NEW MODELS FOR REDUCTION OF THE VLBI DATA

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ABSTRACT. For improvement of accuracy of the VLBI reduction new models of motion the VLBI stations due to atmospheric loading were developed. New models of the atmospheric radio refraction and tropospheric delay are based on numerical integration of the index of refraction depending on the local surface pressure, temperature and the partial pressure of water vapor.

1. INTRODUCTION

One of the methods to improve the accuracy of radio interferometry is development of new models for the delay and delay rate observables. The delay and delay rate models are the sum of components: geometry of baseline and source, clock and the atmosphere. The model of the station motion due to atmospheric loading and nutation theory (geometry components) were improved and used in the ARIADNA software. Atmospheric loading was computed by convolving a Green's function with the surface atmospheric pressure distribution.

Model of motion of each the VLBI station is represented as sum of annual, semi-annual and semi-diurnal components. In addition to this deterministic signal the vertical site displacement is corrected for the local pressure anomaly.

Effects of the atmosphere on the radio wave propagation are the atmospheric radio refraction and additional tropospheric delay. New model of the atmospheric radio refraction which is more precise at low elevation angles was implemented. The developed algorithm was used for calculation of the additional tropospheric delay. Instead of use of the mapping functions the path length through the troposphere was calculated by the numerical integration of the index of refraction which depends on the surface pressure, temperature and the partial pressure of water vapor.

2. DISPLACEMENT DUE TO THE ATMOSPHERIC LOADING

Effects of atmospheric loading can be computed by convolving a Green's function with distribution of surface atmospheric pressure. Green's function describe the response of an elastic Earth to a point load on its surface.

The vertical u_v and tangential u_θ displacement can be written as Farrell's elastic Green's functions (Farrell, 1972):

$$u_v = \frac{m(\theta,\lambda)}{M} R \sum_n h'_n P_n(\cos\psi), \tag{1}$$

$$u_\theta = \frac{m(\theta,\lambda)}{M} R \sum_n l'_n \frac{\partial P_n(\cos\psi)}{\partial\psi}, \tag{2}$$

where $m(\theta,\lambda)$ is mass of a point load with co-latitude θ and longitude λ; M, R are mass and radius of the Earth, h'_n, l'_n are the loading numbers, ψ is the arc between load and point of measurement of surface displacement. Mass of load can be found from local pressure $p(\theta,\lambda)$:

$$m(\theta,\lambda) = \frac{1}{g}[p(\theta,\lambda) - \overline{p(\theta,\lambda)}]ds,$$

where g is the gravitational acceleration, $\overline{p(\theta,\lambda)}$ is mean surface pressure over a region with square equals to ds.

Components of horizontal displacement in the eastern u_e and northern u_n directions are equal to

$$u_e = -u_\theta \sin\varphi, \quad u_n = -u_\theta \cos\varphi, \tag{3}$$

where φ is azimuth of load from a point of measurement.

Total displacement in point of measurement can be found by summation of u_v, u_e, u_n for different $m(\theta,\lambda)$ over the Earth's surface.

Displacements of each VLBI station were computed using four times daily global surface pressure values on a $5° \times 5°$ grid. The NCEP spherical harmonic coefficients data were used. The coefficients can be found on site of the SBA (SBA, 2003) :

$$p(\theta,\lambda) = (2-\delta_{0m}) \sum_{m=0}^{M} \sum_{n=m}^{N+J} (a_n^m \cos m\lambda - b_n^m \sin m\lambda) P_n^m(\cos\theta),$$

where δ_{0m} is the Kronecker delta function, and $J = 0$ or M depending on whether the truncation is triangular or rhomboidal, respectively. Surface displacements are calculated using an Earth model in which the oceans respond as an inverted barometer to atmospheric pressure loading.

As example the vertical, eastern and northern displacement of Fortaleza station due to the atmospheric loading are shown on left part of Fig.1 and correspondence of displacement and local pressure (right part).

The largest variation (peak-to-peak) in the radial displacement are of order of 10 mm and occur on different timescales: from 12 hours to 1 year. It corresponds pressure variations over a point of measurement of order of 20–30 mbar. Horizontal displacements have amplitude of order of 1 mm. Main period of variations is equal to 1 year. Semi-diurnal displacements are observed for equatorial stations.

Model of vertical displacement is represented by sum of three terms and additional term that corresponds a linear regression with coefficient k of local pressure variations ΔP and displacement (right upper plot on Fig.1). Only periodic terms are included in model of horizontal displacement because there are no correlations with local pressure variations:

$$u_r = \sum_{i=1}^{3}[(a_r)_i \cos(Arg_i) + (b_r)_i \sin(Arg_i)] + k\Delta P,$$

$$u_{e,n} = \sum_{i=1}^{3}[(a_{e,n})_i \cos(Arg_i) + (b_{e,n})_i \sin(Arg_i)],$$

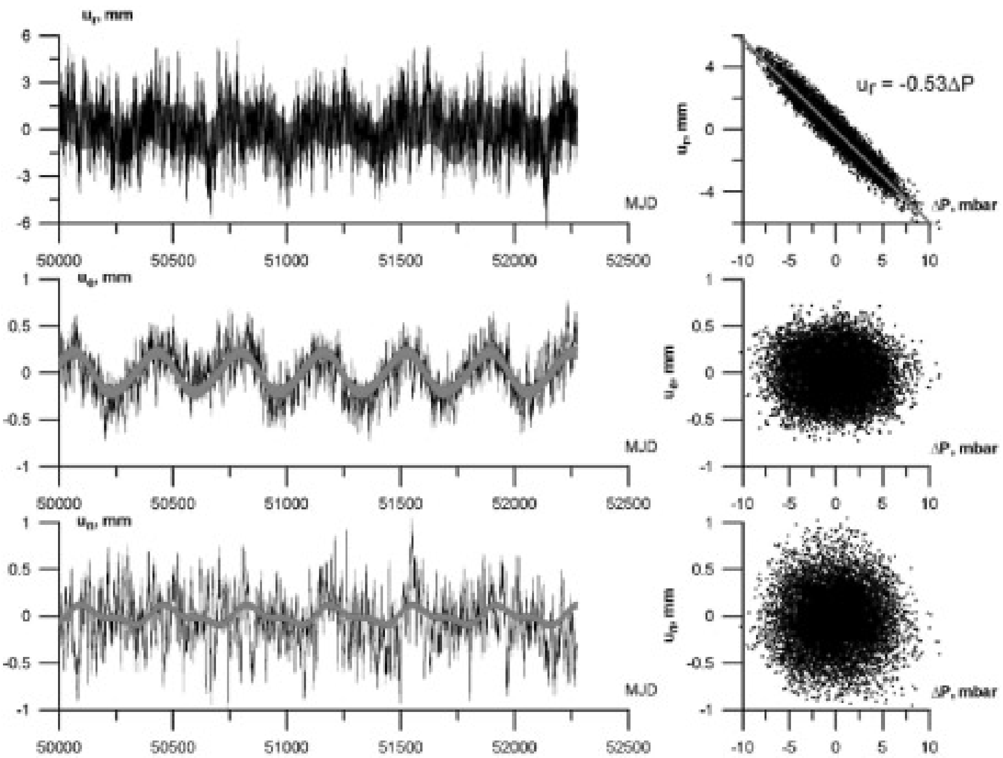

Figure 1: Displacement of Fortaleza station.

where $Arg_i = 2\pi/T_i\Delta t$, $T_1 = 1$ year, $T_2 = 1/2$ year,$T_3 = 1/2$ day,$\Delta t = MJD - 44239.0$ (Jan.1 1980).

Coefficients k, $(a_r)_i, (b_r)_i$, $(a_{e,n})_i, (b_{e,n})_i$, $i = 1, 2, 3$ were calculated for each VLBI station and can be sent by author on request.

3. TROPOSPHERIC DELAY AND REFRACTION

In order to calculate tropospheric delay and refraction the model of standard atmosphere is used. In spherical symmetric atmosphere the additional signalTs path between two layers with radius S_1 and S_2 is

$$S = \int_{S_1}^{S_2} (n-1) \sec\theta ds,$$

where n is the index of refraction and θ is zenith angle of the observed source. As shown by Murray (1983) the index of refraction for radio waves depends on density of air ρ and density of water vapour ρ_w:

$$S = S_d + S_w = \int_{S_1}^{S_2} \beta_d \rho \sec\theta ds + \int_{S_1}^{S_2} (\beta_w - \beta_d)\rho_w \sec\theta ds,$$

where β_w, β_d are parameters depending on the surface pressure, temperature and the partial pressure of water vapor.

For $\theta = 0°$ we have zenith tropospheric delay Z_d or Z_w and for $\theta \neq 0°$ we can write:

$$S = Z_d + \Delta Z_d + Z_w + \Delta Z_w = Z_d\Big(1 + \frac{\Delta Z_d}{Z_d}\Big) + Z_w\Big(1 + \frac{\Delta Z_w}{Z_w}\Big) = Z_d \cdot F_d + Z_w \cdot F_w, \qquad (4)$$

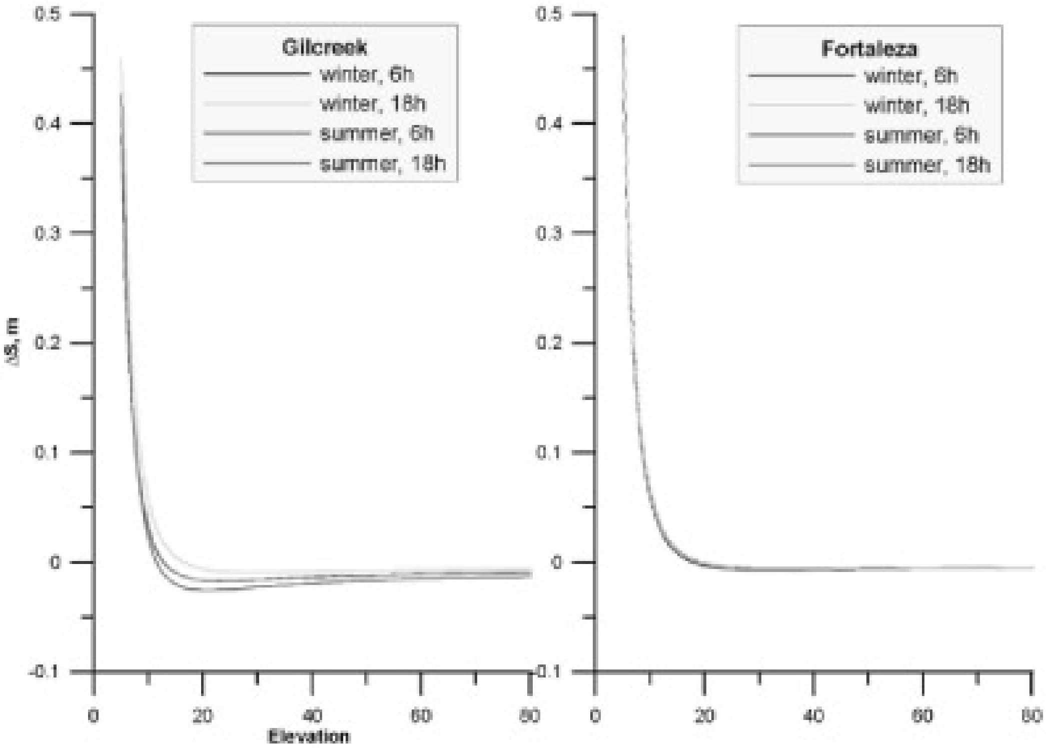

Figure 2: Tropospheric delay differences (direct calculation minus the Niell's mapping functions) for Gilcreek and Fortaleza stations.

where F_d, F_w are the mapping functions. Tropospheric delay was written in form (4) in order to compare results with traditional approach when delay in the troposphere is calculated as product of zenith delay and the mapping function.

Tropospheric delay was calculated by numerical integration of the index of refraction depending on the local surface pressure, temperature and the partial pressure of water vapor. Difference of tropospheric delay that was obtained by numerical integration and by calculation of the Niell's mapping functions is shown on Fig. 2 for equatorial (Fortaleza) and northern (Gilcreek) stations.

One can see from Fig.2 that there is diurnal variation of tropospheric delay for Gilcreek. Amplitude of this variation can reach 2-3 cm for elevation angles in range $10° \div 30°$.

Radio refraction Δz in spherical symmetric atmosphere is equal to

$$\Delta z = \int_{\theta_0}^{\theta_1} \frac{d\ln n}{d\ln s}\left(1 + \frac{d\ln n}{d\ln s}\right)^{-1} d\theta,$$

where θ_0 is apparent zenith angle of the source, θ_1 is zenith angle of the source when the atmosphere is absent, s is the length of radio wave path. The empirical model used in CALC software was compared with values Δz (Fig. 3).

There is significant difference for low elevation angles between two models: for angles $< 10°$ this difference can reach $150 \div 200$ arcsec.

3. CONCLUSIONS

New models of displacements of the VLBI sites were calculated. Tropospheric delay and radio refraction were calculated by numerical integration of the index of refraction depending on the local surface pressure, temperature and the partial pressure of water vapor.

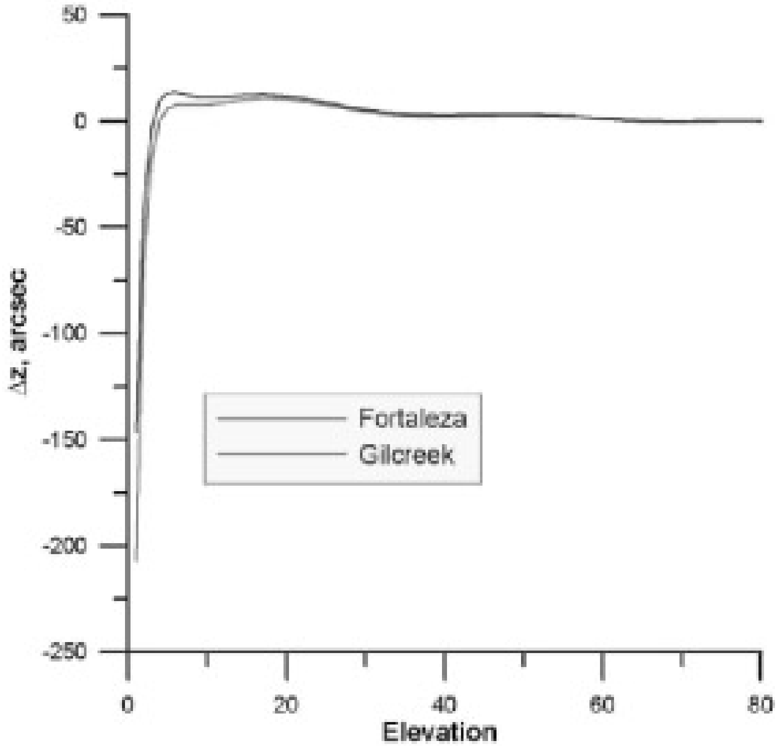

Figure 3: Radio refraction differences (direct calculation minus empirical model used in CALC software) for Gilcreek and Fortaleza stations.

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4. REFERENCES

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