LIGHT DEFLECTION AND TIME TRANSFER TO THE POST-POST-MINKOWSKIAN ORDER USING SYNGE'S WORLD FUNCTION

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ABSTRACT. Astrometric missions like GAIA and SIM will require microarcsecond accuracy in the measurement of the direction of light rays. At this level of precision, it is necessary to develop a relativistic modelling of the light deflection up to the order G^2, G being Newton's gravitationnal constant. We emphasize that this modelling may be achieved without integrating the differential equations of null geodesics, by improving the method of the world function developed by Synge. As an example, we give a detailed calculation of the light deflection in a static, spherically symmetric space-time considered in the post-post-Minkowskian approximation. The world function also makes it very easy to determine the time transfer function.

1. INTRODUCTION

With advances in technology, great improvements in astrometric accuracy will certainly be made in the near future. Two major space astrometric missions are already planned to measure the positions of celestial objects with typical uncertainties in the range 1-10 μas (microarcsecond). The Global Astrometric Interferometer for Astrophysics (GAIA, Perryman *et al.* 2001) mission, to be launched not later than 2012, will be capable to measure the positions of 25 million stars with a global uncertainty better than 10 μas and a few million with an uncertainty better than 4 μas. The Space Interferometric Mission (SIM, Danner and Unwin 1999) will be capable of at least 4 μas accuracy for 3000 quasars and fundamental stars.

For these missions, and for those which will follow them, all relativistic effects contributing to the deflection of light must be determined at a level of 1 μas, or significantly better. It was shown that this level of accuracy can be achieved by the GAIA mission if one retains only the perturbative terms of the background metric which are of the first order with respect to the Newtonian gravitational constant G (the so-called post-Minkowskian approximation). However, it must be emphasized that this approximation is sufficient only because the light rays received by GAIA will be propagating sufficiently far from the Sun. If we want to build a relativistic model for any light ray propagating through the Solar System, it will be necessary to take into account terms of order G^2 in the metric. The aim of the present paper is to furnish a general method allowing the calculation of the deflection of light within the post-post-Minkowskian approximation of any metric.

Since the pioneering works (see, e.g., Will 1988 and Refs. therein), a lot of papers investigated the deflection of light and/or the time delay effects in the linearized, weak-field limit of general relativity (Klioner 1991, Klioner and Kopeikin 1992, Kopeikin 1997, Kopeikin and Schäfer 1999, Ciufolini and Ricci 2002). In (Kopeikin and Schäfer 1999) especially, one finds an exhaustive determination of light rays in the field of an arbitrary large number of moving point masses in terms of retarded Liénard-Wichert potentials.

In contrast with the generality of the results obtained at the first order, the investigations undertaken at the second order in G have been confined to static spherically symmetric fields (see Epstein and Shapiro 1980, Fischbach and Freeman 1980 for the deflection angles, Klioner and Kopeikin 1992, Richter and Matzner 1982 and 1983, Brumberg 1987, for the time delay and light bending).

In the above-mentioned studies, the deflection of light and the time delay are calculated by integrating the differential equations of null geodesics. This procedure is workable as long as one contents oneself with analyzing the effects in the first-order post-Minkowskian approximation or considering the case of a static spherically symmetric field. However, analytical or numerical integrations of the geodesic equations require very cumbersome calculations when the effects in G^2 are taken into account. So we explore an alternative method, based on the two-point world function, as developed by Synge (Synge 1964). This method presents the decisive advantage to avoid the solution of the null geodesic equations. Moreover, the knowledge of the world function allows an explicit determination of the travel time of a photon. We summarize here the results that we have obtained for a general metric within the second post-Minkowskian approximation and we apply them to a static, spherically-symmetric ds^2 containing three post-Minkowskian parameters. Thus we recover with simple quadratures formulae obtained in John 1975 with heavy calculations.

In the following, we suppose that space-time is covered by a global quasi-Cartesian coordinate system $x^\mu = (x^0, x^i) = (ct, \mathbf{x})$, c being the speed of light in a vacuum. The signature of the metric is $(+---)$.

2. THE WORLD FUNCTION

Consider two points x_A and x_B in a given space-time endowed with a metric $g_{\mu\nu}$ and assume that x_A and x_B are connected by a unique geodesic path Γ. Throughout this paper, λ denotes the unique affine parameter along Γ which fulfills the boundary conditions $\lambda_A = 0$ and $\lambda_B = 1$. The so-called world-function of space-time (Synge 1964) is the two-point function $\Omega(x_A, x_B)$ defined by

$$\Omega(x_A, x_B) = \frac{1}{2}\int_0^1 g_{\mu\nu}(x^\alpha(\lambda))\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}d\lambda\,, \tag{1}$$

the integral being taken along Γ. It is easily seen that $\Omega(x_A, x_B) = \varepsilon[s_{AB}]^2/2$, where s_{AB} is the geodesic distance between x_A and x_B and $\varepsilon = 1, 0, -1$ for timelike, null and spacelike geodesics, respectively. It results from Eq. (1) that the world-function $\Omega(x_A, x_B)$ is unchanged if we perform any admissible coordinate transformation.

The utility of the world-function for our purpose comes from the following properties (Synge 1964) :

i) The world function satisfies the Hamilton-Jacobi equations

$$\frac{1}{2}\, g^{\alpha\beta}(x_A)\frac{\partial\Omega}{\partial x_A^\alpha}(x_A, x_B)\frac{\partial\Omega}{\partial x_A^\beta}(x_A, x_B) = \Omega(x_A, x_B)\,, \tag{2}$$

$$\frac{1}{2}\, g^{\alpha\beta}(x_B)\frac{\partial\Omega}{\partial x_B^\alpha}(x_A,x_B)\frac{\partial\Omega}{\partial x_B^\beta}(x_A,x_B) = \Omega(x_A,x_B)\,. \tag{3}$$

ii) The vectors $(dx^\alpha/d\lambda)_A$ and $(dx^\alpha/d\lambda)_B$ tangent to the geodesic Γ respectively at x_A and x_B are given by

$$\left(g_{\alpha\beta}\frac{dx^\beta}{d\lambda}\right)_A = -\frac{\partial\Omega}{\partial x_A^\alpha}\,, \quad \left(g_{\alpha\beta}\frac{dx^\beta}{d\lambda}\right)_B = \frac{\partial\Omega}{\partial x_B^\alpha}\,. \tag{4}$$

As a consequence, if $\Omega(x_A,x_B)$ is explicitly known, the determination of these vectors does not require the integration of the differential equations of the geodesic.

iii) Points x_A and x_B may be linked by a light ray if and only if the condition

$$\Omega(x_A,x_B) = 0 \tag{5}$$

is fulfilled. Thus, $\Omega(x_A,x) = 0$ is the equation of the light cone $\mathcal{C}(x_A)$ at x_A. This fundamental property shows that the knowledge of $\Omega(x_A,x_B)$ yields (at least in principle) the knowledge of the travel time $t_B - t_A$ of a photon connecting two points x_A and x_B. It must be pointed out, however, that solving the equation $\Omega(ct_A,\boldsymbol{x}_A,ct_B,\boldsymbol{x}_B) = 0$ for t_B yields two distinct solutions t_B^+ and t_B^- since the timelike curve $x^i = x_B^i$ cuts the light cone $\mathcal{C}(x_A)$ at two points x_B^+ and x_B^-, x_B^+ being in the future of x_B^-. In the present paper, we shall always regard x_A as the point of emission of the photon and x_B as the point of reception, and we shall write t_B for t_B^+. With this convention, $t_B - t_A$ may be considered either as a function of t_A, $\boldsymbol{x}_A$, $\boldsymbol{x}_B$, or as a function of t_B, $\boldsymbol{x}_A$, $\boldsymbol{x}_B$. So we put

$$t_B^+ - t_A = \mathcal{T}_e(t_A,\boldsymbol{x}_A,\boldsymbol{x}_B) = \mathcal{T}_r(t_B,\boldsymbol{x}_A,\boldsymbol{x}_B)\,, \tag{6}$$

and we call $\mathcal{T}_e(t_A,\boldsymbol{x}_A,\boldsymbol{x}_B)$ the emission time transfer function and $\mathcal{T}_r(t_B,\boldsymbol{x}_A,\boldsymbol{x}_B)$ the reception time transfer function.

Consider now a stationary space-time. In this case, we use exclusively coordinates (x^μ) such that the metric does not depend on x^0. Then, the world-function is a function of $x_B^0 - x_A^0$, $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$, and (6) reduces to a relation of the form

$$t_B^+ - t_A = \mathcal{T}(\boldsymbol{x}_A,\boldsymbol{x}_B)\,. \tag{7}$$

The knowledge of $\mathcal{T}$ enables to determine the direction of light rays since a comparaison between Eq. (5) and Eq. (7) immediately shows that the vectors $(l^\mu)_A$ and $(l^\mu)_B$ defined by their covariant components

$$(l_0)_A = 1, \quad (l_i)_A = c\frac{\partial}{\partial x_A^i}\mathcal{T}(\boldsymbol{x}_A,\boldsymbol{x}_B)\,, \tag{8}$$

$$(l_0)_B = 1, \quad (l_i)_B = -c\frac{\partial}{\partial x_B^i}\mathcal{T}(\boldsymbol{x}_A,\boldsymbol{x}_B)\,, \tag{9}$$

are tangent to the ray at x_A and x_B, respectively. It must be pointed out that these tangent vectors correspond to an affine parameter such that $l_0 = 1$ along the ray (note that such a parameter does not coincide with λ). Generally, extracting the time transfer formula Eq. (7) from Eq. (5), next using Eqs. (8)-(9) will be more straightforward than deriving the vectors tangent at x_A and x_B from Eq. (4), next imposing the constraint (5).

In the present work, we assume that the gravitational potentials may be expanded as

$$g_{\mu\nu} = \eta_{\mu\nu} + Gh_{\mu\nu}^{(1)} + G^2 h_{\mu\nu}^{(2)} + O(G^3)\,. \tag{10}$$

As a consequence, the world function admits an expansion as follows

$$\Omega(x_A,x_B) = \Omega^{(0)}(x_A,x_B) + G\Omega^{(1)}(x_A,x_B) + G^2\Omega^{(2)}(x_A,x_B) + O(G^3)\,, \tag{11}$$

where $\Omega^{(0)}$ is the Minkowskian world function

$$\Omega^{(0)}(x_A, x_B) = \frac{1}{2}\eta_{\mu\nu}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu) \tag{12}$$

and $\Omega^{(1)}$ is given by (Synge 1964, Linet and Teyssandier 2002)

$$\Omega^{(1)}(x_A, x_B) = \frac{1}{2}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu)\int_0^1 h_{\mu\nu}^{(1)}(x_{(0)}(\lambda))d\lambda\,. \tag{13}$$

Using a method inspired by Buchdahl 1990, we show elsewhere (Le Poncin-Lafitte *et al.* 2004) that $\Omega^{(2)}(x_A, x_B)$ may be written in the form

$$\begin{aligned}\Omega^{(2)}(x_A, x_B) &= \frac{1}{2}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu)\\ &\times \int_0^1 \Big[\, h_{\mu\nu}^{(2)}(x_{(0)}(\lambda)) - \eta^{\rho\sigma}\, h_{\mu\rho}^{(1)}(x_{(0)}(\lambda))\, h_{\nu\sigma}^{(1)}(x_{(0)}(\lambda))\\ &\qquad -\eta^{\rho\sigma}\,\frac{\partial\Omega^{(1)}}{\partial x^\rho}(x_A, x_{(0)}(\lambda))\, h_{\mu\nu,\sigma}^{(1)}(x_{(0)}(\lambda))\Big]\, d\lambda\\ &\qquad +\frac{1}{2}\eta^{\mu\nu}\,\frac{\partial\Omega^{(1)}}{\partial x_B^\mu}(x_A, x_B)\,\frac{\partial\Omega^{(1)}}{\partial x_B^\nu}(x_A, x_B)\,,\end{aligned} \tag{14}$$

where the line integrals are taken along the unperturbed geodesic connecting x_A and x_B defined by the parametric equations

$$x_{(0)}^\alpha(\lambda) = (x_B^\alpha - x_A^\alpha)\lambda + x_A^\alpha\,, \quad 0 \le \lambda \le 1\,. \tag{15}$$

3. APPLICATION TO A STATIC, SPHERICALLY SYMMETRIC BODY

In order to apply the general results obtained above, let us determine the world function and the time transfer function in the second post-Minkowskian approximation when points x_A and x_B are both outside a static, spherically symmetric body of mass M. We suppose that the metric components may be written as

$$\begin{aligned} h_{00}^{(1)} &= -\frac{2M}{c^2 r}\,, \quad h_{0i}^{(1)} = 0\,, \quad h_{ij}^{(1)} = -\frac{2\gamma M}{c^2 r}\delta_{ij}\,,\\ h_{00}^{(2)} &= \frac{2\beta M^2}{c^4 r^2}\,, \quad h_{0i}^{(2)} = 0\,, \quad h_{ij}^{(2)} = -\frac{3\delta M^2}{2c^4 r^2}\delta_{ij}\,,\end{aligned} \tag{16}$$

where β and γ are the usual post-Newtonian parameters and δ is a post-post-Newtonian parameter (in general relativity, $\beta = \gamma = \delta = 1$). Furthermore, we suppose that x_A and x_B are such that the connecting geodesic path is entirely in the exterior space. We use the notations

$$r = |\boldsymbol{x}|\,, \quad r_A = |\boldsymbol{x}_A|\,, \quad r_B = |\boldsymbol{x}_B|, \quad R_{AB} = |\boldsymbol{x}_B - \boldsymbol{x}_A|\,.$$

It is easily seen that $G\Omega^{(1)}(x_A, x_B)$ is the term due to the mass in the multipole expansion of the world function given by Eq. (56) in Linet and Teyssandier 2002 (see also John 1975) :

$$\Omega^{(1)}(x_A^0, \boldsymbol{x}_A, x_B^0, \boldsymbol{x}_B) = -\frac{M}{c^2}\left[(x_B^0 - x_A^0)^2 + \gamma R_{AB}^2\right] F(\boldsymbol{x}_A, \boldsymbol{x}_B)\,, \tag{18}$$

where

$$F(\boldsymbol{x}_A, \boldsymbol{x}_B) = \int_0^1 \frac{d\lambda}{|\boldsymbol{x}_{(0)}(\lambda)|} = \frac{1}{R_{AB}} \ln\left(\frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}}\right) . \tag{19}$$

For the integrals of second order involving $h^{(2)}_{\mu\nu}$ and terms quadratic in $h^{(1)}_{\mu\nu}$, we find

$$\begin{aligned} &\frac{1}{2}(x^\mu_B - x^\mu_A)(x^\nu_B - x^\nu_A) \int_0^1 \left[h^{(2)}_{\mu\nu}(x_{(0)}(\lambda)) - \eta^{\rho\sigma}\, h^{(1)}_{\mu\rho}(x_{(0)}(\lambda))\, h^{(1)}_{\nu\sigma}(x_{(0)}(\lambda))\right] d\lambda \\ &\quad = \frac{M^2}{c^4}\left[(\beta - 2)(x^0_B - x^0_A)^2 + \left(2\gamma^2 - \frac{3\delta}{4}\right) R^2_{AB}\right] E(\boldsymbol{x}_A, \boldsymbol{x}_B) \ , \end{aligned} \tag{20}$$

where $E(\boldsymbol{x}_A, \boldsymbol{x}_B)$ is given by

$$E(\boldsymbol{x}_A, \boldsymbol{x}_B) = \int_0^1 \frac{d\lambda}{|\boldsymbol{x}_{(0)}(\lambda)|^2} = \frac{\arccos(\boldsymbol{n}_A \cdot \boldsymbol{n}_B)}{r_A r_B \sqrt{1 - (\boldsymbol{n}_A \cdot \boldsymbol{n}_B)^2}} , \tag{21}$$

$\boldsymbol{n}_A$ and $\boldsymbol{n}_B$ being defined as

$$\boldsymbol{n}_A = \boldsymbol{x}_A / r_A \, , \qquad \boldsymbol{n}_B = \boldsymbol{x}_B / r_B \, . \tag{22}$$

For the integral involving the gradient of $\Omega^{(1)}$, we obtain

$$\begin{aligned} &-\frac{1}{2}(x^\mu_B - x^\mu_A)(x^\nu_B - x^\nu_A) \int_0^1 \eta^{\rho\sigma}\, \frac{\partial \Omega^{(1)}}{\partial x^\rho}(x_A, x_{(0)}(\lambda))\, h^{(1)}_{\mu\nu,\sigma}(x_{(0)}(\lambda)) d\lambda \\ &\quad = \frac{M^2}{c^4}\left[(x^0_B - x^0_A)^2 + \gamma R^2_{AB}\right] \left[\left(1 + \frac{r_A}{r_B}\right) \frac{1}{R^2_{AB}} \frac{(x^0_B - x^0_A)^2 + \gamma R^2_{AB}}{r_A r_B + (\boldsymbol{x}_A \cdot \boldsymbol{x}_B)} \right. \\ &\quad \left. -2\gamma E(\boldsymbol{x}_A, \boldsymbol{x}_B) - \frac{(x^0_B - x^0_A)^2 - \gamma R^2_{AB}}{R^2_{AB}} \frac{F(\boldsymbol{x}_A, \boldsymbol{x}_B)}{r_B} \right] . \end{aligned} \tag{23}$$

Substituting Eqs. (20) and (23) into Eq. (14), and then carrying out the calculation of the square of the gradient of $\Omega^{(1)}(x_A, x)$, we obtain an expression for the world function as follows

$$\begin{aligned} \Omega(x_A, x_B) &= \frac{1}{2}(x^0_B - x^0_A)^2 - \frac{1}{2} R^2_{AB} \\ &- \frac{GM}{c^2}\left[(x^0_B - x^0_A)^2 + \gamma R^2_{AB}\right] F(\boldsymbol{x}_A, \boldsymbol{x}_B) \\ &+ \frac{G^2 M^2}{c^4}\left\{ \frac{(x^0_B - x^0_A)^4}{R^2_{AB}} \left[\frac{1}{r_A r_B + (\boldsymbol{x}_A \cdot \boldsymbol{x}_B)} - \frac{1}{2} F^2(\boldsymbol{x}_A, \boldsymbol{x}_B)\right] \right. \\ &+ (x^0_B - x^0_A)^2 \left[\frac{2\gamma}{r_A r_B + (\boldsymbol{x}_A \cdot \boldsymbol{x}_B)} - (2 - \beta + 2\gamma) E(\boldsymbol{x}_A, \boldsymbol{x}_B) \right. \\ &\qquad \left. + (2 + \gamma) F^2(\boldsymbol{x}_A, \boldsymbol{x}_B)\right] \\ &\left. + R^2_{AB} \left[\frac{\gamma^2}{r_A r_B + (\boldsymbol{x}_A \cdot \boldsymbol{x}_B)} - \frac{3\delta}{4} E(\boldsymbol{x}_A, \boldsymbol{x}_B) - \frac{\gamma^2}{2} F^2(\boldsymbol{x}_A, \boldsymbol{x}_B)\right]\right\} + O(G^3) . \end{aligned} \tag{24}$$

Now substituting for $\Omega(x_A, x_B)$ from Eq. (24) into Eq. (5) and then solving this equation thus obtained for $x^0_B - x^0_A$, we find for the time transfer function in the second post-Minkowskian approximation

$$\begin{aligned} \mathcal{T}(\boldsymbol{x}_A, \boldsymbol{x}_B) &= \frac{R_{AB}}{c} + \frac{(\gamma + 1) GM}{c^3} \ln\left(\frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}}\right) \\ &\quad + \frac{G^2 M^2 R_{AB}}{c^5} \left[\kappa E(\boldsymbol{x}_A, \boldsymbol{x}_B) - \frac{(1+\gamma)^2}{r_A r_B + (\boldsymbol{x}_A \cdot \boldsymbol{x}_B)}\right] , \end{aligned} \tag{25}$$

where

$$\kappa = 2 - \beta + 2\gamma + \frac{3\delta}{4}\,. \tag{26}$$

For $\gamma = \beta = \delta = 1$, we recover the expression of time transfer found by Brumberg (1987) within general relativity. Now, substituting for $\mathcal{T}$ from Eq. (25) into Eqs. (8)-(9) and then putting $\mathbf{N}_{AB} = (\boldsymbol{x}_B - \boldsymbol{x}_A)/R_{AB}$ yield for the covariant components of the vectors tangent at $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$

$$\mathbf{l}_A = (l_i)_A = -\mathbf{N}_{AB} + G\mathbf{l}_{(1)}(\boldsymbol{x}_A, \boldsymbol{x}_B) + G^2\mathbf{l}_{(2)}(\boldsymbol{x}_A, \boldsymbol{x}_B) + O(G^3)\,, \tag{27}$$

$$\mathbf{l}_B = (l_i)_B = -\mathbf{N}_{AB} - G\mathbf{l}_{(1)}(\boldsymbol{x}_B, \boldsymbol{x}_A) - G^2\mathbf{l}_{(2)}(\boldsymbol{x}_B, \boldsymbol{x}_A) + O(G^3)\,, \tag{28}$$

where contributions $\mathbf{l}_{(1)}$ are given in Linet and Teyssandier (2002)

$$\mathbf{l}_{(1)}(\boldsymbol{x}_A, \boldsymbol{x}_B) = -(\gamma+1)\frac{M}{c^2}\frac{(r_A + r_B)\mathbf{N}_{AB} + R_{AB}\boldsymbol{n}_A}{r_A r_B(1 + \boldsymbol{n}_A.\boldsymbol{n}_B)}\,. \tag{29}$$

For the post-post-Minkowskian effect we find

$$\begin{aligned}
\mathbf{l}_{(2)}(\boldsymbol{x}_A, \boldsymbol{x}_B) \quad &= \quad \frac{M^2}{c^4 r_A r_B}\left[\frac{(1+\gamma)^2}{1+\boldsymbol{n}_A.\boldsymbol{n}_B} - \kappa r_A r_B E(\boldsymbol{x}_A, \boldsymbol{x}_B)\right]\mathbf{N}_{AB} && (30)\\
&+\frac{M^2}{c^4 r_A r_B}\frac{R_{AB}}{r_A}\left[\frac{(1+\gamma)^2}{[1+\boldsymbol{n}_A.\boldsymbol{n}_B]^2} - \kappa\frac{r_A r_B E(\boldsymbol{x}_A, \boldsymbol{x}_B) - \boldsymbol{n}_A.\boldsymbol{n}_B}{1-(\boldsymbol{n}_A.\boldsymbol{n}_B)^2}\right]\boldsymbol{n}_A\\
&+\frac{M^2}{c^4 r_A r_B}\frac{R_{AB}}{r_A}\left[\frac{(1+\gamma)^2}{[1+\boldsymbol{n}_A.\boldsymbol{n}_B]^2} - \kappa\frac{1-(\boldsymbol{n}_A.\boldsymbol{n}_B) r_A r_B E(\boldsymbol{x}_A, \boldsymbol{x}_B)}{1-(\boldsymbol{n}_A.\boldsymbol{n}_B)^2}\right]\boldsymbol{n}_B\,.\\
& && (31)
\end{aligned}$$

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