

# CONTRIBUTION OF SLR RESULTS TO LLR ANALYSIS

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**ABSTRACT.** Terrestrial stations of Satellite Laser Ranging (SLR) can be used for Lunar Laser Ranging (LLR). Currently, Moon's echoes are regularly provided by the two stations : OCA (Grasse, France) and McDonald observatory (Texas, USA). The quality of the density and the distribution of SLR global tracking network associated with the sub-centimeter precision of this technique allows to determine the time series for positions of the stations in the International Terrestrial Reference Frame (ITRF). The global accuracy is better than 5 mm (Coulot et al. 2003). We show here different attempts we tried to improve the Lunar Laser Ranging residuals (O-C) by taking account of time series for the positions of Grasse and McDonald LLR stations determined by SLR technique.

## 1. INTRODUCTION

LLR data of the last decade have a sub-centimeter accuracy. Hence, to compute LLR residuals, we need to have the coordinates of the laser ranging stations in a terrestrial reference system with at least the same accuracy. The coordinates of laser ranging stations that we currently use are ITRF coordinates. Each of these ITRF coordinates are the sum of a constant value corresponding to the position of the station at J2000 and a time derivative coefficient corresponding to the tectonic plate motions. But some other local motions can also occur (seasonal motions, non predictable motions, ... ). For this reason a new ITRF representation is evolving towards time series for the coordinates of terrestrial stations together with time series of Earth's orientation parameters. The work started by GEMINI team of OCA (Coulot et al. 2003) is a first step to determine these time series by SLR technique. We try here to implement these series in our model for the computation of the LLR residuals.

First, we rapidly remind the laser ranging technique and we illustrate the LLR residuals that we got when we do not take account of the time series for the positions of the stations. Next, we use the time series of Grasse determined by SLR technique and we derive a simple model that we have to add to the classical ITRF coordinates of Grasse to reduce LLR residuals. At last, we show the consequences of this implementation on LLR residuals.

## 2. LASER RANGING TECHNIQUES

Satellite laser ranging and lunar laser ranging techniques are very similar. The principle of these techniques is based on measurements of the time propagation of a light pulse between a station on the Earth and a target which is either a satellite or one of the four retro-reflectors settled on the lunar surface by Apollo and Lunakhod missions. A laser pulse is emitted through a telescope in the direction of one of these targets and the starting time of the pulse is measured. The Moon’s retro-reflectors are corner cube arrays. A satellite which allows us to perform laser ranging measurements has also one corner cube at least on its surface. The interest of glass corner cube is to reflect the laser pulse in the precise opposite direction of its arrival. In our case, it reflects the laser pulse toward the Earth. Some of these return laser photons are detected by the same telescope and the returning time is measured. The time interval between the starting and returning times allows us to deduce the distance between the terrestrial laser station and the target. The main difference between LLR and SLR techniques is due to the distance between the Earth and the Moon which is larger than between the Earth and a satellite. For instance, Lageos is fifty times closer to the Earth than the Moon. This difference in the distances, the laser divergence and the atmosphere effects, explain that the number of returning photons from the Moon’s retro-reflectors is drastically smaller than from a satellite. It is why only few laser stations are able to shoot the Moon’s retro-reflectors.

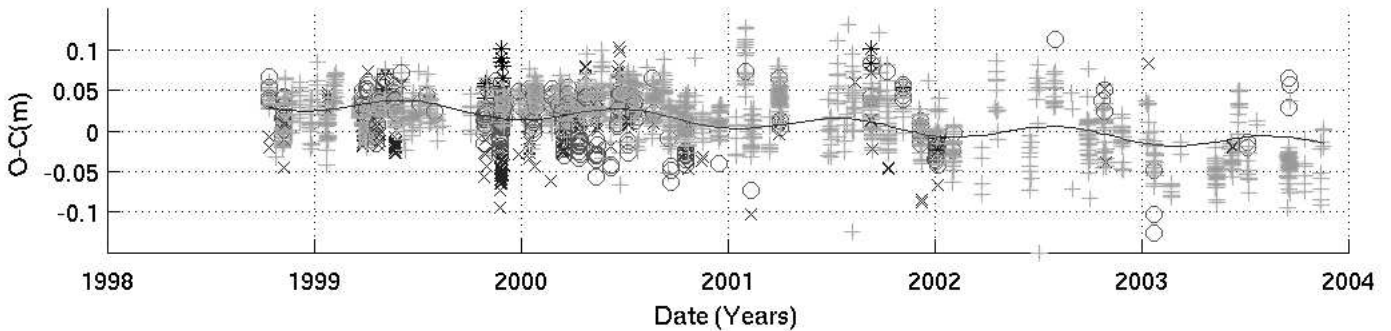


Figure 1: Lunar laser ranging O-C (Grasse station)

Figure 1 shows our LLR residuals for the period 1998-2004. The used LLR observations are data from Grasse station. The corresponding LLR residuals obtained with McDonald are not shown on Figure 1 but are very similar to Grasse’s ones. The computed part of these LLR residuals are provided by a software developed by J. Chapront, M. Chapront-Touzé and G. Francou. It is based on ELP/MPP02 ephemeris for the orbital motion of the Moon and on M. Moons’ libration model with some analytical and numerical complements for the orbital and rotational motions of the Moon. See Chapront et al. (2002) for more details. In this paragraph, the laser ranging station positions used in this modelling are usual ITRF coordinates. The “O”, “X”, “+” and “\*” marks on Figure 1 represent LLR residuals obtained with Apollo XI, Apollo XIV, Apollo XV and Lunakhod 2 retro-reflectors respectively. The line crossing on Figure 1 is a very simple model fitted to LLR residuals (O-C) derived from a weighted least squares software. Using the year as a time unit and  $t = 0$  at J2000, the resulting model in meter is :

$$(O - C)_{aj.1} = 0.02266 - 0.01028 * t - 0.00913 \cos(2\pi t) + 0.00139 \sin(2\pi t)$$

The number of LLR data, the mean values and the RMS of our LLR residuals for each retro-reflectors and for both Grasse and McDonald are given in Table 1. The total mean values are not really centred on zero because it was resulting from a fit performed on a much longer interval of time.

	Grasse station data			McDonald station data		
Retro-reflectors	Data Nb.	Mean value(m)	RMS(m)	Data Nb.	Mean value(m)	RMS(m)
Apollo XI	241	0.012	0.034	188	-0.013	0.058
Apollo XIV	165	-0.009	0.040	222	-0.001	0.048
Apollo XV	1899	0.018	0.032	1881	-0.004	0.043
Lunakhod 2	19	0.057	0.032	10	-0.018	0.022
Total	2324	0.015	0.034	188	-0.004	0.045

Table 1: Mean values and RMS of Lunar laser ranging residuals for each retro-reflector provided by Grasse laser station since 1998 and by McDonald laser station since 1994.

### 3. TIME SERIES OF GRASSE SATELLITE LASER RANGING STATION POSITIONS

Here, we do not explain how the time series for the positions of satellite laser ranging stations were determined by GEMINI team (see Coulot et al. (2003) for this) but we simply display the results for Grasse. On Figure 2, we show the X,Y and Z coordinates of Grasse station in ITRF determined by SLR technique each eight days since 1998. This date corresponds to the beginning of satellite laser ranging with Lageos at Grasse.

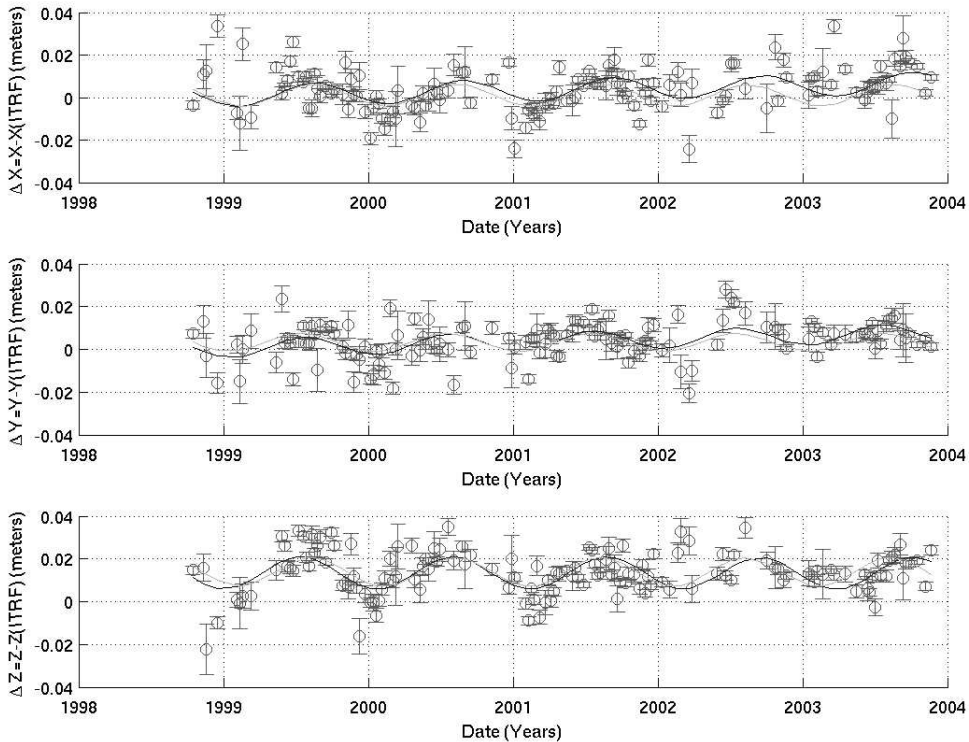


Figure 2: Time series of Grasse station positions in ITRF 2000

By a frequency analysis performed with the software named FAMOUS (Mignard, 2003). We find as a main periodic term in each time series of the station coordinates a term close to a one year period. We find, for X, Y and Z respectively, periodic terms:  $1.0324 \pm 0.0249$  years,  $0.9944 \pm 0.0251$  year and  $1.0584 \pm 0.0694$  years and with signal-to-noise ratios: 4.7, 4.0 and 4.9. The annual periodic term is probably due to seasonal variations of continental water storage. To take account of the uncertainties of each eight days determination of station positions into the model of these time series, we fit a one order polynomial in time and an annual term, with

the aid of a weighted least squares. The resulting models for each coordinates of Grasse station are :

$$\begin{aligned}\Delta X &= 0.00104 + 0.00173 * t - 0.00357 \cos(2\pi t) - 0.00353 \sin(2\pi t) \\ \Delta Y &= 0.00327 + 0.00103 * t - 0.00399 \cos(2\pi t) - 0.00100 \sin(2\pi t) \\ \Delta Z &= 0.01385 - 0.00057 * t - 0.00517 \cos(2\pi t) - 0.00415 \sin(2\pi t)\end{aligned}$$

We first add this model to the ITRF coordinates of Grasse station and we compute once again LLR residuals. This operation does not really improve RMS of LLR residuals. If we try to fit a model as in paragraph 2 to these new residuals, we get the following resulting approximation that we must compare with the previous one above:

$$(O - C)_{aj,2} = 0.02266 - 0.00836 * t - 0.01218 \cos(2\pi t) + 0.00565 \sin(2\pi t)$$

We see, that we don't have any improvement of annual term coefficients. Nevertheless, we have an improvement of the time derivative coefficient of 2 mm/year on the 7 years of this analysis.

We perform the same analysis with time series of McDonald positions. The fit is more complicated mainly because the frequency analysis of McDonald series doesn't display similar frequencies in each of its three coordinates. As for Grasse data, the RMS of McDonald lunar laser ranging residuals are not significantly improved by taking account of the time series of McDonald obtained by SLR techniques.

#### 4. CONCLUSION

The introduction of time series for the positions of SLR stations into the LLR reductions is justified by two main reasons: The first one is that since 1993 for McDonald and since 1998 for Grasse, Lageos Satellite laser ranging observations (used to get time series of station positions) and LLR observations have been done in a concomitant way with the same instrument and the same laser ranging technique. The second one is that the amplitude of variation for the positions of the laser stations are of the same order that the actual LLR residuals RMS (the annual amplitude of Grasse laser station position is about 2 cm and the Grasse LLR residuals RMS between 1998 and 2004 is around 3.4 cm).

Unfortunately, this first attempt to take into account the local variations of laser station positions do not really improve the RMS of LLR residuals. One reason could be that some larger effects (as lunar librations, retro-reflector orientations, atmosphere effects, etc ...) are not modeled with a sufficient accuracy. One another reason could be that we don't take account of geocenter motion in LLR reduction while it is taken into account in the determination of the station positions.

However, it remains an interesting result: the time derivative coefficient of LLR residuals is reduced by 2 mm/year between 1998-2004 when taking account of time series for the positions of SLR stations determined by SLR analysis.

#### REFERENCES

- Coulot, D. et al.: 2003, SF2A "Semaine de l'Astrophysique française", Eds.: Combes, F. et al., EdP-Sciences, Conference Series, p.51.  
 Chapront J., Chapront-Touzé, M., Francou G.: 2002, Astron. & Astroph., 387, p.700.  
 Mignard, F.: 2003, FAMOUS (Frequency Analysis Mapping On Unusual Sampling)