THE DEFINITION OF THE FORCED NUTATIONS BY THE FINITE ELEMENT METHOD

M.V. LUBKOV
Poltava Gravimetrical Observatory of National Academy of Sciences of Ukraine
Myasoyedova 27/29, 36029 Poltava, Ukraine
e-mail: pgo@poltava.ukrtel.net

ABSTRACT.

It is presented the variational finite element approach for modelling EOP. As for approbation of this approach it has made the definition of the luni-solar forced nutation of the simplified dynamical earth model. Three main nutation terms, obtaining according to presented above method from PREM-model of the Earth, have compared with respective data of Molodensky and Wahr.

1. THE FORMULATION OF THE PROBLEM

Let us consider, that Earth is two axes ellipsoid of revolution, has absolutely rigid inner core, stratificated liquid outer core, and also has elastic isotropic nonhomogenous mantle and crust. Liquid core can oscillate relatively the mantle. The Earth is the self-gravitating, hydrostatically prestressed body. It rotates around own axis, taking under influence from the luni-solar attraction. The influence of an ocean and atmospheric loads not takes into account.

We shall neglect the flattening and the rotational effects in the elastic shell, as they are vanishing small, and shall take into account influence of relative, Euler, Coriolis, centrifugal forces on the elliptical liquid core. Then, using operation for eliminating static meanings of the stress tensor in the elastic shell and static pressure in the liquid core, obtained from hydrostatically equilibrium conditions of the Earth, come to the equilibrium equation in the elastic shell and to the motion equation in the liquid core relatively tidal actions, presented in Tisserand reference system (X,Y,Z) of the mantle:

$$0 = grad(V_e + V_1 + u_R g(R)) - div\vec{u} grad W_g + \frac{1}{\rho} div \hat{P}_1; \qquad (1)$$

$$\ddot{\vec{u}} + 2\vec{\Omega} \times \dot{\vec{u}} = grad[V_e + V_1 + u_R g(R) + (1 + \frac{\sigma}{\Omega}\phi)] - 2\frac{\sigma}{\Omega}\frac{\partial \phi}{\partial z}\vec{e}_z - \frac{1}{\rho} grad p_1. \qquad (2)$$

Where $V_e = K(xz\cos\sigma t + yz\sin\sigma t)$ - tesseral part of tidal wave potential; $\phi = -\Omega^2\varepsilon(xz\cos\sigma t + yz\sin\sigma t)$ - changing of centrifugal potential due to nutation; $V_1 = kV_e$ - potential due to tidal deformations; W_g - self-gravitating potential; ρ - density; ε - polar motion radius of the tidal wave, σ - frequency of the tidal wave; R - radius of the earth point; u_R - radial displacement component; $g(R)$ - gravity acceleration; $\hat{P}_1$ - changing stress tensor, due to tidal shell deformation; p_1 - tidal pressure changing in the liquid core, k - Love number.

Let us assume that vibrations into liquid core are results from the forced tidal frequency σ. Make up Lagrang functionals for the elastic shell and for the liquid core respectively in the displacement form in the cylindrical coordinate system (z, r, φ), where axis r coincides with the Tisserand axis Z:

$$E_1 = \pi \iint_{F_s} [c_1(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 4c_2\varepsilon_{zr}^2 + 2c_3(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})] r dz dr -$$
$$-\pi \iint_{F_s} [2(1+k)Krw + w(\frac{\partial w}{\partial z}\cos\alpha + 2\frac{\partial u}{\partial z}\sin\alpha) g(R) + 2w(w\cos\alpha + 2u\sin\alpha) \times \qquad (3)$$
$$\times g_R' \cos\alpha - 2w(\frac{\partial w}{\partial z} + 2(\frac{\partial u}{\partial r})) g(R)\cos\alpha + 2(1+k)Kzu + u(2\frac{\partial w}{\partial r}\cos\alpha + \frac{\partial u}{\partial r}\sin\alpha) \times$$
$$\times g(R) + 2u(2w\cos\alpha + u\sin\alpha) g_R' \sin\alpha - 2u(2\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} + \frac{u}{r}) g(R)\sin\alpha] \rho r dz dr;$$

$$E_2 = \pi \iint_{F_s} [c_4(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 2c_4(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})] r dz dr -$$
$$-\pi \iint_{F_s} [\sigma(\sigma + 2\Omega) w^2 + \sigma^2 u^2] \rho r dz dr - 2\pi \iint_{F_s} [\Omega(\Omega+\sigma)\varepsilon w + \Omega(\Omega-\sigma)\varepsilon u] \rho r dz dr + \qquad (4)$$
$$+\pi \iint_{F_s} [2(1+k)Krw + w(\frac{\partial w}{\partial z}\cos\alpha + 2\frac{\partial u}{\partial z}\sin\alpha) g(R) + 2w(w\cos\alpha + 2u\sin\alpha) g_R' \cos\alpha +$$
$$+2(1+k)Kzu + u(2\frac{\partial w}{\partial r}\cos\alpha + \frac{\partial u}{\partial r}\sin\alpha) g(R) + 2u(2w\cos\alpha + u\sin\alpha) g_R' \sin\alpha] \rho r dz dr;$$

Here $c_1 = \frac{\lambda+4\mu}{3}$; $c_2 = \mu$; $c_3 = \frac{\lambda-2\mu}{3}$; $c_4 = \frac{\lambda}{3}$;λ, μ - Lame coefficients; ε_{ij} - strain tensor components; w, u - displacement components along axes z, r respectively; F_s - meridian cross section area of the Earth; $\cos\alpha = \frac{z}{R}$; $\sin\alpha = \frac{z}{R}$.

2. THE FINITE ELEMENT METHOD RESOLVING PROBLEM

For resolving system of the equations (1, 2), taking into account rigidity of the inner core and absence of the surface earth loads, let us apply the finite element method, based on the variational Lagrang principal in the displacement form, which tends to resolving system of the variational equations such as:

$$\delta E_1(u_i) = 0; \qquad \delta E_2(u_i) = 0. \qquad (5)$$

For resolving system of the equations (5), used 8-node isoparametric quadrilateral curve finite element.

In this short presentation we omit the finite element procedure of resolving these variational equations, we are only present meanings of the main nutation terms, defining according to described above method from PREM-model of the Earth, and also the respective data of Molodensky(1961) and Wahr(1981).

	Dynamical model	Molodensky	Wahr
18,6 year: $\Delta\Theta$	9",2052	9",2044	9",2025
$\Delta\Psi\sin\Theta$	-6", 8454	-6", 8441	-6", 8416
Semiannual:$\Delta\Theta$	0", 5713	0", 5719	0", 5736
$\Delta\Psi\sin\Theta$	-0", 5229	-0", 5232	-0", 5245
Fortnightly:$\Delta\Theta$	0", 0968	0", 0972	0", 0977
$\Delta\Psi\sin\Theta$	-0", 0893	-0", 0899	-0", 0905