EXCITATION FUNCTION RECONSTRUCTION USING OBSERVATIONS OF THE POLAR MOTION OF THE EARTH

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ABSTRACT. Reconstruction of the excitation functions from the observations of the motion of the Earth's pole was performed with use of Jeffreys-Wilson filter, regularization, corrective smoothing in the frequency domain. Corrective smoothing procedures found preferable for solving the inverse problem of reconstruction of the excitation functions from observations. Excitation functions were reconstructed since 1900 year for chandler and annual components of the polar motion, divided from each other and separated from noise with use of singular spectral analysis (SSA). Excitation was predicted with use of SSA and neural networks (NN). Kalman filter was used for prediction of the trajectory of the pole.

1. DYNAMICAL MODELLING AND RECONSTRUCTION OF THE CAUSES

Reconstruction of the excitation functions from the observations of the Earth rotation belongs to the class of the ill-posed inverse problems. So far as different input excitations can produce the motion along the observed trajectory, a priori assumptions should be made. The errors of observations can cause a big deviation of the evaluated excitation from the real one. That's why it was recommended to use the corrective procedures for solving the ill-posed problems [Tikhonov et al., 1977]. The motion of the pole can be described by the equation [Yatskiv, 2000]

$$\frac{i}{\sigma_c}\frac{dm(t)}{dt} + m(t) = \chi(t), \tag{1}$$

where $\sigma_c = 2\pi F_c(1 + i/2Q)$. It was suggested to use the values $F_c = 0.843$ cycles per year and $Q = 175$ [Vicente, Wilson, 2002]. The frequency characteristic of the system (1) is given by the expression

$$L(f) = \frac{\sigma_c}{\sigma_c - 2\pi f}. \tag{2}$$

Fig. 1 represents the gain-frequency (GFCh) and phase-frequency (PFCh) characteristics of the system given by (2). The resonance at the chandler frequency can be well seen. When an excitation transfers from one frequency half plane to another, divided by the frequency $\sigma_c/2\pi$, the phase of the polar motion changes by π.

For reconstruction of $\chi(t)$ Wilson suggested the filter [Vicente, Wilson, 2002]

$$\chi(t) = \frac{ie^{-i\pi F_c \Delta t}}{\sigma_c \Delta t}\left[m_{t+\frac{\Delta t}{2}} - e^{i\sigma_c \Delta t} m_{t-\frac{\Delta t}{2}}\right], \tag{3}$$

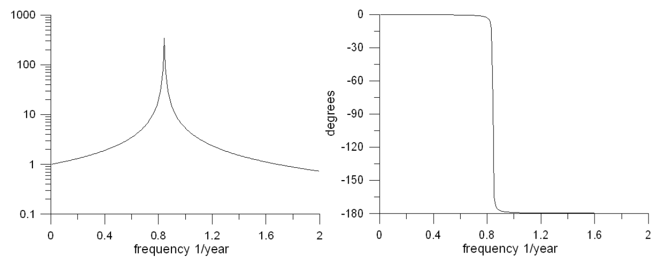

Figure 1: The gain-frequency (left) and phase-frequency (right) characteristics of the rotating Earth dynamical system.

where Δt is a time interval between the equidistant observation's read outs. This filter can be derived from the general solution of the equation (1) with use of the approximate trapezium formula for numerical integration and averaging of two neighbor read outs of excitation. Excitation reconstruction with use of (3) from observations published in EOPC01 bulletin was performed and represented by Fig. 2. It can be seen, that till the 70s years the main composition of the "excitation" is determined by the noises.

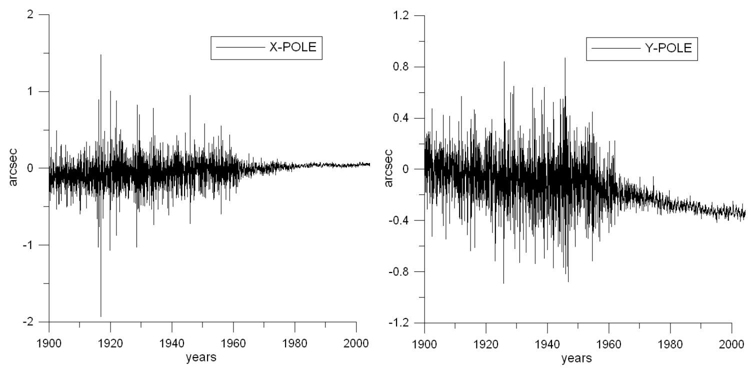

Figure 2: Excitation function reconstructed with use of Wilson filter.

Wilson filter (3) does not imply any corrective operation, if not consider averaging by two points. But it is very desirable to use it. The corrective operation is implied, for instance, by the regularization technique. In a simple assumption, that an input signal belongs to the integrated with squares function space L_2, regularizing weight function can be analytically derived

$$h_{reg}(t,\alpha) = F^{-1}\left(\frac{L'}{L'L+\alpha}\right) = \frac{1}{\alpha\tau}e^{-\frac{t}{\tau}}\cos\frac{t}{\tau\sqrt{\alpha}}, \tag{4}$$

where F^{-1} is an inverse Furier-transformation, touch denotes a complex conjugation, α is a regularization parameter, $\tau = i/\sigma_c$ - the time constant of a system. It's difficult to use the window (4) in the time domain. The regularizing procedure was performed in the frequency domain, the result converted to the time domain is represented by Fig. 3 for two values of regularization parameter [Tikhonov et al., 1977].

Other method can be used is the corrective smoothing, suggested by V.L. Panteleev [Panteleev, 2001]. We can transform the Panteleev smoothing window

$$\psi(t) = \frac{\omega_0}{2\sqrt{2}}e^{-\frac{w_0|t|}{\sqrt{2}}}\left(\cos\left(\frac{\omega_0 t}{\sqrt{2}}\right) + \sin\left(\frac{\omega_0|t|}{\sqrt{2}}\right)\right) \tag{5}$$

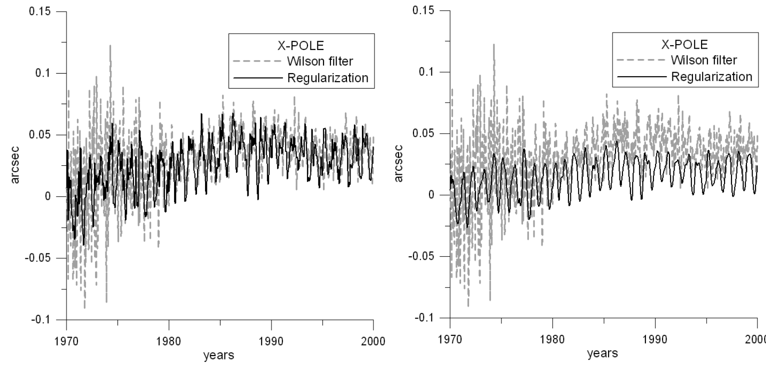

Figure 3: The X-component of the excitation function, reconstructed with use of regularization ($\alpha = 0.1$ to the left, $\alpha = 1$ to the right) and Wilson filter.

with parameter ω_0 through the system

$$\psi_{corr} = \frac{i}{\sigma_c}\dot{\psi} + \psi,$$

which gives us the corrective window. It allow us to reconstruct $\chi(t)$ resuming the influence of errors at the same time. Let's mention also that (5) transformed into the form

$$\psi(t) = \frac{a}{2} exp(-a|t|)(\cos(at) - \sin(a|t|))$$

with parameter a can be used as a wavelet basis.

The SSA method [Golyadina, 2004] allows to separate the time series of the polar motion into components: trend, chandler, annual oscillations (Fig. 4, left) and to exclude noise, which mostly determines the dispersion of excitation on Fig. 2. Reconstructed components of the annual and chandler excitation since 1900 year separated by SSA are represented by Fig. 4, right in comparison with smoothed data upon the earthquakes, evaluated from the USGS catalog. The annual excitation is mostly determined by the atmosphere processes [Salstein, 2000], and the excitation function for it does not correlates with seismicity. But correlation can be seen for the component of excitation function, which corresponds to the chandler motion. Probably, the cause of the chandler motion also influences on the regime of seismicity of the Earth.

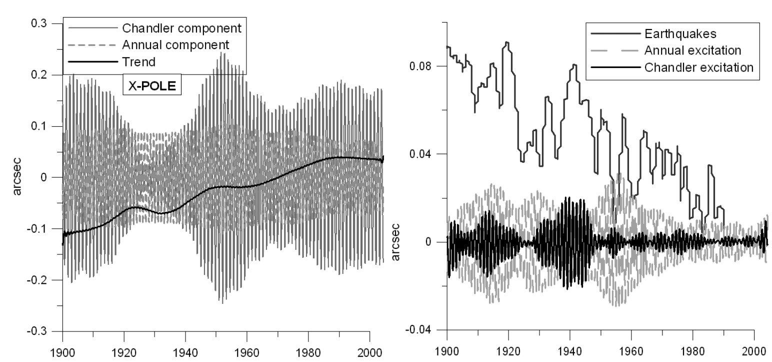

Figure 4: The components of the trend, chandler and annual oscillations of the X-coordinate of the pole picked out by SSA (to the left) and comparison of the corresponding components of the excitation function with the earthquakes (to the right).

At the next stage the forecast of excitation function with use of SSA and NN method was performed. Components freed from noise and separated from each other by SSA were predicted with use of 3-layer NN and joined together (Fig. 5, left).

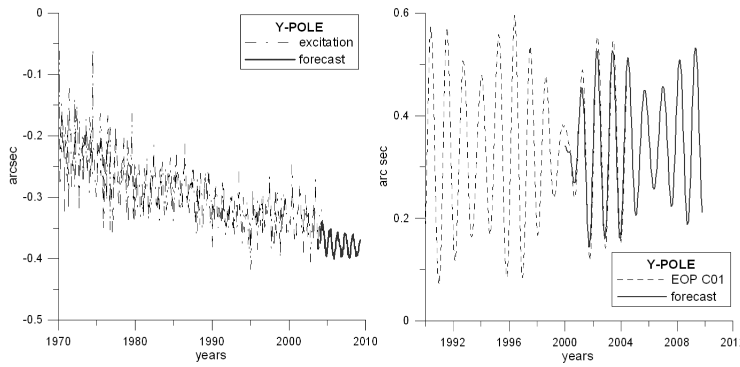

Figure 5: Forecast of the excitation by SSA and NN method (left) and prediction of the coordinates of the pole by the Kalman filter (right).

After the forecast of the excitation function $\chi(t)$ had been made, we used Kalman filter to evaluate the trajectory of the polar motion [Panteleev, 2001], [Gubanov, 1997]. The 5-year prediction is represented by Fig. 5, right. Long-time prediction for chandler component gave us possibility to suppose that it will probably relax in 2010-2020 years.

2. CONCLUSIONS

The comparison of different methods of excitation function reconstruction from the observations of the polar motion shown, that difficulties connected with this ill-posed inverse problem can be solved by using the corrective smoothing procedures. Excitation functions were reconstructed from the 1900 year for the annual and chandler components divided from each other and separated from noise with use of SSA. Their comparison with different processes can be useful for understanding the nature of the annual and chandler oscillations. It was found effective to involve singular spectral analysis, neural networks and Kalman filter for prediction of EOP.

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