NEW HARMONIC DEVELOPMENT OF THE EARTH TIDE-GENERATING POTENTIAL

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EXTENDED ABSTRACT. *(The complete paper is published in the Journal of Geodesy, 2004,* **77***, 829-838)*

Doodson (1921) first performed an accurate representation of the Earth tide-generating potential (TGP) by harmonic series. Subsequent expansions were done by Cartwright and Tayler (1971), Cartwright and Edden (1973), Büllesfeld (1985), Xi (1987, 1989), and Tamura (1987, 1995). The latest and to date most accurate harmonic developments of the TGP have been made by Hartmann and Wenzel (1994, 1995) and Roosbeek (1996).

The classical representation of the Earth TGP generated by external attracting bodies (the Moon, Sun, planets) at an arbitrary point P on the Earth's surface at epoch t is

$$V(t) = \sum_j \mu_j \sum_{n=2}^{\infty} \frac{r^n}{r_j^{n+1}(t)} P_n \left(\cos \psi_j(t)\right) \tag{1}$$

where V is the value of the TGP at P; r is the geocentric distance to P; μ_j, r_j are, respectively, the gravitational parameter and geocentric distance to the j^{th} body; ψ_j is the angle between P and the j^{th} body as seen from the Earth center; P_n is the Legendre polynomial of degree n.

The expression (1) is expanded in our study as

$$V(t) = \sum_{n=2}^{\infty} \sum_{m=0}^{n} V_{nm}(t) = \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{r}{R_E}\right)^n \bar{P}_{nm} \left(\sin \varphi'\right) \left[C_{nm}(t) \cos m\theta^{(A)}(t) + S_{nm}(t) \sin m\theta^{(A)}(t)\right] \tag{2}$$

where

$$C_{nm}(t) = \frac{1}{2n+1} \sum_j \frac{\mu_j}{R_E} \left(\frac{R_E}{r_j(t)}\right)^{n+1} \bar{P}_{nm} \left(\sin \delta_j(t)\right) \cos m\alpha_j^{(A)}(t) \tag{3}$$

$$S_{nm}(t) = \frac{1}{2n+1} \sum_j \frac{\mu_j}{R_E} \left(\frac{R_E}{r_j(t)}\right)^{n+1} \bar{P}_{nm} \left(\sin \delta_j(t)\right) \sin m\alpha_j^{(A)}(t) \tag{4}$$

and R_E is the mean Earth equatorial radius; $\alpha_j^{(A)}(t)$, $\delta_j(t)$ are, respectively, the instantaneous right ascension and declination of the j^{th} body referred to the true geoequator of epoch t with an origin point A - that being the projection of the mean equinox of date; $\theta^{(A)}(t)$ is the local sidereal time at P reckoned from the same point A - so that it is related to the Earth fixed east longitude (from Greenwich) λ of P simply as

$$\theta^{(A)}(t) = \lambda + GMST \tag{5}$$

($GMST$ is Greenwich Mean Sidereal Time defined by a well-known expression by Aoki et al., 1982); φ' is the geocentric latitude of the point P; and $\bar{P}_{nm}$ is the normalized associated Legendre function related to the unnormalized one P_{nm} as

$$\bar{P}_{nm} = N_{nm} P_{nm} \tag{6}$$

where

$$N_{nm} = \sqrt{\frac{\delta_m (2n+1)(n-m)!}{(n+m)!}}$$

and

$$\delta_m = \begin{cases} 1, & if \quad m = 0 \\ 2, & if \quad m \neq 0. \end{cases}$$

The classical expression for the TGP (1) has to be completed by some additional terms reflecting the main effect of Earth's flattening (Wilhelm 1983; Dahlen 1993; Hartmann and Wenzel 1995; Roosbeek 1996) which can be re-written as follows

$$V^{fl}(t) = \frac{r}{R_E} \left(\bar{P}_{10} \left(\sin \varphi'\right) C_{10}(t) + \bar{P}_{11} \left(\sin \varphi'\right) \left[C_{11}(t) \cos \theta^{(A)}(t) + S_{11}(t) \sin \theta^{(A)}(t) \right] \right) \tag{7}$$

where

$$C_{10}(t) = \sqrt{\frac{15}{7}} \frac{\bar{J}_2}{R_E} \sum_j \mu_j \left(\frac{R_E}{r_j(t)} \right)^4 \bar{P}_{30} \left(\sin \delta_j(t)\right) \tag{8}$$

$$C_{11}(t) = \sqrt{\frac{10}{7}} \frac{\bar{J}_2}{R_E} \sum_j \mu_j \left(\frac{R_E}{r_j(t)} \right)^4 \bar{P}_{31} \left(\sin \delta_j(t)\right) \cos \alpha_j^{(A)}(t) \tag{9}$$

$$S_{11}(t) = \sqrt{\frac{10}{7}} \frac{\bar{J}_2}{R_E} \sum_j \mu_j \left(\frac{R_E}{r_j(t)} \right)^4 \bar{P}_{31} \left(\sin \delta_j(t)\right) \sin \alpha_j^{(A)}(t) \tag{10}$$

$\bar{J}_2$ is the normalized value for the dynamical form-factor of the Earth ($\bar{J}_2 = J_2/N_{20}$).

The coefficients $C_{nm}(t)$, $S_{nm}(t)$ contain information about instantaneous positions of the attracting bodies at every epoch t at which one calculates the TGP value $V(t) + V^{fl}(t)$. Angles $\alpha_j^{(A)}(t)$ and $\delta_j(t)$ in expressions (3), (4), (8)-(10) are reckoned along and from the true geoequator of the epoch t, so the relevant values for the coefficients $C_{nm}(t)$, $S_{nm}(t)$ fully take into account the effects of both precision and nutation in obliquity. Because of the choice for the origin point A nutation in longitude is not included to the values for $C_{nm}(t)$, $S_{nm}(t)$. Otherwise, one would have to repeatedly take the same effect into account in (5), i.e. substitute there Greenwich True Sidereal Time for Greenwich Mean Sidereal Time, what is more complicated. (When expanding (1) to (2) one gets just differences $\theta^{(A)}(t) - \alpha_j^{(A)}(t)$ between the local sidereal time at P and the right ascension of every perturbing body.)

Having harmonic expansions for $C_{nm}(t)$, $S_{nm}(t)$ one can further calculate the time-dependent values of the TGP at an arbitrary point $P(r, \phi', \lambda)$ on the Earth's surface by using relations (2), (5) and (7). The tidal acceleration along the Earth radius (or "the gravity tide") is obtained as the radial derivative of the TGP

$$g(t) \equiv \frac{\partial \left(V(t) + V^{fl}(t) \right)}{\partial r} = \sum_{n=2}^{\infty} \frac{n}{r} \sum_{m=0}^{n} V_{nm}(t) + \frac{1}{r} V^{fl}(t). \tag{11}$$

In our work the coefficients $C_{nm}(t)$, $S_{nm}(t)$ have been directly expanded to finite second-order Poisson series of the following form

$$C_{nm}(t) \left[S_{nm}(t) \right] \approx \sum_{k=1}^{N} \left[\left(A_{k0}^c + A_{k1}^c t + A_{k2}^c t^2 \right) \cos \omega_k(t) + \left(A_{k0}^s + A_{k1}^s t + A_{k2}^s t^2 \right) \sin \omega_k(t) \right] \tag{12}$$

where A^c_{k0}, A^c_{k1},...,A^s_{k2} are constants, and $\omega_k(t)$ are some pre-defined arguments which are assumed to be forth-degree polynomials of time

$$\omega_k(t) = \nu_k t + \nu_{k2}t^2 + \nu_{k3}t^3 + \nu_{k4}t^4. \qquad (13)$$

For that we first calculated numerical values for the coefficients $C_{nm}(t)$, $S_{nm}(t)$ of the TGP expansion according to (3), (4), (8)-(10) at every six hours within 1000-3000. The latest JPL long-term ephemeris DE/LE-406 (Standish, 1998) was employed as a source of the Moon, Sun and planets coordinates. When calculating the coefficients we used values for the planetary gravitational parameters from Standish (1998) and values for $\bar{J}_2$ and R_E from the IERS Conventions (McCarthy and Petit, 2003). (The value for the latter constant which further has to be used in (2) and (7) along with expansions of the coefficients is 6378136.3 m.)

The arguments (13) in expansion (12) were selected as follows. From Simon et al. (1994) we took complete fourth-order polynomial expressions for mean longitude of the ascending node of the Moon Ω, for Delaunay variables D, l', l, F (mean elongation of the Moon from the Sun, mean anomaly of the Sun, mean anomaly of the Moon, and mean longitude of the Moon subtracted by Ω, respectively), and for mean longitudes of Venus, Jupiter, Mars, Saturn and Mercury. (As Hartmann and Wenzel (1994) showed the attraction of other planets is negligible when calculating the TGP.) The set of arguments of the Moon, Sun and planets motion referred to the mean ecliptic and equinox of date was chosen using the latest value of the precession constant. Then we approximately evaluated a spectrum of the tabulated numerical values of $C_{nm}(t)$, $S_{nm}(t)$ at numerous combinations of multipliers of the arguments' frequencies by using classical FFT procedure. After that, every wave in the spectrum which had a preliminary amplitude exceeding or equal to minimum level $10^{-8}m^2/s^2$ was expanded to Poisson series by using the improved technique of spectral analysis (Kudryavtsev, 2004). Analysis proves that only coefficients $C_{nm}(t)$, $S_{nm}(t)$ of degree $n \leq 6$ have amplitudes increasing the chosen minimum level. The total number of waves included in the final spectrum of the TGP, named KSM03, is equal to 26,753 - what is the total sum of all N in expansions (12) made for every coefficient.

The complete set of coefficients of KSM03 development of the TGP can be found at http://lnfm1.sai.msu.ru/neb/ksm/tgp/coeff.zip. (The description of the data format is done in file http://lnfm1.sai.msu.ru/neb/ksm/tgp/readme.pdf.)

The accuracy of KSM03 expansion of the Earth TGP has been checked by computation of the gravity tide values (11) at a mid-latitude station. As the latter we choose Black Forest Observatory (BFO) Schiltach: $r = 6366836.9$ m, $\phi = 48.3306°$N, $\lambda = 8.3300°$E at which Hartmann and Wenzel (1995) and Roosbeek (1996) also computed the tidal gravity by using their expansions of the TGP. First, we calculated the total tidal gravity at that station by means of strict expressions (2)-(4) and (7)-(11) where the Moon, Sun and planets spherical coordinates were computed using the most precise JPL ephemeris DE/LE-405 (Standish, 1998). The gravity tides at BFO were calculated at every hour within the whole time span covered by that ephemeris, 1600-2200. Then we calculated the gravity tides at the same point and at the same set of epochs by using KSM03 expansion of the TGP and compared the results with the exact values. The maximal deviation between the two sets of data at any epoch within the whole time span of six hundred years length does not exceed 0.39 nGal (1 nGal=$10^{-11}m/s^2$). The corresponding r.m.s. difference between the data over the same interval is less than 0.025 nGal. It exceeds the accuracy of any previously made harmonic development of the TGP in time domain by a factor of at least three.

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