THE BCRS, GCRS AND THE CLASSICAL ASTRONOMICAL REFERENCE SYSTEM

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ABSTRACT. The roles of the BCRS and the GCRS are reviewed. Problems related with the traditional astronomical equinox based reference system in the framework of relativity are discussed.

1. THE BCRS AND THE GCRS

According to IAU2000-Resolution B1.3 (Soffel et al., 2003) the Barycentric Celestial Reference System (BCRS) with coordinates $(t, \mathbf{x})$ is defined by a metric tensor of the form

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}) \\ g_{0i} &= -\frac{4}{c^3} w^i + \mathcal{O}(c^{-5}) \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2}{c^2} w \right) + \mathcal{O}(c^{-4}) \,. \end{aligned} \tag{1}$$

Here, the gravito-electric potential w generalizes the usual Newtonian gravitational potential U and the gravito-magnetic potential w^i describes gravitational effects resulting from moving gravitational sources. The order symbols in equation 1 indicate that the validity of the metric tensor is restricted to the first post-Newtonian approximation of Einstein's theory of gravity. By the choice of the metric tensor the orientation of spatial BCRS coordinates is fixed only up to some constant, i.e., time independent rotation. One might think of this orientation to be determined by the ICRS/ICRF. For special purposes, however, the spatial axes might differ from these basic ones by a constant rotation matrix and the orientation, e.g., given by some 'ecliptic of some fixed epoch'.

IAU2000-Resolution B1.3 also specifies the Geocentric Celestial Reference System (GCRS) with coordinates $(T, \mathbf{X})$ by a corresponding geocentric metric tensor, where external bodies contribute only tidal terms that grow with increasing coordinate distance from the geocenter. One can show that because of the acceleration of the geocenter the GCRS is only a local system that cannot be extended to infinity.

Both systems, the BCRS and the GCRS might be considered as quasi-inertial with respect to rotational motion. However, if relativity is taken into account the BCRS and the GCRS are actually quite different systems that are related by some complicated 4-dimensional space-time transformation (some generalized Lorentz-transformation; e.g., Soffel et al., 2003). Both systems have their own specific roles. The BCRS serves as basis for solar-system ephemerides, for the definition of the ecliptic, for interplanetary spacecraft navigation. It is the fundamental system where concepts such as proper motion or radial velocity of stars can be defined and where the relation with remote objects (quasars etc.), hence to the ICRS, is given directly. On the other hand the GCRS is used for the description of physical processes and time scales in the vicinity of the Earth, the dynamics of the Earth itself, for the introduction of potential coefficients for the Earth's gravity field, for satellite theory etc. Concepts such as rotation axis, CIP, CIO or equator are basically defined in the GCRS whose spatial coordinates differ from those of the ITRS by a time dependent rotation matrix only. To stress again: the barycenter and the geocenter carry their own celestial (quasi-inertial) system.

2. PROBLEMS WITH THE CLASSICAL ASTRONOMICAL EQUINOX BASED SYSTEM IN THE FRAMEWORK OF RELATIVITY

The orientation of some BCRS$[E_0]$ (assuming the standard BCRS to be oriented according to the ICRS) according to some (ccordinate) fixed ecliptic of some epoch presents no problem. One might then construct a corresponding GCRS$[E_0]$ as kinematically non-rotating and thus oriented according to the BCRS$[E_0]$. Some equator, 'GCRS-ecliptic' (e.g., as $X-Y$-plane) and equinox can then be defined as coordinate quantities in the GCRS. If such a construction, however, in virtue of the complicated 4-dimensional space-time transformation between the BCRS and the GCRS, would yield some useful generalization of the corresponding classical quantities is unclear.

The problem becomes much more serious if some dynamical equinox based system is considered. Even in the Newtonian framework different historical concepts for some moving mean ecliptic exist: some *inertial mean ecliptic* that usually is associated with the name of LeVerrier and some *rotating mean ecliptic*, associated with Newcomb. Note, that the corresponding mean equinoxes differ in right ascension by about 90 mas. For references the reader is referred to (Standish 1981, Kinoshita and Aoki 1983). It is obvious that some dynamical mean ecliptic might be defined as some $t = TCB$ dependent BCRS spatial coordinate plane.

In principle one might invent many ways of mapping such a plane into the GCRS. Let us consider the $(t, \mathbf{x}) \longleftrightarrow (T, \mathbf{X})$ transformation (the relation between the BCRS and the GCRS) in the form

$$
\begin{aligned}
T &= t - \frac{1}{c^2}\left(A(t) + \mathbf{v}_E \cdot \mathbf{r}_E\right) + \dots \qquad (2)\\
\mathbf{X} &= \mathbf{r}_E\left(1 + \frac{1}{c^2} w_{\rm ext}(\mathbf{x}_E)\right) + \frac{1}{2c^2}\mathbf{v}_E(\mathbf{v}_E \cdot \mathbf{r}_E) + \dots
\end{aligned}
$$

Here, $\mathbf{r}_E = \mathbf{x} - \mathbf{x}_E$, $\mathbf{x}_E$ and $\mathbf{v}_E$ are the BCRS coordinate position and velocity of the geocenter and $w_{\rm ext}$ is the gravitational potential of all solar system bodies apart from the Earth. Such a transformation maps 4 space-time coordinates of an event. In that manner one might map the various events $(t, \mathbf{x}_{\rm eclip}(t))$ related with some dynamical BCRS ecliptic into the GCRS. However, a $t =$const. BCRS coordinate plane (mean ecliptic) will NOT be mapped into some $T =$const. GCRS plane and this naive mapping idea does not lead to something useful.

Another idea is due to George Kaplan (Kaplan 2003). One might consider only the orientation of some BCRS (mean) ecliptic as given by some angular momentum vector, defining the corresponding ecliptic pole. Normalize this vector with $\epsilon \to 0 : \mathbf{n}_\epsilon(t)$. Add the position vector of the geocenter to obtain $\mathbf{e}_\epsilon(t)$ and then transform the event $(t, \mathbf{e}_\epsilon(t))$ into the GCRS. Because of the small quantity ϵ only terms linear in $\mathbf{r}_E$ in the transformation are needed and the transformed spatial vector reads

$$\mathbf{E}_\epsilon = \left(1 + \frac{w_{\rm ext}(\mathbf{x}_E)}{c^2}\right)\mathbf{n}_\epsilon + \frac{1}{2c^2}\mathbf{v}_E(\mathbf{v}_E \cdot \mathbf{n}_\epsilon) + \dots \tag{3}$$

Since the ecliptic is usually defined with respect to the Earth-Moon barycenter the second term will not be zero in principle. Actually the whole idea of transforming a direction from the BCRS to the GCRS and to face aberration terms is questionable. Note that the GCRS was NOT constructed by transforming some BCRS-pole.

Another idea would be to transform the BCRS solar-system ephemerides into the GCRS and to define some GCRS-ecliptic there. Still another idea would be to take the spherical angles of some BCRS ecliptic pole at time $t = TCB$ as spherical angles of some ecliptic pole in the GCRS at time $T = TCG$ at the geocenter (i.e., to take the same angles both in the BCRS and in the GCRS).

Possibly for the purpose of historical continuity one might think of such coordinate games. It is obvious, however, that in the spirit of relativity the new paradigm with quantities such as the CIP or the CIO being defined in the GCRS is preferred. Actually the real problem is this: if one is concerned about relativity one needs some overall consistent post-Newtonian scheme to describe Earth's rotation. Only then a definition of some GCRS-ecliptic becomes meaningful and a precession matrix can be defined to post-Newtonian order. However, such a consistent post-Newtonian scheme for the description of Earth's rotation does presently not exist. Clearly these problems have to be investigated in more detail.

3. REFERENCES

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