NUTATIONS AND PRECESSION OF ELASTIC EARTH IN ANGLE-ACTION VARIABLES

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1. INTRODUCTION

Since the classical works of Laplace, Tisserand or Pontecoulant till the modern solutions of the Earth rotation (Kinoshita, 1977), most of the theories of the rotation of solar system bodies assume the simplest unperturbed motion, a steady rotation around the axis of largest inertia so that the angle θ between the angular momentum and the polar axis of inertia vanishes in the zeroth order approximation. Such an assumption is accurate enough in the Earth case, which justifies the usual approaches to the Earth rotation, although it is worth to remark that allowing a non-zero value of the angle θ can bring into light new dynamical effects. As an example, we can point out the paper (Barkin, 2000) in which an initial value of the angle $\theta = 0''175$ has been used to investigate long-periodic variations in the Earth pole motion.

However, the motion of some celestial bodies does not satisfy that condition. Namely, the observed orientation of the Venus pole reveals a large value of the free obliquity amplitude $\theta = 2^\circ 1$ compared to the nominal forced amplitude of 0.5° (Yoder, 1995). The motion of irregular asteroids or spacecrafts requires the Euler-Poinsot representation even in the zeroth approximation. Therefore, to study the later problems or enhance the available solutions to the Earth, Venus or Mars rotation, it becomes necessary or useful the development of analytical perturbation theories relying on the unperturbed Euler-Poinsot motion of a triaxial rigid body, with arbitrary tensor of inertia and initial conditions. Convenient expressions for such a solution can be found in (Kinoshita, 1977; Barkin, 1992, 1998). A generalization to the case of a body weakly deformed by its own rotation was presented in the paper (Barkin et al., 1996) and we will refer to it as Euler-Chandler motion. A perturbation solution using the angle-action variables associated to the Chandler-Euler problem was developed in (Barkin, 1998). In this presentation we report on the main effects on the free Earth rotation and the solution of the forced Earth rotation based on the said Chandler-Euler unperturbed motion.

2. EULER-CHANDLER UNPERTURBED MOTION

The unperturbed rotation of a celestial body weakly deformed by its own rotation can be reduced to the classical Euler-Poinsot motion of an ideal rigid body with suitable, different moments of inertia, referred to as Euler-Chandler motion since it gathers the Chandler polar motion. The complete solution of the problem (Andoyer's variables, components of angular velocity in the body and space frames, etc.) is expressed in terms of elliptical and Θ-functions, and Fourier series in the angle-action variables. We remark the following characteristics of the solution: 1. The known difference between Euler and Chandler periods is recovered in it (304.4 and 433.2 days). 2. A new phenomenon is the appearance of a difference of eccentricities of Euler and Chandler polar trajectories (the corresponding geometrical eccentricities are 0.00328 and 0.00462). 3. Non-uniformity of Chandler motion. 4. Small variation of the Earth angular velocity with half a Chandler period. 5. Changes of the projection of the Earth's angular momentum on its polar axis. 6. Extreme values of the angles between relevant pairs of vectors are derived: angular momentum vector and polar axis of inertia of the Earth, angular momentum and angular velocity. 9. Variations of the moment of inertia of the Earth about rotation axis.

3. ANALYTICAL SOLUTION BY PERTURBATION METHODS

The theory of the perturbed rotation of a deformable Earth is approached using angle-action variables. The expression of the force function requires to perform real or complex expansions of quadratic functions of the direction cosines, that have been carried out in detail for the second order harmonic of the Earth-Moon (and Earth-Sun) interactions. First order perturbations caused by the lunisolar gravitational attraction are derived as trigonometric series whose arguments are linear combinations of the angles and the arguments of the lunar orbital motion. Their coefficients are expressed in terms of elliptical integrals and elliptical and hyperbolic functions of the initial values of the action variables and some elastic parameter k. The approach is useful to get more insight into the effects of the mantle elasticity on the Earth rotation and can be specialized to recover the nutation series derived in elastic Andoyer variables (Getino and Ferrandiz, 1991). The role of elasticity on polar motion, nutations and precession is discussed, focusing on the following: 1. Contribution of the gravitational attraction of Moon and Sun to the mean angular velocity and to Chandler period. 2. Influence of elasticity on the precession constant value. 3. Contribution of the mantle elasticity to the amplitudes of first order perturbations in angle-action variables. All the results are analytical and applicable to the study of the rotation of other solar system bodies (Venus, single or double asteroids, satellites with irregular shape, comets, etc.).

Acknowledgements. Barkin's work was supported by Spanish grant SAB2000-0235 and Program of Alicante University "Senior Professor" 2005. Partial support of Spanish projects AYA2005-8109 and ACOMP06/120 from Generalitat Valenciana is also acknowledged.

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