ABOUT WOBBLE EXCITATION IN THE CASE OF TRIAXIAL EARTH ROTATION

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Earth rotation theory is still challenged by an inadequate predictions for all normal modes excluding nutation. The origin of discrepancies between theory and observations we are going to show here may appears due to misunderstanding of constraints exerted on derivation of linearized Munk and Macdonald equations. They proposed two of them: 1) to replace the real Earth with 3-axis ellipsoid of inertia on the model with 2-axis ellipsoid; 2) to neglect all small terms in Liouville system.

Let's focus on the first one. An exchange of 3-axis body on 2-axis body has to make sure that trajectories of both will be close at any time because $(B - A)$ is small. Is it really true?

The answer must be sufficiently rigorous to exclude any disturbance from ordinally used mathematical procedures such as approximation, expansion, etc. The best way doing it is to analyse Euler solution for 3-axis free rigid body rotation. All the work then will be merely accurate computation of analytical solution components. Such an approach differs from well known papers of Kinoshita, Fukushima, Getino, S. Molodensky, Souchay and others by concentrating on achievement of extreme precision.

Euler system for the rotation of rigid 3-axis body

$$\begin{cases} A\dot{\omega}_1 + (C-B)\omega_2\omega_3 & = & 0, \\ B\dot{\omega}_2 - (C-A)\omega_3\omega_1 & = & 0, \\ C\dot{\omega}_3 + (B-A)\omega_1\omega_2 & = & 0 \end{cases} \tag{1}$$

has as it's well known the next solution

$$\begin{cases} \omega_1 & = & \sqrt{\dfrac{2EC-G^2}{A(C-A)}}cn(u), \\ \omega_2 & = & \sqrt{\dfrac{2EC-G^2}{B(C-B)}}sn(u), \\ \omega_3 & = & \sqrt{\dfrac{G^2-2EA}{C(C-A)}}dn(u), \end{cases} \tag{2}$$

where $sn(u)$, $cn(u)$ are elliptic sine and cosine and $dn(u) = \sqrt{1-k^2sn^2(u)}$ – an elliptic tangent, $k^2 = \dfrac{(B-A)(2EC-G^2)}{(B-C)(G^2-2EA)} = \text{const}$ – module of elliptical functions, E, G – integration constants.

Due to the strict limitation on paper size we save in the text only main formulae, details can be found in textbooks of Whittaker, Goldstein, Landau, etc. By the way general picture of free motion is easy seen from (2) – properties of elliptical functions determine motion features. Here we can review only several. 1) 3-axis body pole moves along elliptical trajectory; 2) ω_3 component of 3-axis body has its own variation while 2-axis ω_3 is constant; 3) all ω_i have spectra with infinite number of discrete lines. The more is $(B - A)$ the more distinct are differences. In the case of Earth it is hard work to compute elliptical function values with high precision. The problem was solved by comparing of three various algorithm results.

The important property of motion are easy seen also from parametric expansion of elliptical functions (here for sine):

$$sn(u) = \frac{2\pi}{\sqrt{m}\,K} \sum_{n=0}^{\infty} \frac{q^{n+\frac{1}{2}}}{1 - q^{2n+1}} \sin(2n+1)\nu; \qquad m = k^2, \tag{3}$$

where $q = e^{-\pi K'/K}$ – parameter of expansion, $4K$ – period of polar motion.

Due to the very small value of q solution (2) has a very steep form of power spectra as for polar motion and LOD.

Thus, an application of constraint $A = B$ leads to an exchange of model with infinite power spectrum on the model with single line on eigenfrequency. While free rotating 3-axis body does not differ practically from 2-axis one – trajectory discrepancies are less than 1 mm, situation is drastically changed if mass motion exists. Symmetrical 2-axis body modes are excited in main resonance only, 3-axis body – on infinite set of resonances and everyone can distort as amplitude and phase of motion. Such a profound difference can not be ignored even for body with small equatorial ellipticity due to nonlinear effects.

Is it right then to ignore other small terms in Liouville equations? To validate second constraint it is necessary to perform numerical integration of perturbed 3-axis body equations. This problem is harder then previous one and will be considered elsewhere.

CONCLUSIONS

An investigation of proposal $A = B$ as constraint in derivation of Munk and MacDonald system of equations revealed that 3-axis body rotation and 2-axis body rotation are SIMILAR only while $(B-A)$ is small but NOT EQUIVALENT dynamically. Both have profoundly different excitation structures. Linear system of equations is unable to describe all properties of rotation of the real Earth.

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