SECOND-ORDER TERMS IN THE EARTH'S NUTATION

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ABSTRACT. Given the accuracy of the observations, the effects of the second-order terms in the equations of the Earth's rotation have now to be considered. The effects of the zonal deformations have been considered by several authors (Souchay & Folgueira 1999, Mathews et al. 2002, Lambert & Capitaine 2004) but they are only one effect among several second-order effects. This study investigates these second-order terms. The computation yields an almost complete cancellation of the different contributions so that the net effect of the second-order terms is no more than a few tens of microarcseconds.

1. DYNAMICAL EQUATIONS AND SECOND-ORDER TORQUE

The motion of the rotation axis $\vec{\omega} = \Omega(m_1, m_2, 1 + m_3)$, of a stratified Earth with fluid core is given by the angular momentum conservation law, written in the rotating frame :

$$\begin{aligned} \dot{m} - i\sigma_{\rm r} m + \frac{\dot{c} + i\Omega c}{A} + \frac{A_{\rm f}}{A}(\dot{m}_{\rm f} + i\Omega m_{\rm f}) &= \frac{\Gamma}{A\Omega} \\ \dot{m} + \dot{m}_{\rm f} - i\sigma_{\rm f} m_{\rm f} + i\Omega m_{\rm f} + \frac{\dot{c}_{\rm f}}{A_{\rm f}} &= 0 \end{aligned} \tag{5}$$

with $m = m_1 + im_2$, $c = c_{13} + ic_{23}$, A is the equatorial mean moment of inertia and σ_r the Euler frequency. Quantities relevant to the core are subscripted by f. The lunisolar potential is :

$$V_{lm} = \frac{1}{3}\Omega^2 r^2 \mathrm{Re}(\phi_{lm} Y_{lm}) \tag{6}$$

where $Y_{lm} = P_{lm}(\cos\theta)\exp(-im\lambda)$ for any point within the Earth at geocentric distance r, terrestrial longitude λ and colatitude θ. The second-order torque on the Earth can be written, using the definition of the moments of inertia c_{ij} :

$$\begin{aligned} \Gamma_{20} &= -i\Omega^2\phi_{20}c \\ \Gamma_{21} &= \Omega^2\phi^\star_{21}c_{12} + i\Omega^2\phi^{\rm re}_{21}(c_{33} - c_{11}) - \Omega^2\phi^{\rm im}_{21}(c_{33} - c_{22}) \\ \Gamma_{22} &= 2i\Omega^2\phi_{22}c^\star \end{aligned} \tag{7}$$

The deformability of the Earth is taken into account using McCullagh's theorem. One gets :

$$\begin{aligned} c_{11}+c_{22}-2c_{33} &= -2\kappa A\phi_{20} \\ c_{13}+ic_{23} &= \kappa A\phi_{21} \\ c_{22}-c_{11}+ic_{12} &= -4\kappa A\phi_{22}^{\mathrm{re}}+2i\kappa A\phi_{22}^{\mathrm{im}} \\ c_{11}+c_{22}+c_{33} &= 0 \end{aligned} \tag{8}$$

where $\kappa = ka^5\Omega^2/3GA$ is the secular Love number, a is the mean equatorial radius of the Earth and k is the static Love number. The zonal deformations are taken into accound accurately through their effects on the length-of-day (see Lambert & Capitaine 2004 for this part of the computation). The Love number k is obviously depending upon the frequency and includes a small imaginary part accounting for the anelasticity.

Introducing these relationships into (7), one gets, for an homogeneous Earth :

$$\begin{aligned} \Gamma_{20} &= iA\Omega^2\kappa\phi_{20}m - iA\Omega^2\kappa\phi_{20}\phi_{21} \\ \Gamma_{21} &= iA\Omega^2\kappa\phi_{20}\phi_{21} - 2iA\Omega^2\kappa\phi_{22}\phi_{21}^{\star} \\ \Gamma_{22} &= -2iA\Omega^2\kappa\phi_{22}m^{\star} + 2iA\Omega^2\kappa\phi_{22}\phi_{21}^{\star} \end{aligned} \tag{9}$$

A numerical evaluation shows that the terms containing quantities m and m_{f} are very small (order of magnitude of 1 μas). The remaining terms cancel out so that the net effect is close to zero (not completely zero because of the frequency dependence of κ). For an anelastic Earth with fluid core, one has, with $C_{\mathrm{eff}} = C_{\mathrm{m}}/(1-\gamma C_{\mathrm{f}}/\kappa C)$:

$$\begin{aligned} \Gamma_{20} &= iA\Omega^2\kappa\phi_{20}m - iA\Omega^2\kappa\phi_{20}\phi_{21} + iA\Omega^2\xi\phi_{20}m_{\mathrm{f}} \\ \Gamma_{21} &= -\frac{3}{2}i\Omega^2 C_{\mathrm{eff}} m_3^{z}\phi_{21} - 2iA\Omega^2\kappa\phi_{22}\phi_{21}^{\star} \\ \Gamma_{22} &= -2iA\Omega^2\kappa\phi_{22}m^{\star} + 2iA\Omega^2\kappa\phi_{22}\phi_{21}^{\star} - 2iA\Omega^2\xi\phi_{22}m_{\mathrm{f}}^{\star} \end{aligned} \tag{10}$$

2. RESULTS

The zonal tides effects on the nutation investigated in previous studies (Souchay & Folgueira 1999, Mathews et al. 2002, Lambert & Capitaine 2004) are not the only second-order effects. Mathews (2003) pointed out that reciprocal effects should cancel out one each other so that the net effect is considerably lowered.

Indeed, in this study, we establish that the remainder is due to the rotational effects, to the frequency dependence of κ, to the anelasticity and to the fluid core. The only effect above 1 μas is on the 18.6-y nutation (37 μas in longitude and -1 μas in obliquity). The precession in longitude is changed by -518 μas/c.

This computation was revised after discussions which occured during the meeting with A. Escapa. The sign of one contribution was erroneous, leading to wrong values of the final amplitudes. These results agree with those of Mathews (2003) and Escapa et al. (2004), the latter work using a Hamiltonian approach.

3. REFERENCES

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