EARTH ROTATION BASED ON THE CELESTIAL COORDINATES OF THE CELESTIAL INTERMEDIATE POLE

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ABSTRACT. We report on the semi-analytical part of the Descartes-nutation project (see Folgueira et al. 2005, and this Volume, Session 2.3) devoted to the integration of the dynamical equations of Earth's rotation in terms of the X, Y celestial coordinates of the Celestial Intermediate Pole (CIP) and Earth Rotation Angle (ERA). We first explain how the Earth's rotational equations have been developed as functions of those variables. We then describe the integration method that has been used to get the semi-analytical solution for an axially symmetric Earth (Capitaine et al. 2006a) and we report on tests of the efficiency of the method. We finally describe how this approach has be used (Capitaine et al. 2006b) to get new series for the X, Y CIP coordinates that best represent the rigid Earth precession-nutation of the CIP equator.

1. INTRODUCTION

The series for the X, Y coordinates of the celestial intermediate pole (CIP) unit vector in the geocentric celestial reference system (GCRS) that are currently available have been derived from expressions for the IAU 2000A nutation for the classical nutation angles (Mathews et al. 2002) and either the IAU 2000 precession (Capitaine et al. 2003a and IERS Conventions 2003) or P03 precession (Capitaine et al. 2003b) for the classical precession angles.

The work described in this paper aims at obtaining the X, Y series directly as solutions of the equations for Earth rotation (Capitaine et al. 2006a and b). The first part has consisted in (i) establishing the equations in terms of X, Y, (ii) developing an integration method, (iii) testing the efficiency of the method and the accuracy of the solution and (iv) extending the approach to a non-rigid Earth. The second part has consisted in computing the rigid Earth solution using the expression for the external torque acting on the Earth based on the best currently available semi-analytical solutions for the orbital motions of the Moon, the Sun and the planets. All the semi-analytical computations performed in this work have been based on the software package GREGOIRE developed by Chapront (2003) devoted to Fourier and Poisson series manipulations.

2. THE EQUATIONS OF EARTH ROTATION AS FUNCTION OF X AND Y

The equations for Earth rotation are based on the equations for the Earth's angular momentum balance in space but require, for obtaining a rigorous solution in an appropriate way, to be written explicitly as function of the components ω_1, ω_2, ω_3 of the instantaneous rotation vector in the terrestrial system (i.e. Euler's kinematical equations) where the inertia momenta A, B and C, have the simplest formulations.

Expressing $\omega_1, \omega_2, \omega_3$ as functions of the transformation parameters between the International Terrestrial System (ITRS) and the GCRS, allows us to obtain the rigorous equations of Earth rotation in terms of X, Y and ERA ($= \theta$). This requires using the GCRS-to-ITRS transformation as recommended by IAU 2000 Resolution B1.8 (IAU Transactions 2000), based on the Celestial Intermediate Origin (CIO) (i.e. the new name recommended by the IAU NFA Working Group (2006) for the Celestial Ephemeris Origin, also called "non-rotating origin" (Guinot 1979)). As we are looking for the solution relative to the motion in space of the CIP and we are considering the axially symmetric case, the ITRS motion of the CIP can be omitted, which reduces the relationship to:

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot{\Theta} \end{pmatrix} + \frac{\mathbf{R}}{Z} \begin{pmatrix} -\dot{Y} - X\,\dot{s} \\ \dot{X} - Y\,\dot{s} \\ Z\,\dot{s} \end{pmatrix}. \tag{1}$$

where $\mathbf{R} = R_3(\Theta)$, $Z = \sqrt{1-(X^2+Y^2)}$ and $\Theta = \theta - s$, with $\dot{\theta} = \Omega$ (the mean angular velocity of the Earth) and s is the distance along the CIP equator between the point Σ and the CIO (see Fig. 1). We denote CIRS' the intermediate system defined by the CIP and the point Σ.

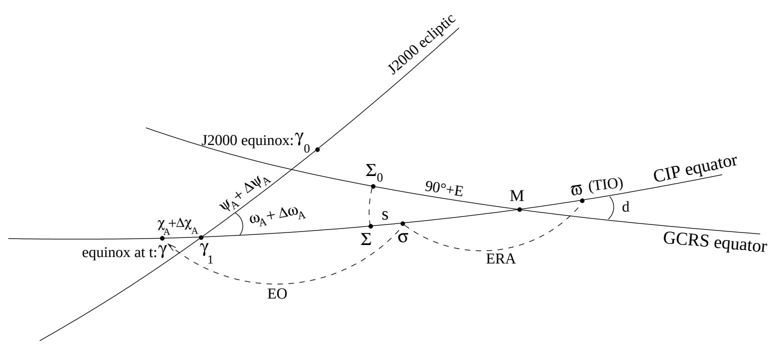

Figure 1: Relationship between various points: Σ_0 is the GCRS origin, M is the ascending node of the CIP equator on the GCRS equator, Σ is the point on the CIP equator such that $\widehat{\Sigma M} = \widehat{\Sigma_0 M}$, γ_0 is the J2000 equinox, γ is the equinox of date and γ_1 is the ascending node of the J2000 ecliptic on the CIP equator. ERA$=\widehat{\sigma\varpi}$ is the Earth rotation angle (θ), EO$=\widehat{\sigma\gamma}$ is the equation of the origins and $\widehat{\gamma_1\gamma}$ is the precession of the ecliptic along the CIP equator.

For an axially symmetric rigid Earth, an appropriate form for practical integration is:

$$\begin{aligned} -\ddot{Y} + (C/A)\,\Omega\,\dot{X} &= (L/A) + F'' \\ \ddot{X} + (C/A)\,\Omega\,\dot{Y} &= (M/A) + G'', \end{aligned} \tag{2}$$

where only the prominent terms have been retained in the first member, the other terms (F'' and G''), which are of the second order in the X and Y quantities, having been moved to the second member; L and M are the CIRS' components of the external torque.

3. METHOD FOR SOLVING THE EQUATIONS

Equations (2) can be integrated in a semi-analytical way by successive approximations. We have used the method of "variations of parameters" described in Woolard (1953) and Bretagnon et al. (1997). This consists in using the solution for the reduced system and get a particular solution of the general equations as an expression with the same form, but with the constants of integration transformed into time-dependent quantities.

The solutions of the reduced equations have the following form:

$$\begin{aligned} \dot{X} &= -K'_c \sin\sigma t + K'_s \cos\sigma t \\ \dot{Y} &= K'_s \sin\sigma t + K'_c \cos\sigma t \end{aligned} \tag{3}$$

where $\sigma = (C/A)\,\Omega$ is the Euler frequency in the CIRS' and K'_s and K'_c are the constants of integration of this free motion.

In order that the particular solution for $X\,,Y$ verifies Eqs (2), the quantities $K'_s(t)$ and $K'_c(t)$ should be derived from the following equations:

$$\begin{aligned} \dot{K}'_s &= -\frac{L}{A}\sin\sigma t + \frac{M}{A}\cos\sigma t - F''\sin\sigma t + G''\cos\sigma t \\ \dot{K}'_c &= -\frac{L}{A}\cos\sigma t - \frac{M}{A}\sin\sigma t - F''\cos\sigma t - G''\sin\sigma t\,. \end{aligned} \tag{4}$$

The solution for X and Y can thus been obtained by a quadrature of Eqs. (3) with substituting the expressions for $K'_s(t)$ and $K'_c(t)$ derived from Eqs. (4). The final solution results from an iterative process that ensures that the solution converges to the required level of accuracy.

4. TEST OF THE INTEGRATION METHOD

We have performed a number of semi-analytical simulations for testing the approach and the integration process described in the previous sections. These simulations have consisted in computing the expression of the (pseudo-) IAU 2000 torque using (i) the rotational equations for the Euler angles (Woolard 1953) and (ii) the IAU 2000A precession-nutation series for the Euler angles. The (pseudo-) torque components obtained in this way are in the intermediate system linked to the CIP and the point γ_1 (see Fig. 1). They have been transformed into components in the CIRS'. The rotational equations (2) based on those (pseudo-) torque components have then been integrated to get series for X and Y.

These simulations have shown that the integration method is efficient in providing a solution for $X\,,Y$ that converges at a 0.01 μas level after only a very few iterations. Comparison of this solution with respect to the current IAU 2000 expressions for $X\,,Y$ have shown that its accuracy is compliant with that of the current IAU 2000A precession-nutation model. This validates (i) the rotational equations in terms of $X\,,Y$ established in this work (c.f. Sect. 2) and (ii) the integration process described in Sect. 3.

5. EXTENSION OF THE APPROACH TO A DEFORMABLE EARTH

We have extended this approach to a model of a deformable Earth compliant with the P03 precession solution of Capitaine et al. (2003b) that includes the contribution of the secular variation of the dynamical ellipticity of the Earth. We have shown that the solutions for a deformable Earth can be obtained in a form similar to that for a rigid Earth with using dynamical equations expressed as functions of $X\,,Y$ that include the additional contributions from the tidal deformation of the Earth, the J_2 rate variation and the rotational deformation.

6. SOLUTION FOR A RIGID EARTH

In a further step, we have computed the X , Y solutions of the dynamical equations of Earth rotation for an axially symmetric rigid Earth model corresponding to the solutions VSOP87 for the orbital motions of the Earth and planets and ELP2000 for the Moon.

We have first developed the semi-analytical expressions for the components of the external torque in the CIRS', based on the solutions VSOP87 and ELP2000, and we have then integrated the dynamical equations in terms of X , Y (i.e. Eqs. (2)) using the method described in Sect. 3.

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