EVOLUTION OF A TWO-PLANETARY REGULAR SYSTEM ON A COSMOGONIC TIME SCALE

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ABSTRACT. For the planetary three-body problem we use Jacobian coordinates, introduce two systems of osculating elements, construct the Hamiltonian expansions in the Poisson series in all elements. Further we construct the averaged Hamiltonian by the Hori — Deprit method with accuracy upto second order with respect to the small parameter, the generating function, change of variables formulae, and right-hand sides of averaged equations. The averaged equations for the Sun – Jupiter – Saturn system are integrated numerically at the time-scale of 10 Gyr. The motion turns out to be almost periodical. The low and upper limits for averaged eccentricities are 0.016, 0.051 (Jupiter), 0.020, 0.079 (Saturn), and for averaged inclinations to the ecliptic plane are 1.3°, 2.0° (Jupiter), 0.73°, 2.5° (Saturn).

1. AVERAGED PLANETARY THREE-BODY PROBLEM

This work continues researches beginning in (Kholshevnikov, Greb, & Kuznetsov 2001; 2002). We consider the planetary three-body problem the Sun – Jupiter – Saturn. We use Jacobian coordinates as best-fitting. Small parameter μ is equal to 10^{-3}. Let us represent the Hamiltonian as a sum of the unperturbed part h_0 and the perturbed one μh_1. The Hamiltonian h_0 depends on semi-major axes only. The Hamiltonian h_1 may be thought of as a constant factor having dimension of velocity squared and a dimensionless part h_2.

The disturbing function h_2 is presented as Poisson series $h_2 = \sum A_{kn} x^k \cos ny$, where x are positional elements, y are angular ones, A_{kn} are numerical coefficients, k and n are multi-indices. The summation is taken over non-negative k_s and integer n_s, $s = 1, ..., 6$. We obtain limits of the Poisson series summation ensuring the Hamiltonian accurate up to μ^σ.

We use two systems of osculating elements. The first system is close to the Keplerian one. In the second system denominators arising in a process of averaging transforms are extremely simple. On the other hand, it has a deficiency, mixing several elements of all planets.

The rational version of the Poisson series processor PSP (Brumberg 1995; Ivanova 1995) is used to construct the expansion of disturbing Hamiltonian h_2 into Poisson series. The expansion of disturbing Hamiltonian is processed up to μ^2. The summation is taken over $k_1 + \cdots + k_6 \leq 6$, $|n_s| \leq 15$ $(s = 1, \ldots, 6)$. For each of the osculating elements system two variants of the expansion are constructed. The first variant deals with numerical values of parameters (masses, mean values of semi-major axes, ...) corresponding to the system the Sun – Jupiter – Saturn. The second one deals with their litteral expressions depending on parameters of the system. The expansions with numerical data contain 61086 terms. The expansions with litteral parameters contain 182744 terms for the first system and 183227 terms for the second one.

The Hori – Deprit method (Lie transforms method) is used to construct the averaged Hamiltonian $H(X, Y)$. This method is based on Poisson brackets that allows us to use non-canonical elements writing down the Poisson brackets in the corresponding system of phase variables (Kholshevnikov & Greb 2001). The Hamiltonian $h(x, y)$ is averaged over the fast variables. The averaged Hamiltonian H is presented by power series in the small parameter μ upto μ^2. For calculations we use the rational version of the echeloned Poisson series processor EPSP (Ivanova 2001). Transformations are made for both systems of elements with numerical parameters. Two approximations of Hori — Deprit method are made. The generating function, change of variables formulae between averaged and osculating systems, and right-hand sides of averaged equations of motion are obtained.

2. BEHAVIOUR OF THE SUN — JUPITER — SATURN SYSTEM ON 10 Gyr

The averaged equations are integrated numerically at the time-scale of 10 Gyr. The equations for slow variables are integrated by Everhart and Runge-Kutta high order methods. The equations for fast variables are integrated by spline interpolation method. Accuracy of integration is detected by computation of integrals of energy and area.

The motion turns out to be almost periodical. The low and upper limits for averaged eccentricities are 0.016, 0.051 (Jupiter), 0.020, 0.079 (Saturn), and for averaged inclinations are 1.3°, 2.0° (Jupiter), 0.73°, 2.5° (Saturn). Evolution of the ascending nodes longitudes with respect to the ecliptical plane and the arguments of pericentre turns out to be secular. Evolution of the ascending node longitudes depends on the base plane. Difference between the ascending nodes longitudes of Jupiter and Saturn with respect to Laplace plane is equal to 180° exactly.

Estimates of the Liapunovian Exponents for the system the Sun – Jupiter – Saturn are obtained. The corresponding Liapunovian Time turns out to be 14 Myr (Jupiter) and 10 Myr (Saturn).

Our results are qualitatively in agreement with ones obtained by (Laskar 1994; Murray, & Dermott 1999; Ito, & Tanikawa 2002).

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