THE EQUATIONS OF THE EARTH'S ROTATION IN THE FRAMEWORK OF THE IAU 2000 RESOLUTIONS

M. FOLGUEIRA[1,2] N. CAPITAINE[2], J. SOUCHAY[2]

[1] Instituto de Astronomía y Geodesia (UCM-CSIC),
Facultad de CC. Matemáticas. Universidad Complutense de Madrid.
Plaza de Ciencias, 3. Ciudad Universitaria. 28040 Madrid. Spain.
E-mail: martafl@mat.ucm.es

[2] SYRTE/UMR8630-CNRS, Observatoire de Paris
61 avenue de l'Observatoire, 75014 Paris. France.
E-mail: capitain@danof.obspm.fr; Jean.Souchay@obspm.fr

ABSTRACT. This paper describes the objectives of the European DESCARTES sub-project entitled "Advances in the integration of the equations of the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions" and reports on preliminary results.

1. THE EUROPEAN DESCARTES SUB-PROJECT

For many applications in Celestial Mechanics and Astrometry, the x, y-coordinates (denoted X, Y) of the Celestial Intermediate Pole (CIP) unit vector in the Geocentric Celestial Reference System (GCRS), are extremely useful. They are for example the parameters to which the VLBI observations are directly sensitive. It is therefore important to obtain the equations of the Earth's rotation as function of these parameters. This would allow us to provide a precession-nutation model directly in the form recommended by the IAU 2000 Resolutions. With these considerations in mind, the European DESCARTES sub-project entitled: "Advances in the integration of the equations of the Earth's rotation in the framework of the new parameters adopted by the IAU 2000 Resolutions" has been undertaken.

The main objectives of the project can be summarized as follows:

1. discussing the most suitable sets of variables to be used for integrating the equations of the Earth rotation,
2. obtening the equations of the Earth rotation explicitly expressed with the Earth Orientation Parameters (EOP) that are recommended by the IAU 2000 Resolutions,
3. integrating these equations in order to provide the results with an accuracy compliant with that required by the IAU 2000 Resolutions,
4. comparing the new solutions for X and Y with those derived indirectly from the IAU 2000 solutions for the classical $\Delta\psi$ and $\Delta\varepsilon$ variables,
5. transforming the solution for a rigid Earth to the non-rigid Earth,
6. comparing the solution obtained analytically with the observed parameters.

2. VARIATIONAL EQUATIONS IN TERMS OF THE COORDINATES (X, Y) OF THE CIP IN THE GCRS

The differential equations of the rotational motion of a rigid Earth in the framework of the IAU 2000 Resolutions have been derived from a direct application of the Euler's dynamical equations. The developments have led to equations which can be written in a compact form as:

$$\begin{cases} F_1\,\ddot{X} + F_2\,\ddot{Y} + F_3\,\dot{X}^2 + F_4\,\dot{Y}^2 + F_5\,\dot{X}\dot{Y} + F_6\,\dot{X} + F_7\,\dot{Y} = \frac{L}{\bar{A}} \\ G_1\,\ddot{X} + G_2\,\ddot{Y} + G_3\,\dot{X}^2 + G_4\,\dot{Y}^2 + G_5\,\dot{X}\dot{Y} + G_6\,\dot{X} + G_7\,\dot{Y} = \frac{M}{\bar{A}} \end{cases} \tag{1}$$

where F_i and G_i are functions of X and Y, L and M are the components of the external torque in the GCRS and $\bar{A}$ is for $\frac{A+B}{2}$, A and B being the principal moments of inertia of the rigid Earth.

From a numerical evaluation of the order of magnitude of the terms in (1), it can be seen that the prominent terms are $F_2\,\ddot{Y}$ and $F_6\,\dot{X}$ in the first equation and $G_1\,\ddot{X}$ and $G_7\,\dot{Y}$ in the second one. The expressions of F_2, F_6, G_1 and G_7 as functions of the X and Y variables, are:

$$\begin{aligned} F_2 &= -1 - \frac{1}{2}Y^2 - \frac{3}{8}X^2Y^2 \\ F_6 &= \frac{C\Omega}{\bar{A}}\left\{1 + \frac{1}{2}X^2 + \frac{3}{8}X^4 + \frac{3}{8}X^2Y^2\right\} \\ G_1 &= 1 + \frac{1}{2}X^2 + \frac{3}{8}X^4 + \frac{3}{8}X^2Y^2 \\ G_7 &= \frac{C\Omega}{\bar{A}}\left\{1 + \frac{1}{2}Y^2 + \frac{3}{8}X^2Y^2\right\} \end{aligned} \tag{2}$$

The variational equations (1) can then be integrated by either an analytical or a numerical method in order to obtain the solutions for X and Y in the absence of external forces. These solutions will be used as approximations for the study of the perturbed problem in terms of X and Y, which is under development in the frame of this project.

Acknowledgments. The research was carried out in the Department SYRTE (*Systèmes de Référence Temps Espace*) at Observatoire de Paris and received a financial support from Descartes Prize Allowance (2004) (M. Folgueira), for which we express our sincere appreciation.

3. REFERENCES

Capitaine, N., Guinot, B. and Souchay, J., 1986, Celest. Mech. 39, 283–307.

Capitaine, N., 1990, Celest. Mech. 48, 127–143.

IERS Conventions (2003), D.D. McCarthy & G. Petit (eds.), (IERS Technical Note 32), Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004. 127 pp.