VLBI DELAY MODEL FOR RADIO SOURCES AT FINITE DISTANCE

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ABSTRACT. A relativistic VLBI delay model for radio sources at finite distance was derived as an expansion of standard VLBI delay model (consensus model). The effect of curved wave front was taken into account up to the second order of the term $(V_2(T_2 - T_1)/R)$ by solving the delay equation with Halley's method. The VLBI observation delay in Terrestrial Time is expressed with terrestrial coordinates of observation stations and with radio source coordinates in a TDB-frame. The precision of the new delay model is 1 ps for all the Earth-based VLBI observations from Earth satellites to galactic objects. In case the radio source is farther than 10 pc away, an approximated expression of the delay model provides correction terms to adapt the consensus model for finite-distance radio sources very easily.

1. INTRODUCTION

The Very Long Baseline Interferometry (VLBI) is a powerful tool of astronomy and space geodesy with the highest angular resolution. The VLBI technique has been also used for the spacecraft navigation as its engineering application (e.g. Border, 1982). The consensus model, which is currently used as the standard VLBI delay model in the world VLBI community (MacCarthy and Petit, 2003), was derived based on the plane-wave approximation by ignoring the effect of sources' distance (Eubanks, 1991). Therefore it is inaccurate if the radio sources are at finite distance, e.g. pulsars, maser sources in our galaxy, and the difference is intolerable in the solar system. An iterative scheme to obtain the VLBI delay for finite-distance radio sources were investigated Fukushima(1994). Klioner (1991) proposed VLBI delay model with analytical formula with 1 ps accuracy. Although the delay time and the baseline vectors of these models are described in the solar system Barycentric Celestial Reference Frame (BCRF), whose time like argument is TCB (Seidelmann and Fukushima, 1992). Thus user have to convert the quantity of delay time and baseline vectors between the reference frame consistent with Terrestrial Time (TT) to that in the BCRF in that model. Although the 4-dimensional transformation is not always so simple and obvious for non-specialist. Moyer (2000) provides a VLBI delay equation for spacecraft observation, which need to be solved via numerical procedure. His model is composed of a series of procedure to get time of flight by numerical iterative solution of light time equation in BCRF or GCRF. Thus the epoch in BCRF have to be converted to station time, whose time scale is TT. However, the approach taking numerical difference of two legs of ray paths from radio sources to observers is not practical for extra solar radio sources to get accuracy of 1 ps. Motivated by potential needs to analyze the VLBI observation of interplanetary missions and galactic sources, we developed a purely analytical model of VLBI delay for radio sources at finite distance expressed by TT time scale.

2. VLBI DELAY MODEL REPRESENTED IN TERRESTRIAL TIME

The new VLBI delay model for radio source as extension of consensus model is given by

$$(\mathrm{TT}_2 - \mathrm{TT}_1)_{\mathrm{Finite}} = \frac{-\left[1 - 2\frac{U_{\mathrm{E}}}{c^2} - \frac{\vec{V}_{\mathrm{E}}^2 + 2\vec{V}_{\mathrm{E}}\cdot\vec{w}_2}{2c^2}\right]\frac{\vec{K}\cdot\vec{b}}{c} - \frac{\vec{V}_{\mathrm{E}}\cdot\vec{b}}{c^2}\left[1 + \widehat{\vec{R}}_2\cdot\frac{\vec{V}_2}{c} - \frac{(\vec{V}_{\mathrm{E}} + 2\vec{w}_2)\cdot\vec{K}}{2c}\right] + \Delta T_{g,21}}{(1 + \widehat{\vec{R}}_2\cdot\frac{\vec{V}_2}{c})(1+H)}, \quad (1)$$

where the effect of curved wavefront is concentrated in the pseudo unit vector $\vec{K}$ defined by $\vec{K} = (\vec{R}_1 + \vec{R}_2)/(R_1 + R_2)$, $\vec{R}_i = \vec{X}_0(T_0) - \vec{X}_i(T_1)$, $i = 1, 2$. Notation 0,1, and 2 are radio source, station 1 and 2, respectively. Due to limited space of this article, please refer to a paper (Sekido and Fukushima 2005) for definitions of variables and detail of the derivation. For practical reasons, our model is intended to express delay time interval in Terrestrial Time (TT) with the baseline vector obtained by rotation transformation from that in Terrestrial Reference Frame (TRF), which is fixed to the earth, to the celestial reference system (CRS) as described in IERS conventions (MacCarthy and Petit, 2003). Radio source coordinates used in eqn. (1) are supposed to be represented in a planetary ephemeris. The precision of the delay is 1 ps for the Earth-based observation of any radio source in the "space", i.e. at the height above 100 km and more. The effect of curved wavefront was approximated up to the second order of $V_2(T_2 - T_1)/R_2$ by using Halley's method (Danby, 1988), where V_2 and R_2 is geometrical distance between station 2 and the radio source in BCRF.

When the distance to the radio source is more than 10 pc, 1 ps precision of delay prediction is available with the consensus model by just adding following correction terms.

$$c\left[(\mathrm{TT}_2 - \mathrm{TT}_1)_{\mathrm{Finite}} - (\mathrm{TT}_2 - \mathrm{TT}_1)_{\mathrm{IERS}}\right] = (\vec{b}\cdot\vec{p}_M)(1 - \frac{\vec{k}\cdot\vec{V}_2}{c}) - (\vec{k}\cdot\vec{b})\frac{\vec{p}_M\cdot\vec{V}_2}{c} + O(b\varepsilon^2), \quad (2)$$

where $\vec{p}_{\mathrm{M}} \equiv \{\vec{X}_M - (\vec{X}_M\cdot\vec{k})\vec{k}\}/R$, $\vec{X}_M = (\vec{X}_1(T_1) + \vec{X}_2(T_1))/2$, and R is distance to the radio source from solar system barycenter.

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