such a feature can be found in the musical structure, where it has been interpreted as events localized in time. Although it belongs to a more complex structure, a piece of music can be understood as a set of musical notes characterized by four parameters: frequency, time of occurrence, duration and intensity (Daubechies et al.1992, Lau and Weng 1995).

In the last decades, the wavelet technique has been extensively adopted in atmospheric science (Gambis 1992, Krankowski et al. 2005). In the last years numerous attempts were made to compare TEC values obtained from different ground- and space-based observations (Codrescu et al. 2001, Orús et al. 2002, Balehaki et al. 2003). Over the last decade, an extensive database of TEC measurements has become available from GPS and VLBI. The GPS satellites continuously provide TEC data (since 1994) through the world-wide network of GPS ground receivers. Earlier than GPS the VLBI technique has provided TEC data for more than 20 years (since 1984), collected at IVS (Hobiger et al. 2005).

In this work, the Morlet wavelet transform (MWT) is used to determine the wavelet time-frequency spectra of TEC data above one European collocation station - Wettzell. The GPS and VLBI TEC time series of quiet and disturbed ionospheric conditions, utilized in this study, cover one solar cycle (from 1995 to 2003).

2. DATA

The absolute TEC and instrumental biases were estimated using a single site algorithm (Baran et al. 1997). The relationship between the ionospheric delay and TEC, and the difference between the dual-frequency code (P) and phase (Φ) measurements may be written:

$$\Delta P[m] = M \cdot TEC/cos(z') + A_P + \varepsilon_P \qquad \Delta\Phi[m] = M \cdot TEC/cos(z') + A_\Phi + \varepsilon_\Phi, \quad (1)$$

where the scaling factor M converts units of distance (m) to units of TEC (el/m^2), z' is the zenith angle, ε_P , ε_Φ are noise terms, A_P and A_Φ are equipment biases (A_Φ contains the phase ambiguity).

The diurnal variation of the vertical TEC (VTEC) is expressed as the series of harmonic terms:

$$VTEC = a_1 + a_2\Delta\phi + (a_3 + a_4)cos(s) + (a_5 + a_6)cos(2s) + ... + (a_{13} + a_{14})cos(6s) + a_{15}\Delta\phi^2 s, \quad (2)$$

where $s = \pi(LT - 14)/12$ and LT is the local solar time, $\Delta\phi$ is the latitudinal difference between the coordinates of the receiver and the sub-ionospheric point.

The VLBI observables at two different frequencies (2.3 and 8.4 GHz, S- and X-band) are performed in order to calculate the ionospheric delay and consequently to model the ionosphere above each station (Kondo 1991). In the Vienna TEC Model $VTEC_i(t)$ is calculated as a piecewise linear function (Hobiger et al. 2005):

$$VTEC_{VIENNA,i} = offset_i + rate_{i1}(t_1 - t_0) + rate_{i12}(t_2 - t_1) + ... + rate_{in}(t_n - t), \quad (3)$$

where $t \leq t_n$. After calculating the partial derivatives $(\frac{\delta TEC_{VIENNA,i}}{\delta offset_i}\ ;\ \frac{\delta TEC_{VIENNA,i}}{\delta rate_{i,j}})$ a least-squares adjustment allows separation of the VTEC values from constant instrumental offset $\tau_{offset,i}$.

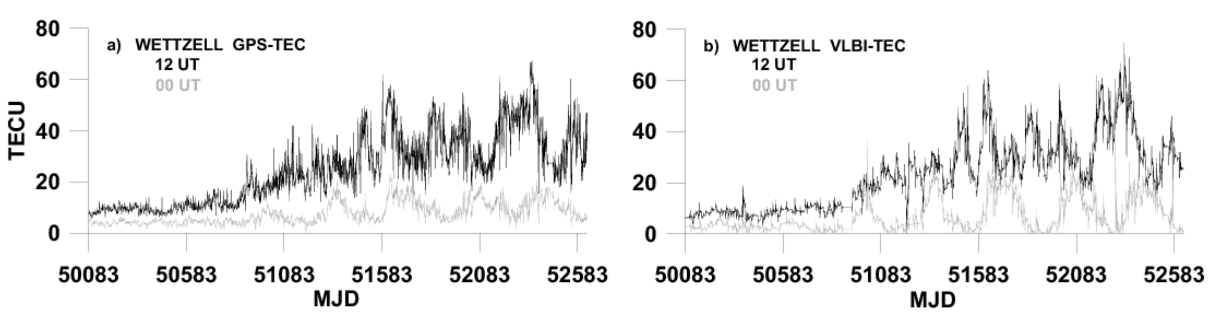

Figure 1: GPS (a) and VLBI TEC (b) measurements observed at noon (black line) and at midnight (grey line) over Wettzell during 1996 till 2002.

The accuracy of the TEC determined from GPS and VLBI data is of the order of 1-3 TECU ($TECU = 10^{16} electrons \cdot m^{-2}$). Figure 1 shows an example of TEC monitoring using GPS and VLBI measurements over the mid-latitude European station Wettzell, i.e. TEC observed at noon and midnight for the period of 1996 to 2002. The day by day variation in TEC shows a strong dependence on solar activity; a very good agreement between diurnal variations of GPS and VLBI TEC observations was obtained.

3. WAVELET ANALYSIS

The wavelet transform coefficients are computed using the inverse discrete Fourier transform (DFT) by the following approximate formula (Chui 1992):

$$S(T,b) \simeq \frac{1}{n}\sqrt{|T|}\sum_{\nu=-n/2+1}^{n/2} \tilde{s}(\nu)\breve{\psi}(2\pi T\nu/n)\exp(i2\pi b\nu/n), \tag{4}$$

where T and $b = 0, 1, ..., n-1$, are the dilation (period) and translation (time shift) parameters, respectively, $\tilde{s}(\nu) = \sum_{k=0}^{n-1} s(k)\exp(-i2\pi\nu k/n)$ is the DFT of the analyzed signal $s(k), k = 0, 1, ..., n-1$, of the total length n, and

$$\breve{\psi}(\omega) = \sigma[\exp(-(\omega-2\pi)^2\sigma^2/2) - \exp(-(\omega-2\pi)^2\sigma^2/4)\exp(-\pi^2\sigma^2)] \tag{5}$$

denotes the continuous Fourier transform (CFT) of the Morlet wavelet function (Schmitz-Hübsch and Schuh 1999) with parameter $\sigma > 0$ which controls the frequency resolution of the wavelet transform. The corresponding time-frequency spectrum is defined as $|S(T,b)|^2$.

4. RESULTS

The MWT time-frequency spectra are shown in Figures 2 - 3. Figure 2 presents the MWT time-frequency spectra of TEC obtained using GPS (left panel) and VLBI (right panel) during the period March till June 2001. The semidiurnal and diurnal oscillations are clearly visible in these spectra. The time-frequency spectrum shows that the diurnal oscillation is more energetic than the semidiurnal one. Figure 3 presents the MWT time-frequency spectra of TEC data over Wettzell station during one period of solar activity (between 1996 and 2002). Semiannual and annual oscillations, respectively, can be seen in these Figures. The annual oscillation was energetic and clearly visible especially during periods of increasing and maximum solar activity. The semiannual oscillation was less energetic than annual oscillation, but it is also easy to distinguish in periods of increasing and maximum solar activity. From Figures 2 and 3 you can see that the time-frequency spectra of GPS-TEC and VLBI-TEC data over Wettzell station from one period of solar activity (between 1996 and 2002) show the same pattern for semidiurnal, diurnal, semiannual and annual oscillations. Results obtained using GPS and VLBI TEC measurements agree very well for all oscillations.

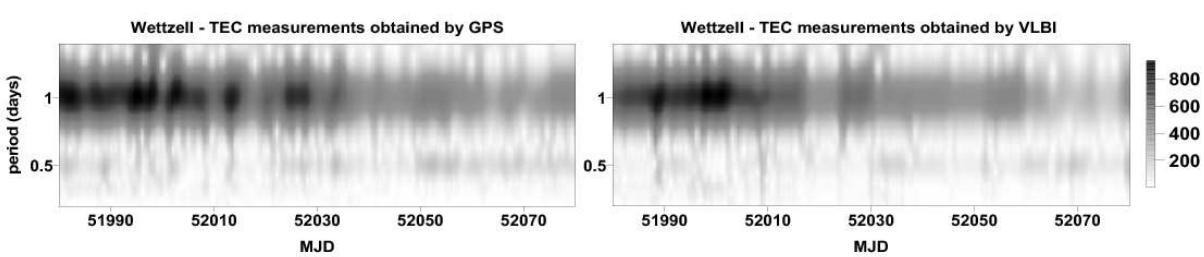

Figure 2: Wavelet time-frequency spectrum of the TEC data obtained using GPS (left panel)and VLBI (right panel) measurements over Wettzell during the period March till June 2001.

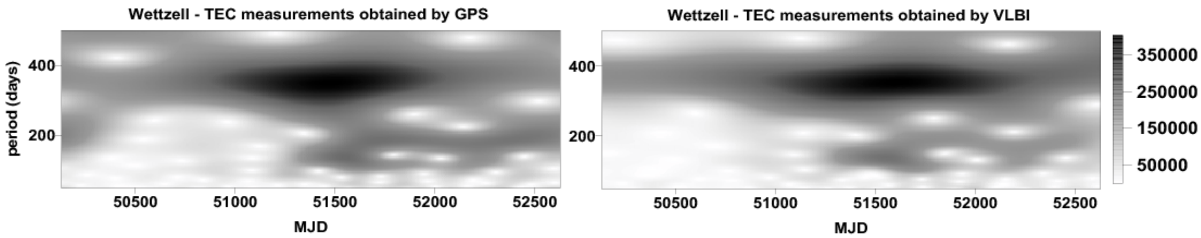

Figure 3: Wavelet time-frequency spectrum of the TEC data obtained using GPS (left panel) and VLBI (right panel) measurements over Wettzell for the period 1996 till 2002.

5. CONCLUSIONS

Information from time-frequency analysis of the TEC time series is very useful for investigation of the irregular variations in these data and also for the comparison of different time series. The time-frequency spectra of GPS-TEC and VLBI-TEC data over one collocation station Wettzell, computed by the MWT method, during one period of solar activity (between 1996 and 2002) show the same portrait for all detected oscillations: semidiurnal, diurnal, semiannual and annual. The information from time-frequency analysis can explain big prediction errors of these data especially during the ionospheric storms. GPS or VLBI TEC measurement will be used in further investigations aiming at creating the predicted TEC maps over Europe.

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