P03-BASED PRECESSION-NUTATION TRANSFORMATIONS

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ABSTRACT. The IAU WG on precession and the ecliptic has recommended the adoption of the P03 models of Capitaine et al. (2003). We discuss methods for generating the rotation matrices that transform celestial to terrestrial coordinates, taking into account frame bias (B), P03 precession (P), P03-adjusted IAU 2000A nutation (N) and Earth rotation. The NPB portion can refer either to the equinox or the celestial intermediate origin (CIO), requiring either the Greenwich sidereal time (GST) or the Earth rotation angle (ERA) as the measure of Earth rotation. The equinox based NPB transformation can be formed using various sequences of rotations, while the CIO based transformation can be formed using series for the X, Y coordinates of the celestial intermediate pole (CIP) and for the CIO locator s; also, either matrix can be computing using series for the x, y, z components of the "rotation vector". Common to both methods is the CIP, which forms the bottom row of the transformation matrix. In the case of the CIO based transformation, the CIO is the top row of the NPB matrix, whereas in the equinox based case it enters via the GST formulation in the form of the equation of the origins (EO). The EO is the difference between ERA and GST and equivalently the distance between the CIO and equinox. The choice of method is dictated by considerations of internal consistency, flexibility and ease of use; the different ways agree at the level of a few microarcseconds over several centuries, and consume similar computing resources.

1. INTRODUCTION

At the 2003 IAU General Assembly, a working group was formed to select models for the precession of the ecliptic and equator that are consistent with dynamical theories to replace the simple rate corrections adopted in 2000 (see Hilton et al. 2005). The new precession will be used with the existing IAU 2000A nutation, and in order to be consistent with the new precession, this requires small ($\sim 10\ \mu$as) corrections for the effects of (i) the change in obliquity from the IAU 1980 ecliptic to the P03 ecliptic and (ii) the secular variation in the Earth's dynamical flattening, not taken into account in the IAU 2000A model. In this paper, based on the recent study of Capitaine & Wallace (2005), we review methods for using the new precession-nutation in practical applications.

The following matrix:

$$\mathbf{M}_{class} = \mathbf{N}\,\mathbf{P}\,\mathbf{B} \tag{1}$$

is needed in two forms, namely the classical form based on the equinox and the new form based on the celestial intermediate origin (CIO). The matrices $\mathbf{B}$, $\mathbf{P}$ and $\mathbf{N}$ are the successive contributions of the frame bias, precession and nutation. In order to predict terrestrial coordinates, or hour angles, formulations for both Greenwich sidereal time and Earth rotation angle are needed. The end-to-end transformation for both forms is between celestial and terrestrial coordinates, represented by the matrix $\mathbf{R}$ in:

$$\begin{aligned}
\mathbf{R} &= R_3(ERA) \cdot \mathbf{M}_{CIO} && (2)\\
&= R_3(ERA) \cdot R_3(-EO) \cdot \mathbf{M}_{\Upsilon} &&\\
&= R_3(GST) \cdot \mathbf{M}_{\Upsilon}. && (3)
\end{aligned}$$

The link between the two methods is the equation of the origins, EO, a quantity somewhat akin to the equation of the equinoxes but including precession as well as nutation. Note that in the equinox based version, (3), the R_3 rotation is a function both of Earth rotation and time, whereas in the CIO based version, (2), the rotation-related and time-related components are kept separate.

2. PRECESSION-NUTATION

The two types of NPB matrices, $\mathbf{M}_{CIO}$ and $\mathbf{M}_{\Upsilon}$, can be generated in a number of ways, including semi-analytical expressions for the CIP location $X(t), Y(t)$ and CIO locator $s(t)$, classical methods using precession and nutation angles, and models for the Euler axis and angle (the "r-vector"). The matrices $\mathbf{M}$ can both be expressed in terms of three Euler angles E, d, $E+\beta$, where E, d are the GCRS polar coordinates of the CIP and the angle β selects the origin of right ascension:

$$\mathbf{M} = R_3(-E-\beta) \cdot R_2(d) \cdot R_3(E), \tag{4}$$

or:

$$\mathbf{M}_{\beta} = R_3(-\beta) \cdot \mathbf{M}_{\Sigma}, \tag{5}$$

where the matrix $\mathbf{M}_{\Sigma}$ is a function of the CIP X, Y, Z:

$$\begin{aligned}
\mathbf{M}_{\Sigma} = {} & R_3(-E) \cdot R_2(d) \cdot R_3(E)\\
= {} & \begin{pmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{pmatrix},
\end{aligned} \tag{6}$$

with:

$$a = 1/(1+\cos d) = 1/(1+Z) = 1/[1+(1-X^2-Y^2)^{1/2}], \tag{7}$$

For the CIO-based matrix $\mathbf{M}_{CIO}$, $\beta = s$; for the equinox-based matrix $\mathbf{M}_{\Upsilon}$, $\beta = -EO + s$. Another convenient way of writing the $\mathbf{M}$ matrices is as three unit vectors $\mathbf{v}$:

$$\mathbf{M}_{CIO} \equiv \begin{pmatrix} \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \end{pmatrix}, \qquad \mathbf{M}_{\Upsilon} \equiv \begin{pmatrix} \mathbf{v}_{\Upsilon} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{\Upsilon} \\ \mathbf{v}_{CIP} \end{pmatrix}. \tag{8}$$

In each case the top row ($\mathbf{v}_{CIO}$ and $\mathbf{v}_{\Upsilon}$) is the the RA origin of date, namely the CIO or the equinox respectively. The bottom row is the GCRS position of the CIP, which of course is common to both formulations.

The most conservative method of forming the equinox based matrix $\mathbf{M}_{\Upsilon}$ is to provide individual rotation matrices for each of the three stages, delivering successively mean place of epoch and mean place of date. In this scheme, the precession stage can use four angles that come directly from P03, or alternatively the traditional z, ζ and θ, giving a total of either ten or nine rotations respectively. The Fukushima-Williams method (Fukushima 2003) instead condenses these into only four rotations, different uses of which deliver the full transformation or stop short at one of the earlier stages.

For the CIO based matrix, the starting-point is the CIP position and the CIO locator s. By (8), any of the above classical methods can be used to obtain X, Y simply by evaluating only the matrix elements $\mathbf{M}(3,1)$ and $\mathbf{M}(3,2)$. However, an efficient and foolproof alternative is semi-analytical series for the coordinates themselves, that deal with frame bias, precession and nutation in a single step.

A radically different approach, capable of generating both $\mathbf{M}_{\Upsilon}$ and $\mathbf{M}_{CIO}$, is to use semi-analytical series to generate the x, y and z coordinates of the “rotation vector”. This is the Euler axis unit vector scaled by the amount of rotation, from which the more familiar rotation matrix can be derived.

3. EARTH ROTATION AND THE ORIGIN OF RIGHT ASCENSION

The choice of equinox or CIO as the longitude zero affects how Earth rotation is expressed, namely as sidereal time or Earth rotation angle. These two measures are related through the equation of the origins (EO), which is the distance between the CIO and the equinox, so that $GST = ERA - EO$.

The CIO is located by the quantity s, through (5) and $\beta = s$. It can be obtained quite readily by numerical integration, but for much faster results in practical applications a series is always used. Series for s itself exist, but a much more concise result is obtained by evaluating the expression $s + XY/2$: see Table 1. Even fewer terms are needed to compute $s + XY/2 + D$, where $D = -Y_2 t^2 (X_1 t/3 + X_{nut})$, but this involves intermediate results from evaluating the X, Y series, a complication that probably outweighs the small performance gains. Comparable numbers of terms are needed to compute the periodic part of the equation of the origins, once the precession and nutation in right ascension are known.

quantity	t^0	t	t^2	t^3	t^4
s	24	125	21	2	0
$s + XY/2$	33	3	25	4	1
$s + XY/2 + D$	33	3	1	1	0
$\Delta\psi \cos\epsilon + EO$	33	1	0	0	0

Table 1: Sizes of the series of periodic terms for generating s and the EO.

4. COMPUTING CONSIDERATIONS

We have compared the different approaches for numerical consistency and the consumption of computing resources. Fig. 1 shows the residual rotation that remains after taking the product of (i) the equinox based transformation using the Fukushima-Williams NPB matrix with GST with (ii) the inverse of the transformation using series for X, Y and s with ERA. Similar levels of consistency are achieved by the r-matrix method. The costs, in both lines of code and computing time, are similar for all the methods, with perhaps a slight edge in favour of the X, Y, s approach.

The best formulation depends on the application. For a very focused application such as IERS VLBI analysis, where only the CIO based paradigm is required and the accuracy objectives are clear, direct use of series for X, Y and s is the most straightforward option. Where a collection of utility software components is to be developed, as for SOFA, a good approach is to select a set of core components – the nutation series, the Fukushima-Williams precession angles and the equation of the origins for example (see Fukushima 2004) – and use it to build the full range of products, both equinox based and CIO based. This not only minimizes the total amount of code required, but also ensures that numerical consistency depends only on rounding errors.

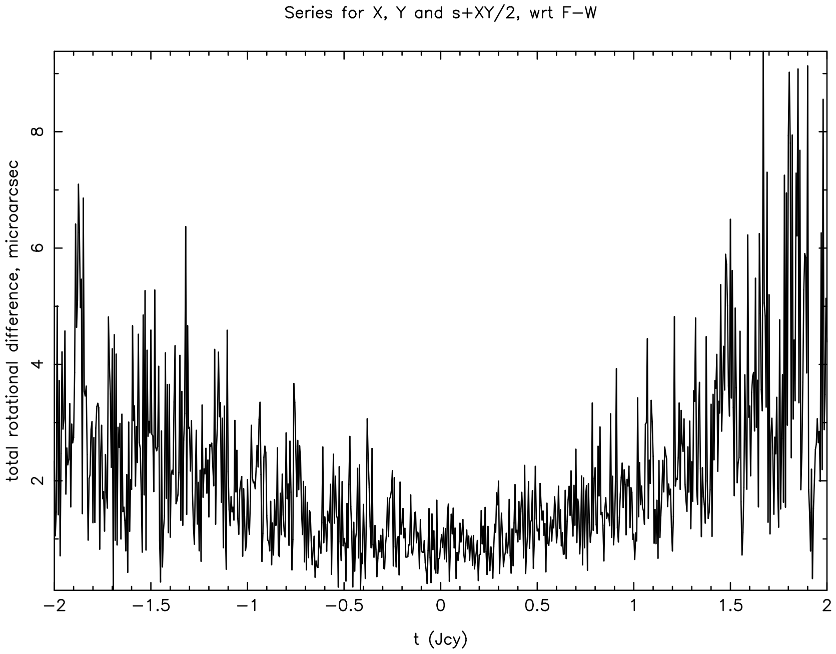

Figure 1: A numerical comparison, based on the P03/IAU 2000A precession-nutation, of two methods of transforming celestial coordinates into terrestrial: (a) series for X, Y and s, with Earth rotation angle compared with (b) precession-nutation angles and sidereal time. The total rotational difference, a product of the resolutions of the expressions used, remains within a few microarcseconds for four centuries.

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