CHOICE OF THE OPTIMAL SPECTRAL ANALYSIS SCHEME FOR INVESTIGATION OF THE EARTH ROTATION PROBLEM

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ABSTRACT. The spectral analysis algorithms for the research and elaboration of numerical and semi-analytical models of the rigid Earth rotation are studied. The optimal algorithm scheme, in respect to the computational time and to the accuracy is determined. It is ALGORITHM - 1A (Figure 1), which investigates the arrays of the differences between the semi-analytical solution and the numerical one using method - A of the spectral analysis (Figure 2) in which the power spectrum is constructed only one time.

1. ALGORITHMS

In this investigation the numerical and semi-analytical models of the rigid Earth rotation are studied by means of four algorithmic schemes. These schemes include two types of the algorithms (Figure 1) and two variants of the spectral analysis methods (Figure 2).

The first type algorithm (ALGORITHM - 1) includes one of two spectral analysis schemes for processing the differences between the Numerical Solution and the Semi-analytical Solution.

The second type algorithm (ALGORITHM - 2) includes one of two spectral analysis schemes for the investigation of the Numerical Solution or the Semi-analytical Solution.

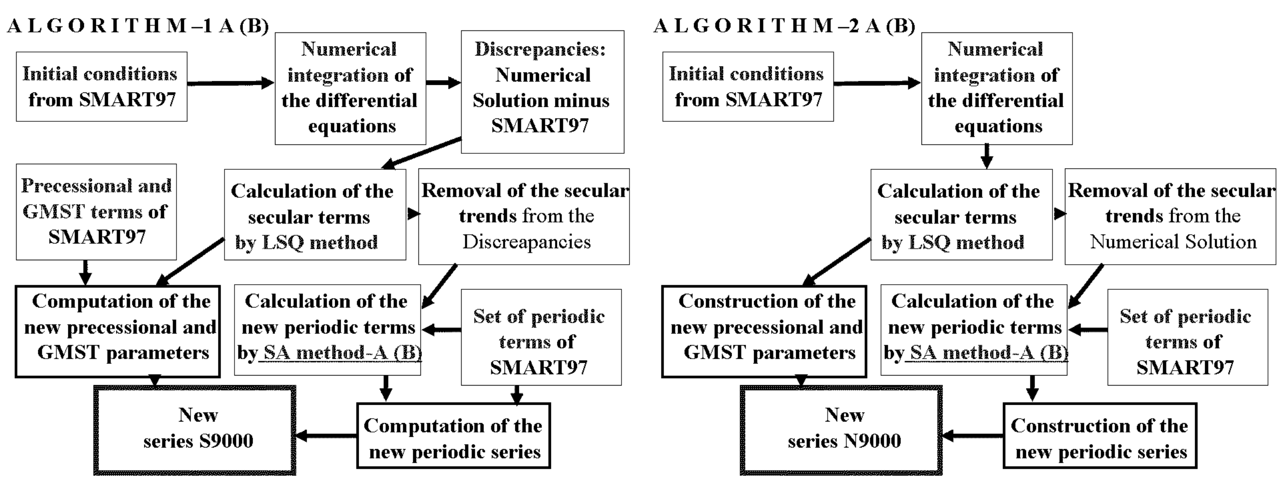

Figure 1: Two types of algorithms

The numerical solution of the rigid Earth rotation problem is the result of the quadruple precision numerical integration of the differential equations of the problem (Eroshkin *et al.*, 2004). The initial conditions are determined by SMART97 solution (Bretagnon *et al.*, 1998). The numerical solution is compared with SMART97 semi-analytical solution over the time interval of 2000 years time interval from AD 1000 to AD 3000 with one-day spacing. The arrays of the differences are constructed only for ALGORITHM - 1. For shortness, the term "ARRAYS" will be used instead of "DIFFERENCES" for ALGORITHM - 1 and "NUMERICAL SOLUTION" for ALGORITHM - 2. The investigation of ARRAYS is carried out by the least squares (LSQ) method and by the spectral analysis (SA) methods:

1. The secular parts of ARRAYS are processed by the LSQ method and are represented by the temporal polynomials of the 6-th degree.
2. New precessional and GMST polynomials are derived as the sums of the calculated secular terms and the precessional and GMST polynomials of SMART97 in the case of ALGORITHM - 1. In the case of ALGORITHM - 2 the computed secular terms are the new precessional and GMST polynomials.
3. The determined polynomials are removed from ARRAYS.
4. The periodic parts of ARRAYS are processed by one of two variants of the SA methods, A or B (Figure 2):

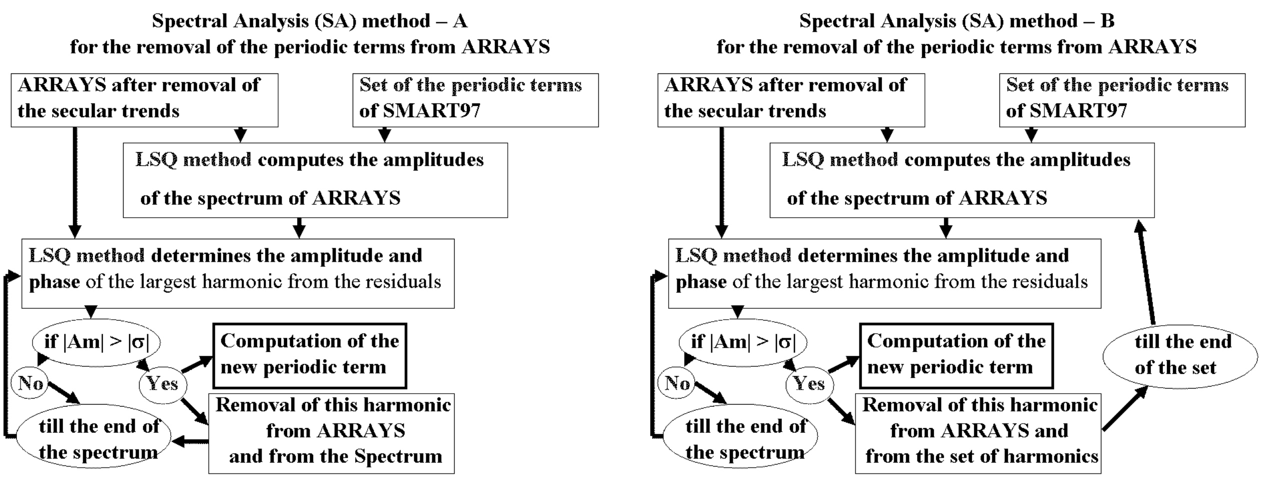

Figure 2: Two variant Spectral Analysis Methods. "Am" means the value of each amplitude of the spectrum and "σ" is the value of its root-mean-square error.

The spectrum of the periodic parts of ARRAYS for SA method - A (ALGORITHMS - 1A, 2A) is constructed one time only. The coefficient of the new periodic term equals the sum of the calculated periodic term coefficient and the coefficient of the corresponding periodic term of the SMART97 in the case of ALGORITHM - 1. In the case of ALGORITHM - 2 the periodic term coefficient is determined directly. The harmonic is removed from ARRAYS and from the spectrum. Starting from the maximum term of the spectrum the procedure is accomplished successively till its least term. The spectra represented in Figure 3 are ranged by terms with the periods from 1.0003 days to 1000 years.

The spectrum of the periodic parts of ARRAYS for SA method - B (ALGORITHMS - 1B, 2B) is constructed anew after every removal of the largest harmonic from ARRAYS. Each constructed spectrum is used for the determination of the new coefficient of the periodic term as for SA method - A. This procedure is performed to the end of the set. The spectra of the terms with periods between 6700 and 6900 days, are shown in Figure 5.

The amplitudes of this spectra are computed by the LSQ method using the argument of the periodic terms of SMART97.

5. New secular and periodic terms form the new high-precision rigid Earth rotation series S9000 in the case of ALGORITHM - 1A (Pashkevich *et al.*, 2004). In the case of ALGORITHM - 2A they are named N9000 series.

Spectra of the harmonic terms, which are close to the main nutational term are depicted in Figure 3 - ψ, θ, ϕ.

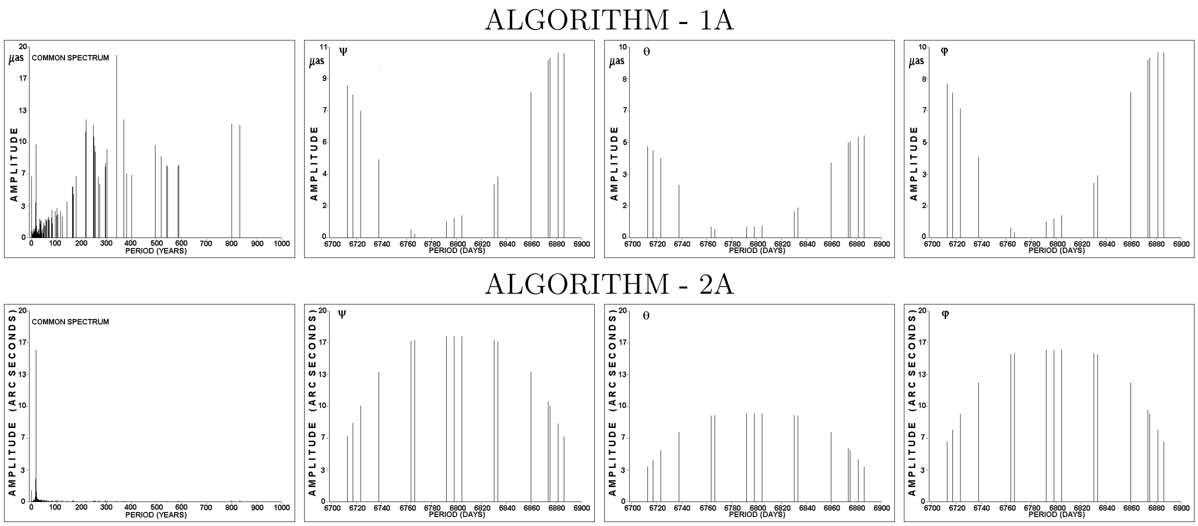

Figure 3: Common spectrum and spectra for Euler angles

Naturally, the values of the amplitudes of the spectra for ALGORITHM - 1A are essentially less than ones for ALGORITHM - 2A. The difference of the spectra for Euler angles (ψ, θ, ϕ)in any chosen algorithm is only values of the amplitudes of the harmonics. The order of removal of the harmonics are similar (Figure 3 - ψ, θ, ϕ). So, the common spectrum can be constructed only once for each of the Euler angles. It makes the process of computation shorter. In each spectrum of ALGORITHM - 2A, there is a set of the harmonics with close frequencies and close amplitudes. This fact reduces essentially the efficiency of ALGORITHM - 2A for the construction of the semi-analytical solution.

The new Numerical Solution (NS) of the problem is performed with the initial conditions determined by S9000 series and N9000 series. The differences between the new NS and S9000 series are less than the differences between the new NS and N9000 series. In the first case they do not surpass 10 μas over 2000-year time interval while in the second case they do not surpass 15 μas over the same time interval (Figure 4). Hence, ALGORITHM - 1A is more accurate than ALGORITHM - 2A.

In Figure 5 the iteration spectrum of the proper rotation angle (ϕ) for ALGORITHM - 2B is depicted for the harmonics with the frequencies close to that of the main nutational term. The amplitudes of the spectrum for ALGORITHM - 2A (Figure 3 - ϕ) are by more than 3 orders of magnitude larger than those for ALGORITHM - 2B. Therefore ALGORITHM - 2B is more accurate than ALGORITHM - 2A. It means that SA method - B is more accurate that SA method - A. The time of a construction of the common spectrum of the periodic terms of ARRAYS (Figure 3) on the computer Intel Pentium IV (2.4 GHz) amounts to 12 hours, with double precision representation of real numbers. The time increases as an exponential function of number of the harmonics.

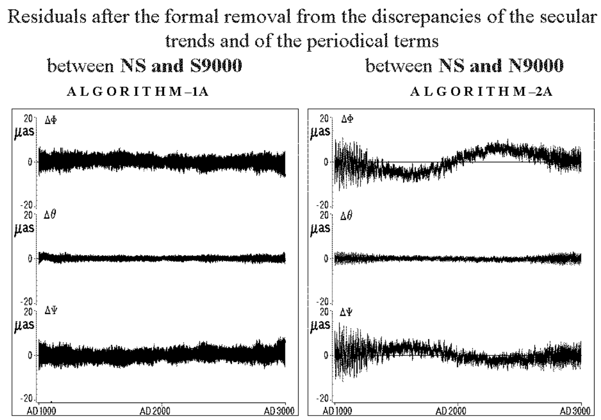

Figure 4

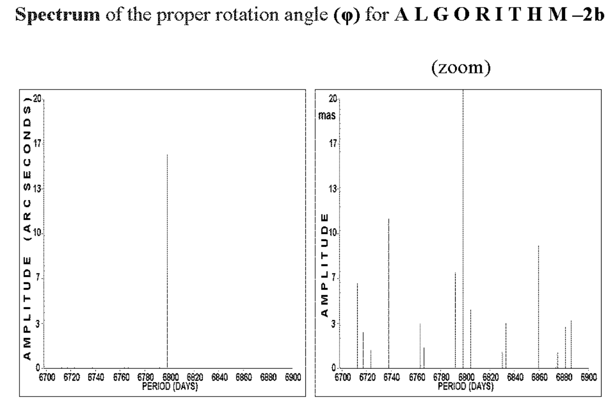

Figure 5

2. SUMMARY AND CONCLUSIONS

1. The secular and periodic terms representing the rigid Earth rotation series S9000 were determined by ALGORITHM - 1A. The differences between NS and S9000 do not surpass 10 μas over 2000 years.

2. The secular and periodic terms representing the rigid Earth rotation series N9000 were determined by ALGORITHM - 2A. The differences between NS and N9000 do not surpass 15 μas over 2000 years.

3. ALGORITHMS - 1B, 2B are very time-consuming, consequently they are not suitable for the investigation of the long time series.

It is proved that ALGORITHM - 1A is optimal with respect to the computational time and the accuracy.

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