

# BASIS FOR A NATIVE RELATIVISTIC SOFTWARE INTEGRATING THE MOTION OF SATELLITES

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## ABSTRACT.

We introduce here a relativistically consistent software called RMI (Relativistic Motion Integrator). We compare it with the GINS software as an example of the classical approach consisting in the Newtonian formalism plus relativistic corrections.

## 1. THE CLASSICAL APPROACH: GINS

GINS [1] is a software routinely used to evaluate gravity potential models (GRIM4, GRIM5, EIGENS 1-2) from the determination of orbit perturbations, or for precise orbit determinations around the Earth (CHAMP, GRACE, JASON...) and around other solar system bodies, e.g. Mars (MGS...). It is based upon the usual formalism used in space geodesy, i.e. it relies upon the classical Newtonian description of motion, to which relativistic corrections are added. The number and type of corrections needed depend upon the precision in the measurements (clock precision and stability). The relativistic corrections on the forces already taken into account in GINS are: the Schwarzschild effect, function of the position and speed of the satellite; the Lens-Thirring effect, due to the rotation of the attracting body; the geodetic precession, function of the chosen coordinate system and due to the non inertial motion of the gravitational source in the solar system. Corrections are also applied on the measurements: a time tag correction, which stems from the transformation between the Universal Time Coordinate and the International Atomic Time, or the time referred to in the ephemeris; a relativistic time delay correction, due to the curvature of space-time. And finally, a correction on the clock frequency, owing to the presence of a gravitational field, is considered, leading to a relativistic Doppler effect. In relativistic theories, these post-Newtonian measurements corrections are a natural consequence of the distinction between proper time and coordinate time.

The classical "Newton + relativistic corrections" method briefly described here faces three major problems. First of all, it ignores that in General Relativity time and space are intimately related. Secondly, a (complete) review of all the corrections is needed in case of a change in conventions (adopted underlying metric), or if precision is gained in measurements. Thirdly, with such a method, one correction can sometimes be counted twice (for example, the reference frequency provided by the GPS satellites is already corrected for the main relativistic effect), if not forgotten. For those reasons, a new approach was suggested.

## 2. THE RELATIVISTIC APPROACH: RMI

In the relativistic approach implemented in RMI [2], the geodesic equations of motion are

directly numerically integrated for a chosen metric, with respect to proper time (for a massive particle). RMI is a prototype software taking only gravitational forces into account. It consists mainly of an integrator, plus separate modules containing metric definitions, the coordinate (space-time) transformations, or routines accessing the ephemeris.

To validate the different relativistic contributions in RMI orbits, the GINS software is used to produce template orbits. In a first step, comparison of RMI orbits with GINS orbits calculated for the Earth gravitational monopole plus a Schwarzschild correction shall validate the main relativistic effect in the RMI software, if the corresponding Schwarzschild metric is chosen. Then, harmonic terms (according to one of the geopotential models used in GINS) can further be added to the monopole term in the Schwarzschild metric used in RMI to test the harmonic contributions of the geopotential. In order to test the Lens-Thirring effect, the corresponding correction is selected from the GINS interface to produce the template orbits, while the Kerr metric is chosen from the RMI interface. With all Solar System masses set to zero except for the Earth and Sun masses, the GCRS (Geocentric Coordinate Reference System) metric provided by IAU (International Astronomical Union) 2000 resolutions [6,7] should include the additional geodetic precession effect when used to calculate RMI orbits. The latter RMI orbits can then be compared to orbits produced by GINS with the corresponding relativistic correction selected. Finally, the complete GCRS metric shall introduce additional contributions from other Solar System bodies in RMI orbits. These latter effects are simply modeled by Newtonian monopole terms in the GINS software, plus a coupling between the Moon and the Earth's flattening. We must notice that the GCRS metric already takes into account an additional relativistic effect, the Thomas precession, which is not modeled in the RMI software.

Once the RMI software is coherently calibrated for all the relativistic effects considered in the GINS software, RMI will go beyond GINS capabilities. Not only will RMI coherently incorporate the latest IAU resolutions [6,7] regarding metric standards, prescription for time transformations [3,4], and latest IAU 2000/IERS 2003 standards on Earth rotation [5,6], but thanks to its separate modules, it will allow to easily update for metric, geopotential model prescriptions. However, the fundamental advantage of RMI over GINS resides in the fact that it coherently incorporates all the relativistic effects at the required metric order chosen.

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