ON THE RESEARCH PROGRESS OF DESCARTES-SUBPROJECT:
"Advances in the integration of the equations of the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions"

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ABSTRACT. This paper reports on the progress of the research European DESCARTES-Subproject entitled: "Advances in the integration of the equations of the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions", describing the scientific approach, the aims and objectives of the work and its successive steps. Firstly, we give a brief overview of the role of the variables in the description of the rotational dynamics of the Earth. Then, we summarize the different mathematical methods used to carry out our investigations, which include analytical, semi-analytical and numerical approaches, in order to obtain the solution with microarcsecond accuracy.

1. INTRODUCTION

The IAU Resolutions passed in 2000 have recommended the use of the Celestial Intermediate Pole (CIP) and of the "new paradigm" to transform between the international terrestrial system (ITRS) and the geocentric celestial reference system (GCRS). The recommended form of the ITRS-to-GCRS transformation is based on the Celestial intermediate origin, CIO (originally called the Celestial ephemeris origin, CEO) to express the Earth rotation angle (ERA) instead of the classical Greenwich sidereal time, and on the ITRS and GCRS CIP coordinates. The IAU 2000A expressions for the GCRS CIP (X, Y) coordinates provided by Capitaine et al. (2003a) have been derived from the IAU 2000A expressions of Mathews et al. (2002) (denoted MHB) for the classical precession and nutation quantities. The IAU 2000A nutation expressions have been generated by the convolution of the MHB 2000 transfer function with the rigid Earth nutation series REN 2000 of Souchay et al. (1999), which are themselves solutions of the rigid Earth's dynamical equations. The IAU 2000 precession component has been derived from a fit of the precession rates in longitude and obliquity to series of Very Long Baseline Interferometry (VLBI) observations. More recently, an updated solution of the dynamical equations of the Earth's precession, denoted P03, has been obtained by Capitaine et al. (2003b) which has been used to obtain the P03 expressions for the CIP (X, Y) coordinates in a similar way.

In our work, we have used a different approach which has consisted in developing the equations for Earth rotation as function of the variables that are directly related to the parameters recommended by the IAU 2000 resolutions.

2. OBJECTIVES AND APPROACHES

The main objective of the project is to establish dynamical equations of the rigid Earth's rotation based on variables that can be *directly* related to the rectangular celestial coordinates of the CIP (X, Y) and that can be integrated in the simplest way in order to provide rigorous solutions. For this purpose, we have followed analytical, semi-analytical and numerical approaches. The comparison between the different types of solutions will allow us to evaluate the theoretical and real accuracy that can be achieved.

A further objective will be to investigate the transformation of the rigid Earth solutions to non-rigid Earth solutions and to compare the analytical and semi-analytical solutions to VLBI observations.

3. METHODOLOGY AND RESULTS

The first step of the project has been to review the sets of variables which are the most appropriate for reaching the objectives described in the previous section.

The rectangular components (ω_1, ω_2, ω_3) of the angular velocity vector along the principal axes of inertia, or alternatively the Euler angles between the figure axes and a fixed reference plane, are basic non-canonical variables (set I) that are classically used for writing the Euler dynamical equations. Expressing these variables as functions of the GCRS CIP coordinates (X, Y) will allow us to get the rotational equations of the Earth as functions of (X, Y).

Folgueira et al. (2006) have reviewed the sets of variables to be used in the Hamiltonian approach in order to get the equations in a compact form allowing us to integrate them in a simplest way and obtain analytical solutions. Two sets of canonical variables (Sets II and III) have been proposed. These variables are represented by the amplitude of the angular-momentum vector ($\overrightarrow{L}$), the X- and Y- components of this vector with respect to the inertial reference system, the x- and y-components of $\overrightarrow{L}$ with respect to the figure axes and their canonically conjugate variables.

Table 1 provides the basic variables and equations and the different techniques of integration which can be applied to solve the differential equations.

VARIABLES	BASIC EQUATIONS	METHOD OF INTEGRATION
Non-canonical: Set I	transformed Euler dynamical equations	• Variation of parameters • Runge-Kutta-Fehlberg
Canonical: Sets II and III	Hamiltonian equations	• Hori-Deprit's averaging perturbation

Table 1: Different forms of basic Earth's rotational equations and integration methods considered in this study.

In the following, we present a summary of the papers related to the different approaches carried out. Table 2 shows the methods of integration, the form of the solutions and the references corresponding to this study.

- (Folgueira et al., 2006)

  In this paper, we have reviewed the canonical and non-canonical variables for describing the rotation of the Earth and the relationships between them. Then, considering as the fixed reference plane the equator J2000.0, instead of the ecliptic of date, we have obtained the rigorous analytical solution in terms of sets II and III, to the first order.

- (Capitaine et al., 2006a)

  The equations of Earth rotation were obtained explicitly in terms of the celestial coordinates $(X,\ Y)$ of the CIP and of the Earth rotation angle (ERA), starting from Euler dynamical equations for a rigid Earth and using expressions for the components of the instantaneous rotation vector (set I) as functions of $(X,\ Y)$. Taking into account the order of magnitude of the different terms of these equations, we got the most appropriate form of the equations for a practical integration depending on the components of the external torque in the celestial intermediate system.

  We have investigated the possible methods of integration for providing semi-analytical solutions for the X and Y variables in the axially symmetric case and we have tested the efficiency of these methods.

  Finally, starting from Liouville equations, we have studied a possible generalization of the equations to the case of a elastic Earth based on integration constants compliant with the P03 precession.

- (Capitaine et al., 2006b)

  In this paper, we integrate the second order differential equations described above by the method of variation of constants. We solve the equations by successive approximations in order to get the semi-analytical solution for the rigid Earth in terms of $(X\,,Y)$. The semi-analytical expressions of the external torque used in the second members of the equations are computed from the theories ELP2000 for the Moon and VSOP87 for the Sun and planets. We compare the resulting semi-analytical solutions for $(X\,,Y)$ with the current IAU 2000 expression of $(X\,,Y)$ derived indirectly from the IAU 2000 solutions for the classical precession and nutation variables. For all the computations, we have used the software package GREGOIRE developed by J. Chapront (2003).

  More details on the two above studies are provided in Capitaine et al. (2006c).

- (Souchay et al., 2006)

  The equations of Earth rotation in terms of the $(X,\ Y)$ CIP coordinates and of the ERA are integrated by fifth-order adaptive step size Runge-Kutta-Fehlberg algorithm. Being a single step procedure, it is relatively stable and hence particularly suitable for the numerical simulation in different problems of Celestial Mechanics and Dynamical Astronomy. The numerical solution is also compared with the semi-analytical solution of Capitaine et al. (2006b). The comparison between these solutions allows us to test the accuracy of the integration method.

METHOD OF INTEGRATION	SOLUTIONS	REFERENCES
Hori-Deprit's averaging perturbation	analytical	(Folgueira et al., 2006)
Variation of parameters	semi-analytical	(Capitaine et al., 2006a,b,c)
Runge-Kutta-Fehlberg	numerical	(Souchay et al., 2006)

Table 2: Methods of integration, associated forms of solutions, and references corresponding to this study.

4. FUTURE PERSPECTIVES

The equations as function of the GCRS CIP $(X,\ Y)$ coordinates as considered in the case of an elastic Earth will be extended to the case of a deformable Earth with a liquid core. This general case will be the aim of the other DESCARTES sub-project entitled "Geophysical effects of adopting the new solutions for the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions". The geophysical parameters will be derived from the comparison of the solutions with respect to VLBI observations.

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REFERENCES

Capitaine, N., Chapront, J., Lambert, S., Wallace, P. T.: 2003a, A&A 400, 1145.
Capitaine, N., Wallace, P. T., Chapront, J.: 2003b, A&A 412, 567.
Capitaine, N., Folgueira, M., Souchay, J.: 2006a, A&A 445, 347.
Capitaine, N., Folgueira, M., Souchay, J.: 2006b, to be submitted to A&A.
Capitaine, N., Folgueira, M., Souchay, J.: 2006c, this issue.
Chapront, J., 2003, Notice, Paris Observatory (January 2003).
Folgueira, M., Souchay, J., Capitaine, N.: 2006, this issue.
Mathews, P.M., Herring, T.A., Buffett B.A., 2002, J. Geophys. Res. 107, B4, 10.1029/2001JB000390.
Souchay, J., Loysel, B., Kinoshita, H., and Folgueira, M., 1999, A&AS 135, 111.
Souchay, J., Capitaine, N., Folgueira, M.: 2006, to be submitted to A&A.