ON THE APPROPRIATE SETS OF VARIABLES FOR THE RIGOROUS STUDY OF THE EARTH'S ROTATION IN THE FRAMEWORK OF IAU 2000 RESOLUTIONS

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1. VARIABLES FOR THE IAU 2000 DESCRIPTION OF THE EARTH'S ROTATION

This paper reviews the properties of different sets of canonical variables that can be used in the description of the Earth's rotational motion. The purpose is to select the most appropriate set of variables which (i) can provide a rigorous analytical solution and (ii) can be directly related to the celestial rectangular coordinates (X, Y) of the Celestial Intermediate Pole (CIP), the use of which is recommended by the IAU 2000 Resolutions on reference systems (IERS Conventions 2003). Selecting the set of variables is an important point to consider since the complexity of the differential equations as well as the method of integration to be used strongly depend on an appropriate selection of the variables. Table 1 provides the definitions of different sets of canonical variables considered in this study.

CANONICAL VARIABLES	DEFINITION
Set I: $(L_2, G_2, H_2, l_2, g_2, h_2)$ (Fukushima, 1994)	• L_2: x-component of the angular-momentum vector $(\overrightarrow{L})$ • G_2: amplitude of the angular-momentum vector • H_2: X-component of the angular-momentum vector • canonically conjugate variables to $(L_2,\ G_2,\ H_2)$
Set II: $(L_3, G_3, H_3, l_3, g_3, h_3)$ (new)	• L_3: y-component of the angular-momentum vector $(\overrightarrow{L})$ • G_3: amplitude of the angular-momentum vector • H_3: Y-component of the angular-momentum vector • canonically conjugate variables to $(L_3,\ G_3,\ H_3)$

Table 1: Sets of canonical variables for the IAU 2000 description of the Earth's rotation

The process of constructing these sets involves several canonical transformations from the initial Andoyer variables. Note that other variables can be used to establish differential equations which are appropriate for finding the semi-analytical and numerical solution of the dynamical equations of the Earth's rotation (Capitaine et al., this Volume). However, in this problem, only canonical variables can allow us to obtain a rigorous analytical solution.

2. PROPERTIES OF SETS I AND II OF CANONICAL VARIABLES

The principal characteristics and consequences of adopting the sets of variables presented in Table 1 for the IAU 2000 description of Earth's rotation are as follows:

- The canonical variables H_2 and H_3 of Table 1 are suitable variables for carrying out the rigorous analytical study of the dynamical equations of the Earth's rotation.
- These variables, H_2 and H_3, also lead to the solution of the perturbed problem, which has, at the first order, the following form that is as simple as that obtained for the classical Andoyer variables:

$$\begin{aligned} \Delta H_2 &= k' \tan h_2 \sum_{i \neq 0} \frac{Q_i}{N_i} \sin \Theta_i \\ \Delta H_3 &= k' \tan h_3 \sum_{i \neq 0} \frac{Q_i}{N_i} \sin \Theta_i \end{aligned}$$

  where Q_i is a function of the angle I between the angular equator and the fixed equator of a given epoch, the argument Θ_i is a linear function of the mean elements of the lunar, solar and planetary orbits, k' is a constant and $N_i = \frac{d\Theta_i}{dt}$.

3. DISCUSSION

The variables of Sets I and II seem to be appropriate ones for carrying out the effective study of the dynamical equations of the Earth's rotation in the framework of the IAU 2000 Resolutions, since they contain $X-$ and $Y-$ components of the angular-momentum vector, H_2 and H_3 respectively. These variables are directly related, by geometrical relations, to the rectangular celestial coordinates of the Celestial Intermediate Pole unit vector which allow us a subsequent determination of the Oppolzer terms. Moreover, the description in terms of canonical variables offers advantages related to the integration of such equations since it is possible to apply the Hori-Deprit's averaging perturbation method, which uses the concept of canonical transformations, unquestionably one of the most powerful tools in the Hamiltonian approach.

Acknowledgments. The research was carried out in the Department of *"Systèmes de Référence Temps Espace" (SYRTE)* of Observatoire de Paris and received financial support from Descartes Prize Allowance (2004-2005) (M. Folgueira), for which we express our sincere appreciation.

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