EQUATORIAL SATELLITE DYNAMICS IN FOCK'S MODEL

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ABSTRACT. We transpose the problem in McGehee-type blow-up coordinates and fully depict the phase-space structure. We find a much more rich phase portrait than in the case of the standard Kepler problem.

1. BASIC EQUATIONS

The equatorial satellite motion in Fock's relativistic field is described (in configuration-momentum coordinates) by

$$\dot{\mathbf{q}} = \mathbf{p}, \qquad \dot{\mathbf{p}} = -\sum\nolimits_{n=1}^{4} (n a_n / |\mathbf{q}|^{n+2}) \mathbf{q},$$

where a_n have well-known expressions (Fock 1959). The equations admit the integrals of angular momentum and energy, respectively:

$$\mathbf{q} \times \mathbf{p} = L \text{ (constant)}; \qquad |\mathbf{p}|^2 /2 - \sum\nolimits_{n=1}^{4} a_n / |\mathbf{q}|^n = h/2 \text{ (constant)}.$$

Passing to standard polar coordinates (r, θ), and using the McGehee-type transformations (McGehee 1974) $x = r^2 \dot{r}$, $y = r^3 \dot{\theta}$, $ds = r^{-3} dt$, we obtain regular equations of motion (blowing up the singularity at $\mathbf{q} = (0,0)$), and the first integrals

$$y = Lr; \qquad x^2 + y^2 = hr^4 + 2 \sum\nolimits_{n=1}^{4} a_n r^{4-n}.$$

2. RESULTS

Starting from the first integrals above, we fully describe the phase portrait in the (r, x)-plane: Figure 1 for $h < 0$, Figure 2 for $h \geq 0$. The corresponding physical orbits in Figure 1 are: M/N = ejection/collision; UE/SE = unstable/stable circular orbit; 1, 2, 4 = ejection-collision orbits; 3a = ejection - unstable circular orbit; 3b = unstable circular orbit - collision; 3c = the very special

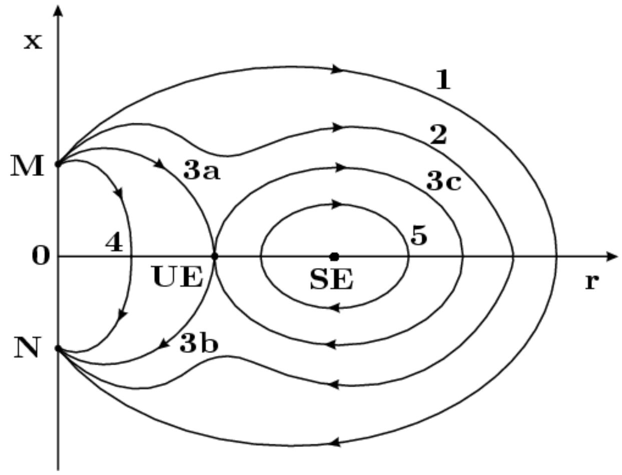

Figure 1: Phase portrait for negative energy

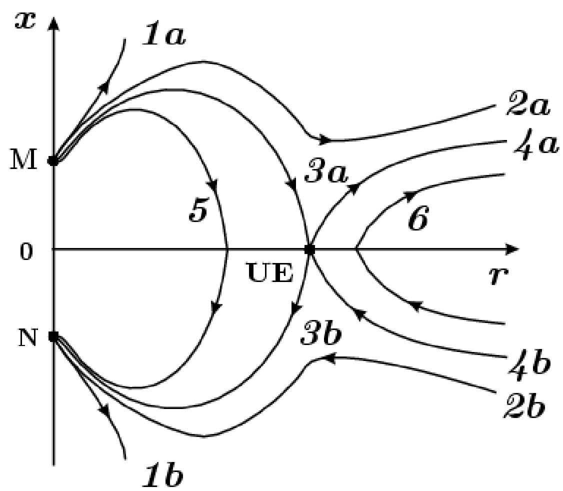

Figure 2: Phase portrait for nonnegative energy

homoclinic orbit; 5 = quasiperiodic and periodic orbits. For Figure 2: 1, 2 (a/b) = ejection-escape / capture-collision; UE, 3a, b = like in Figure 1; 4 (a/b) = UE-escape / capture-UE; 5 = ejection-collision orbits; 6 = capture-escape orbits.

Of course, for satellites, the characteristic trajectories are SE, periodic orbits, UE, quasiperiodic orbits (the two latter ones being met only in Fock's field). But, given the perturbations, any other behaviour described above becomes possible.

REFERENCES

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