THE FREE CORE NUTATION

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ABSTRACT. The International Earth rotation and Reference system Service (IERS) provides observational determinations of the celestial pole offsets that describe quantitatively the difference between the observed direction of the Celestial Intermediate Pole in the celestial reference frame and the direction predicted by the conventional precession-nutation model. The free core nutation is the most significant component of that time series. This motion is due to the fact that the rotation axes of the core and mantle are not aligned, and it is seen in the observations as a periodic variation with a period of 432 days with time-variable amplitude and phase. The IERS is tasked with providing a numerical model for this motion. The current status of the free-core nutation models is reviewed and their accuracy is assessed.

1. INTRODUCTION

Beginning on 1 January 2003, the IAU 2000 Precession-Nutation model and associated celestial pole offsets replaced the IAU 1976 Precession and IAU 1980 Nutation as the IAU-recommended model to be used in the transformation between celestial and terrestrial reference systems. Information regarding these procedures is available in Chapter 5 of the *IERS Conventions 2003* (McCarthy and Petit 2004) available at http://maia.usno.navy.mil/conv2003.html. The nutation in longitude and obliquity is based on the adopted MHB2000 (Mathews, *et al.* 2002) model with the exception of the Free Core Nutation (FCN). Data to model the FCN are to be provided by the International Earth rotation and Reference system Service (IERS).

The IAU 2000 model can be implemented through the use of the IAU2000A model, which is available at ftp://maia.usno.navy.mil/conv2000/chapter5/IAU2000A.f. For those users requiring precision at only the milliarcsecond level, the IAU2000B subroutine is available at ftp://maia.usno.navy.mil/conv2000/chapter5/IAU2000B.f. The IAU has also recommended that VLBI observations be continued in order to improve future nutation models and to measure the unpredictable FCN.

Beginning on 1 January 2003, IERS Bulletins A and B have provided new products in addition to those previously published. These products include the celestial pole offsets $X_{obs} - X_{IAU2000A}$ and $Y_{obs} - Y_{IAU2000A}$ where X_{obs} and Y_{obs} are the observed X, Y coordinates of the Celestial Intermediate Pole in the Geocentric Celestial Reference System, and $X_{IAU2000A}$ and $Y_{IAU2000A}$ are the celestial pole coordinates provided by using the IAU 2000A Precession-Nutation theory. It is expected that celestial pole offsets related to the IAU 1980 Nutation Theory will continue to be published indefinitely. The celestial pole offsets related to the IAU 2000A Precession-Nutation theory are in a separate data file.

The IAU 2000 precession-nutation very adequately models the precession-nutation as observed by very long baseline interferometry, accounting for nearly all of the signal in the celestial pole offsets with respect to the IAU 1980 nutation series. As a result, the celestial pole offsets with respect to the IAU 2000 Precession-Nutation are very much smaller than those calculated with respect to the IAU 1980 mode. The FCN, however, is not accounted for by a model in either the IAU 1980 or the IAU 2000 models. For those users who might want to model the FCN, the IERS currently recommends a model that provides corrections to the expression for the longitude of the equinox and the obliquity (dψ and dϵ).

The relationship between variations in the X, Y coordinates of the Celestial Intermediate Pole in the Geocentric Celestial Reference System and dψ and dϵ are given by

$$\begin{aligned} dX &= d\psi \sin \epsilon_A + (\psi_A \cos \epsilon_0 - \chi_A) d\epsilon, \\ dY &= d\epsilon (\psi_A \cos \epsilon_0 - \chi_A) d\psi \sin \epsilon_A, \end{aligned}$$

where ϵ_A is the obliquity referred to the ecliptic of date, ψ_A is the precession in longitude (Lieske, *et al.* 1977) referred to the ecliptic of epoch, and χ_a is the precession quantity for planetary precession along the equator (Lieske, *et al.* 1977).

2. PHYSICAL NATURE OF THE FCN

The FCN is the most significant periodic component seen in the observations of the celestial pole offsets and is due to the free nutation of the Earth's core. This motion is the result of the fact that the rotation axes of the core and mantle of the Earth are not precisely aligned (Brzezinski and Petrov 1999). In the celestial frame it appears as a retrograde motion of the celestial pole with a period of approximately 430 days (Roosbeek, *et al.* 1999). The exact nature of the FCN is influenced by the flattening of the core-mantle boundary and by any deformations on the surface of that boundary (Mathews, *et al.* 1991). In the terrestrial reference frame this phenomenon would be seen as a diurnal motion, and it may be referred to as the Nearly Diurnal Free Wobble.

3. CURRENT IERS MODEL

The model for the FCN currently provided by the IERS on its web site (see http://www.iers.org/iers/earth/rotation/precnut/table1.html) is that found in the *IERS Conventions (1996)* (McCarthy, 1996) based on the KSV 1996 3 nutation series provided by T. Herring. That model provides corrections to the expression for the longitude of the equinox and the obliquity (dψ and dϵ) calculated using the IAU 1980 nutation model in the form

$$\begin{aligned} d\psi_{FCN} &= a_\psi \sin \nu + b_\psi \cos \nu, \\ d\epsilon_{FCN} &= a_\epsilon \sin \nu + b_\epsilon \cos \nu, \\ \nu &= -2\pi/P \times (JD - 2451545.0), \end{aligned}$$

where a_ψ, b_ψ, a_ϵ, and b_ϵ are tabular values for the epoch of interest, and P= period of the FCN = 433 days.

4. MHB 2000 MODEL

The MHB (Mathews, *et al.* 2002) model provides corrections to the expression for the longitude of the equinox and the obliquity (dψ and dϵ) calculated using the IAU 1980 model in the form

Beginning JD	Year	Month	Day	a	b
2443874.5	1979	1	1	-0.0620	-0.1346
2445700.5	1984	1	1	0.0447	-0.1679
2446431.5	1986	1	1	0.2406	-0.2759
2447161.5	1988	1	1	0.1183	-0.2163
2447892.5	1990	1	1	0.0479	-0.1965
2448622.5	1992	1	1	-0.0796	-0.1321
2449353.5	1994	1	1	-0.0075	-0.1150
2450083.5	1996	1	1	-0.0128	-0.0998
2450814.5	1998	1	1	-0.0263	-0.1122
2451544.5	2000	1	1	0.0519	0.0081
2452061.5	2001	6	1	0.2100	0.1401

Table 1: MHB2000 model parameters.

$$\begin{aligned} d\psi_{FCN} \sin\epsilon &= -A\sin\nu + B\cos\nu, \\ d\epsilon_{FCN} &= -A\cos\nu - B\sin\nu, \\ \nu &= -2\pi(1+f_{FCN})r(JD - 2451545.0) \end{aligned}$$

where A and B are linearly interpolated values for the epoch of interest derived from tabular values shown in Table 1, r = solar to sidereal ratio (1.002737909), and f_{FCN} is the frequency of the FCN (-1.002318109 sidereal days^{-1}).

5. MALKIN MODEL

Malkin (2004) has published another model in the form

$$\begin{aligned} dX &= A(t)\sin[\phi(t-t_0)], \\ dY &= A(t)\cos[\phi(t-t_0)], \end{aligned}$$

where

$$\phi(t) = \int_{t_0}^{t} \frac{2\pi}{P(t)} dt + \phi_0.$$

Numerical values for the model components were not published but a visual comparison of the model with observations is provided in the publication.

6. LAMBERT MODEL

S. Lambert (2004) has also provided a FCN model based on an analysis of the VLBI observations. This model is given in the form

$$\begin{aligned} dX &= A\sin 2\pi\sigma(t-t_0) + B\cos 2\pi\sigma(t-t_0), \\ dY &= -B\sin 2\pi\sigma(t-t_0) + A\cos 2\pi\sigma(t-t_0), \end{aligned}$$

where A and B are linearly interpolated values for the epoch of interest derived from tabular values of a and b, $\sigma = 1/430.21$ days, t is the date (MJD), and $t_0 = 51544.5$. Table 2 provides the numerical values needed to calculate A and B.

Year	a	b
1984	-0.0179	-0.0809
1985	-0.1569	-0.2253
1986	-0.1650	-0.2143
1987	-0.1104	-0.2214
1988	-0.0357	-0.2687
1989	-0.0899	-0.1557
1990	0.0084	-0.1593
1991	0.0418	-0.1277
1992	0.0331	-0.1196
1993	0.0529	-0.0973
1994	0.0154	-0.0692
1995	-0.0189	-0.0748
1996	0.0367	-0.1137
1997	0.0801	-0.0983
1998	0.0249	-0.0622
1999	-0.0595	-0.0442
2000	-0.1043	-0.0017
2001	-0.2024	0.0787
2002	-0.0785	0.1092
2003	0.0036	0.1302

Table 2: Lambert model parameters.

	DX	DY
IERS Web Site	0.000329	0.000296
MHB 2000	0.000253	0.000294
Lambert	0.000247	0.000292

Table 3: Standard deviation of the residuals of FCN Models with respect to observations from January 1984 through August 2004. Entries are standard deviations in seconds of arc.

7. COMPARISON OF MODELS

The three models that provide numerical values for evaluation were compared with VLBI observations of the celestial pole offsets. Figures 1 and 2 show the residuals of each model with respect to the observations. Table 3 provides a comparison of the standard deviations of the differences between the model and the observations. This comparison shows that the Lambert model provides the smallest residuals.

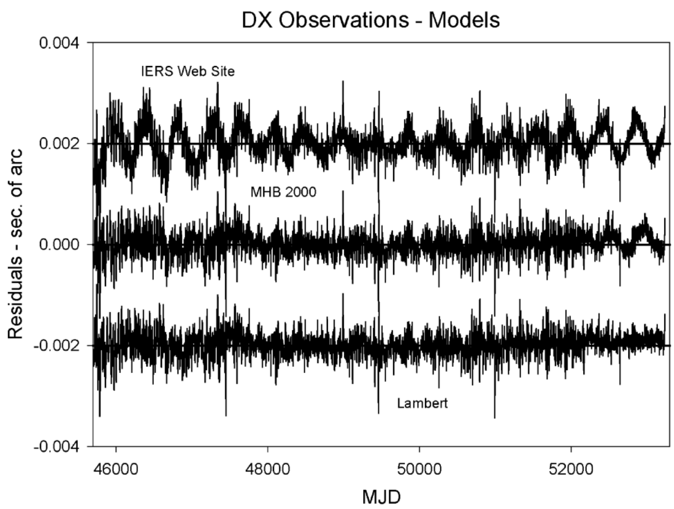

Figure 1: Comparison of FCN models with observations of dX.

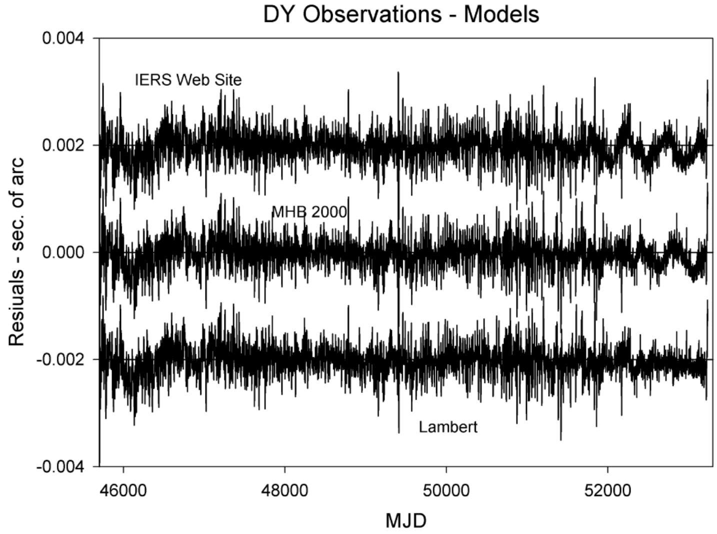

Figure 2: Comparison of FCN models with observations of dY.

8. CONCLUSION

Users that require the most accurate data to transform between the celestial and terrestrial reference systems can make use of the observed values of the celestial pole offsets provided by the IERS. The most significant component of the celestial pole offsets that remains unmodeled by the IAU 2000 precession-nutation model is the FCN. This free motion can be modeled empirically, and the IERS provides a conventional model for this purpose. This conventional model can be improved significantly with the adoption of the Lambert (2004) model.

9. REFERENCES

Brzezinski, A. and Petrov, S. D., 1999, "Observational evidence of the free core nutation and its geophysical excitation," in *Proceedings of the Journées 1998 Systèmes de Référence Spatio-Temporels 1998*, (Paris, Sept. 1998), N. Capitaine (ed.), Observatoire de Paris, 169.

Lambert, S., 2004, private communication.

Lieske, J. H., Lederle, T., Fricke, W., and Morando, B., 1977, "Expressions for the Precession Quantities Based upon the IAU(1976) System of Astronomical Constants," A&A , 58, 1.

Malkin, Z., 2004, "A New Free Core Nutation Model with Variable Amplitude and Period," in *IVS 2004 General Meeting Proc.*, 388-392.

Mathews, P. M., B. A. Buffett, T. A. Herring, and I. I. Shapiro, 1991, "Forced nutations of the Earth: Influence of the inner core dynamics, 1, Theory," J. Geophys. Res. , 96, 8219.

Mathews, P. M., Herring, T. A., and Buffett B. A., 2002, "Modeling of nutation-precession: New nutation series for nonrigid Earth, and insights into the Earth's Interior," J. Geophys. Res. , 107, B4, 10.1029/2001JB000390.

McCarthy, D. D., 1996, *IERS Conventions (1996)*, IERS Technical Note 21, Observatoire de Paris, Paris.

McCarthy, D. D., Petit, G. 2004, *IERS Conventions (2003)*, IERS Technical Note 32, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am main.

Roosbeek, F., Defraigne, P., Feissel, M., and Dehant, V, 1999, "The free core nutation period stays between 431 and 434 sidereal days", Geophys. Res. Lett.,26, 131.