CHARACTERISTICS OF EROS 433 ROTATION

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ABSTRACT. This paper summarizes the contents of recent analytical studies related to the rotation of the asteroid Eros 433 (Souchay et al.,2003; Souchay and Bouquillon,2004). Thanks to the very accurate observational data obtained by the intermediary of the probe mission NEAR (Near Earth Asteroid Rendez-Vous) and detailed by Miller et al.(2002), it is possible to determine with an exceptional accuracy the ephemerides of the rotation of the asteroid, and more specifically the values of the coefficients of precession and nutation. After explaining the parametrization of the problem and the way of calculation, we give the principal results, which enable to modelize Eros' rotation both for long-term (100 years) and very short term (a few hours).

1. INTRODUCTION

As any celestial body, the rotation of Eros can be separated into two independent parts, which must be combined each together : one is the free motion of rotation, i.e., the rotation when not considering any perturbing body, and the other is the forced motion, i.e., the motion caused by the gravitational torque exerted by any external celestial body (Sun, planet, satellite etc...). In the case of Eros, we only consider here the perturbation caused by the Sun. Concerning the free motion, two rotation modes for celestial bodies can be found : one is a rotation along the shortest axis, which generally corresponds to the axis around which the moment of inertia is maximum , the other is along the longest axis, generally with minimum moment of inertia. These modes are respectively called *short axis modes* and *long-axis modes.* Exhaustive studies about these two kinds of rotational free modes can be found, for instance in Kinoshita (1972), starting from an Hamiltonian based theory. The short axis mode is ordinary stable and secularly stable, whereas the long-axis mode is ordinary stable but secularly unstable. In case of dissipation, the long axis mode becomes unstable, whereas the short-axis mode remains stable. Notice that in the solar system, a major part of asteroids undergoes a short-axis rotational mode, although long-axis modes have been found, for example in the case of Toutatis (Burns and Safronov, 1973, Harris,1994) Therefore, although no clear information can be reached from the NEAR's exploration to know if Eros free motion belongs to the first or the second of the categories above, we can assume with no doubt that Eros undergoes a short-axis rotational mode. Therefore we will consider in the following that Eros' rotational motion is around the axis of maximum moment of inertia C, quoted as the *axis of figure*, thus applying the usual terminology available for the planets. Another fundamental bu unknown parameter (in fact this parameter is known only for the Earth) which plays a large place in the rotational mode is the value of the angle (called here J) between the axis of figure and the axis of rotation (see Kinoshita,1977), and which is one of the two parameters enabling us to determine what is called for the Earth, the *polhodie* or polar motion. We can consider that this angle is very close or equal to 0, which is adequate to a condition of minimum of energy.

In order to compute the gravitational torque exerted by the Sun on Eros, which depends on the latitude and the longitude of the Sun with respect to a reference system related to the asteroid, we need to know its orbital osculating elements, which can be found for instance from the MPCORB (Minor Planet Centre Orbit Database). Eros has a semi-major axis close to Mars' one ($a = 1.4583145 A.U.$), and its orbit has a relatively high excentricity ($e = 0.2228487$). This explains why the asteroid can be considered as a Near Earth Asteroid, its perihelion distance being of the order of 1.13 A.U. Notice that the values above have been obtained at 2002, June 5th. Eros' inclination is : $i = 10°.83019$, and its mean motion is $n = 3.5677539 rd/y$, which corresponds to an orbiting period of roughly 1.76 year. All these elements are subject to dramatic changes, due to the possibility of becoming very close to the Earth as well as to Mars.

2. THE NEAR PROBE : A PRECIOUS DATA FOR THE ROTATION

After some technical problems, the NEAR probe was finally inserted into an orbit around Eros, starting from February 14, 2000. A set of several techniques, as radiometric tracking data, optical imaging and laser data, led to very precious and accurate informations about parameters which are fundamental to elaborate ephemerides of the rotation of the celestial object : these are the shape, gravity field and rate of rotation, as well as the location of the polar axis at a given instant (Miller et al.,2002).

For instance, it was possible to elaborate a shape model at 180th. degree order (Zuber et al.,2000), and to determine the gravity field of the asteroid as a combination of spherical harmonics at the order 15 (Miller et al.,2002). It was also shown in this last paper that the very small offset between the center of figure and the center of mass indicates that the asteroid has a very uniform density (± 1%) on a large scale. The rotation rate was measured with a precision of $\pm 0.00023°/d$, that is to say roughly 1"$/d$, whereas the equatorial coordinates α and δ of the pole have been obtained with a precision respectively of $\pm 0.003°$ and $\pm 0.005°$, which gives a global precision for the orientation of the pole in the space of roughly 25" (Miller et al.,2002; Konopliv et al., 2002).

Eros belongs to S-Class asteroids, which can be generally found in the inner part of the main asteroid belt and its albedo is moderate (the geometric value is 0.27). The absolute magnitude of Eros, which means the magnitude at 0° phase angle and 1 AU from the Sun and the Earth, is 11.16.Eros' mass could be estimated to $6.6904 \pm 0.003 \times 10^{15} kg$, corresponding to a volume of $2503 \pm 25 km^3$. This gives an equivalent mass per volume unit equal to $2.67 g/cm^3$ Eros is a very irregular body, with a large variation of the distances from the center of mass to the surface, with 3.19 km as the minimum value, and 17.67 km as the maximum one. The Eros' moments of inertia, which play a fundamental role in the rotation, have been determined at the 4^{th} digit. Their normalized values are : $A = 17.09 km^2; B = 71.79 km^2$ and $C = 74.49 km^2$. As a consequence the dynamical ellipticity, which characterized the flattening of the object in a dynamical point of view, is : $H_d = \frac{2C-(A+B)}{2C} = 0.40341$. This is by two orders of magnitude larger than the dynamical ellipticity of telluric planets as the Earth or Mars. At last the rotation period, which is also fundamental for the determination of the constant of precession and of the coefficients of nutation, is 5.27025547 h.

3. THE PARAMETRIZATION OF EROS'ROTATION

As we are concerned here with the motion of Eros' figure axis in space and by analogy with the case of the Earth, a natural reference frame for the parametrization of this motion should be built from a basic plane and an origin on this plane, which are the Eros orbital plane (E) and the ascending node of (E) with respect to the Eros equator. By analogy with the Earth, this point can be called Eros' vernal equinox, written as γ_{Eros}. For sake of commodity, the basic plane (E) Eros' orbital plane for a given epoch, and not of the date, as it is conventionally the case for the Earth.

In order to determine the mean value of Eros'obliquity ε^0, we have to calculate the coordinates of both the unit vector parallel to the figure axis $\vec{f}$ and the unit vector $\vec{o}$ perpendicular to the orbital plane . The most suitable way to do that is to calculate the coordinates of $\vec{f}$ and $\vec{o}$ with respect to an ecliptic reference system. The equatorial coordinates of the figure axis $\vec{f}$ are given by Miller et al.(2002) : $\alpha_f = 0^h45^{mn}28^{sec}$, $\delta_f = 17^\circ.227$. Through the classical transformations between equatorial coordinates and ecliptic coordinates we obtain easily the ecliptic rectangular coordinates of the unit vector $\vec{f}$ along the figure axis : $\vec{f} = (0.936397, 0.290552, 0.1968248)$. In another way, the ecliptic coordinates of $\vec{o}$ are obtained through the orbital parameters i and Ω: $\vec{o} = (\sin i \sin\Omega, -\sin i\cos\Omega, \cos i) = (-0.1550278, -0.1061713, 0.9821883)$. Therefore, the obliquity ε can be obtained from one among the two formulas below, involving vectorial and scalar products: $\vec{o}\times\vec{f} = \vec{w}\sin\varepsilon$ $\quad$ $\vec{o}\cdot\vec{f} = \cos\varepsilon$ where $\vec{w}$ is the unit vector along the direction of the descending node N' of Eros orbit (E^0) with respect to Eros true equator. Choosing this way of calculation, Souchay et al.(2003) have shown that Eros' mean obliquity at the reference epoch (J2002) is $\varepsilon = 89^\circ.008 \approx 89^\circ 0' 29"$, which means that Eros' figure axis is nearly aligned with Eros orbital plane, in a similar manner as for Uranus.

4. FREE ROTATIONAL MOTION

Eros free rotational motion can be studied from Hamiltonian formalism (Kinoshita,1991), involving such parameters as e and D defined in the following way :

$$e = \frac{1}{2}\Big(1/B - 1/A\Big)D \qquad 1/D = \frac{1}{C} - \frac{1}{2}\Big(\frac{1}{A} + \frac{1}{B}\Big) \tag{1}$$

The very large value of e leads to the use of specific approximated formula for the determination of $n_{\tilde{l}}$ and $n_{\tilde{g}}$ (Kinoshita,1991) which are the frequencies of the polar motion and of the proper angle of rotation, respectively defined from a meridian origin.

$$n_{\tilde{l}} = \frac{G}{D}\sqrt{1-e^2} \times \left[1 - \frac{1}{2(1-e)}j^2\right] + O(j^4) \approx \frac{G}{D}\sqrt{1-e^2} \tag{2}$$

$$n_{\tilde{g}} = \frac{1}{2}(1/A + 1/B)G + \frac{G}{D}(1 - \sqrt{1-e^2}) \times \left[1 + \frac{1}{2}\sqrt{(1+e)/(1-e)}j^2\right] + O(j^4)$$

$$\approx \frac{1}{2}(1/A + 1/B)G + \frac{G}{D}(1 - \sqrt{1-e^2}) \tag{3}$$

Where G is the amplitude of the angular momentum. Notice that $l + g = \Phi$ is the rotation angle, whose the rate has been determined with a remarkable precision (about 2"/day) by Miller et al.(2002), who set the value : $n_{\tilde{l}} + n_{\tilde{g}} = \dot{\Phi} = 28.612732 rd/d$ With the ratio $n_{\tilde{l}}/n_{\tilde{g}} = 1/7.59160$ deduced from the preceding equations, we finally obtain : $n_{\tilde{l}} = -4.3407869 rd/d$ and $n_{\tilde{g}} = 32.953519 rd/d$.

Therefore it is possible to modelize completely the free rotational motion, at the condition that the angle J is known, which is not the case. As an example Souchay and Bouquillon (2004) gave the curve of the free motion for a value $J = 1"$.

5. FORCED MOTION : PRECESSION AND NUTATION

Long periodic variations

The method to calculate the coefficients of nutation and precession is taken from Kinoshita(1977ш and has been applied extensively for the Earth (Souchay et al.,1999)

Finally, the expressions of the nutations in longitude and in obliquity are given by the following formulas :

$$
\begin{aligned}
\Delta\psi = \frac{K\cos I}{2} \times \Bigg[& \left(3e + \frac{27}{8}e^3 + C[\frac{e}{2} - \frac{e^3}{12}]\right)\frac{\sin M}{\dot{M}} \\
& + \left(\frac{9}{2}e^2 + \frac{7}{2}e^4 + C[-1 + \frac{5}{2}e^2 - \frac{41}{48}e^4]\right)\frac{\sin 2M}{2\dot{M}} + \left(\frac{53}{8}e^3 + C[-\frac{7}{2}e + \frac{123}{16}e^3]\right)\frac{\sin 3M}{3\dot{M}} \\
& + \left(\frac{77}{8}e^4 + C[-\frac{17}{2}e^2 + \frac{115}{6}e^4]\right)\frac{\sin 4M}{4\dot{M}} - C\frac{845}{48}e^3\frac{\sin 5M}{5\dot{M}} - C\frac{533}{16}e^4\frac{\sin 6M}{6\dot{M}}\Bigg] \\
& - S(\frac{e}{2} - \frac{e^3}{24})\frac{\cos M}{\dot{M}} + S(1 - \frac{5}{2}e^2 + \frac{37}{48}e^4)\frac{\cos 2M}{2\dot{M}} \\
& + S(\frac{7}{2}e - \frac{123}{16}e^3)\frac{\cos 3M}{3\dot{M}} + S(\frac{17}{2}e^2 - \frac{115}{6}e^4)\frac{\cos 4M}{4\dot{M}} \\
& + S\frac{845}{48}e^3\frac{\cos 5M}{5\dot{M}} + S\frac{533}{16}e^4\frac{\cos 6M}{6\dot{M}}\Bigg]
\end{aligned} \qquad (4)
$$

$$
\begin{aligned}
\Delta\varepsilon = -\frac{K\sin I}{2} \times \Bigg[& (-\frac{e}{2} + \frac{e^3}{24})\frac{C\cos M}{\dot{M}} + (\frac{e}{2} - \frac{e^3}{12})\frac{S\sin M}{\dot{M}} \\
& + (1 - \frac{5}{2}e^2 + \frac{37}{48}e^4)\frac{C\cos 2M}{2\dot{M}} - (1 - \frac{5}{2}e^2 + \frac{41}{48}e^4)\frac{S\sin 2M}{2\dot{M}} \\
& + (\frac{7}{2}e - \frac{123}{16}e^3) \times \frac{(C\cos 3M - S\sin 3M)}{3\dot{M}} + (\frac{17}{2}e^2 - \frac{115}{6}e^4) \times \frac{(C\cos 4M - S\sin 4M)}{4\dot{M}} \\
& + \frac{845}{48}e^3 \times \frac{(C\cos 5M - S\sin 5M)}{5\dot{M}} + \frac{533}{16}e^4 \times \frac{(C\cos 6M - S\sin 6M)}{6\dot{M}}\Bigg]
\end{aligned} \qquad (5))
$$

Where M is Eros' mean anomaly, I the obliquity, and with $K = \frac{3n^2}{\omega_3} \times H_d$ With $H_d = 2C - A - B/2C$. We thus get the numerical value for K : $K = 30404".165/cy$. C and S are terms dependent on slow orbital parameters which can be taken as constant terms for a long time span (Souchay et al.,2003)

We can notice that because of the large value of the eccentricity ($e \approx 0.222$) the sub-harmonics of M remain relatively large, whereas in the case of the nutation of the Earth, for which the eccentricity is small ($e \approx 0.0167$) the corresponding amplitudes decline very fastly.

The Oppolzer terms, which make the difference between the nutations for the axis of angular momentum and the axis of figure have been calculated ion detail by Souchay et al. (2003), and are very small, so that we can assimilate the nutation of the axis of figure to that of the angular momentum.

Finally, we obtain the following numerical results for the nutation respectively in longitude $\Delta\psi$, and in obliquity$\Delta\varepsilon$.

$$\begin{aligned}\Delta\psi = 0".590\sin M + 0".042\cos M - 0".192\sin 2M - 0".166\cos 2M - 0".128\sin 3M \\ -0".087\cos 3M - 0".054\sin 4M - 0".035\cos 4M - 0".024\sin 5M \\ -0".014\cos 5M - 0".008\sin 6M - 0".005\cos 6M \end{aligned} \quad (6)$$

$$\begin{aligned}\Delta\varepsilon = 2".420\sin M - 4".054\cos M - 9".614\sin 2M + 16".037\cos 2M - 5".073\sin 3M \\ +8".464\cos 3M - 2".052\sin 4M + 3".424\cos 4M - 0".853\sin 5M \\ +1".423871\cos 5M - 0".299905\sin 6M + 0".500370\cos 6M \end{aligned} \quad (7)$$

Notice the very large ratio (roughly 27) of the leading amplitude in obliquity, with respect to the leading amplitude in longitude, which is explained by the value very close to 90° of Eros obliquity, and which is the contrary of what happens for the Earth. Moreover, the leading amplitude with respect to the same effect of the Sun on the nutation of the Earth, is also much larger (16" instead of 0.5"). This is due to the large value of Eros' dynamical ellipticity H_d.

Eros' precession is calculated by integrating the constant part of the potential. It is given by the following expression, at the $4^{th\cdot}$ order of the eccentricity :

$$\dot{\psi} \approx K \times \left[1 + \frac{3}{2}e^2 + \frac{15}{8}e^4\right] \cos I \quad (8)$$

We thus obtain the numerical value : $\dot{\psi} = 2".840133/y$. We can thus observe that this rate, which represents the displacement of the mean Eros equinox along the orbit plane, is very small in comparison with the Earth, because of the value of the obliquity I, close to 90°. Thus, according to this precession rate, the Eros equinox accomplishes one revolution in more than 450 000 years, to be compared with the 26 000 years precession cycle for the Earth.

The curve of the figure axis given analytically in this paper has been compared with that given by Miller et al., and the agreement is fairly good (Souchay et al.,2003)

Short periodic variations

The short periodic variations of Eros'rotation are coming from the fact that the asteroid has a strongly triaxial shape. This results in terms in the potential exerted by the Sun with rate close to 2ω, where ω is Eros'rotational rate (Kinoshita,1977;Souchay et al.,1999). These terms have been computed by Souchay and Bouquillon (2004), and the related coefficients of the nutation $\Delta\psi$ are listed in the table 1 below. The coefficients in obliquity are exactly the same with inversion of the sine and cosine parts. The combination of the Fourier terms with close periods leads both for $\Delta\psi$ and $\Delta\varepsilon$ to a beating with amplitudes oscillating between 0 and 0".02.

Table 1: Coefficients of the high-frequency forced nutation in longitude $\Delta\psi^{f}_{forced}$ of Eros. ω is the argument for the sidereal Eros'rotation, and M is Eros' mean anomaly.

2ω	0.0846	0.0000	$2^h38^{mn}06^s$
$2\omega - M$	0.4890	-0.2610	$2^h38^{mn}08^s$
$2\omega + M$	-0.4182	-0.2517	$2^h38^{mn}05^s$
$2\omega - 2M$	-3.4287	2.0655	$2^h38^{mn}10^s$
$2\omega + 2M$	3.3211	1.9900	$2^h38^{mn}03^s$
$2\omega - 3M$	-2.7241	1.6361	$2^h38^{mn}11^s$
$2\omega + 3M$	2.6271	1.5746	$2^h38^{mn}02^s$
$2\omega - 4M$	-1.4715	0.8831	$2^h38^{mn}13^s$
$2\omega + 4M$	1.4636	0.8489	$2^h38^{mn}00^s$
$2\omega - 5M$	-0.7662	0.4592	$2^h38^{mn}15^s$
$2\omega + 5M$	0.7357	0.4409	$2^h37^{mn}58^s$
$2\omega - 6M$	-0.3236	0.1937	$2^h38^{mn}16^s$
$2\omega + 6M$	0.3100	0.1858	$2^h37^{mn}57^s$

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