

# INFLUENCE OF EARTH'S ROTATION RATE AND DEFORMATIONS ON PRECESSION-NUTATION

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**ABSTRACT.** In this study, we investigate, first, the couplings between axial and equatorial components appearing when developping the dynamical equations of the Earth rotation at the second order, and second, the effects of variations of the dynamical ellipticity, giving rise to changes in the gravitational lunisolar torque and therefore in precession-nutation. We provide the computation of these effects for a refined Earth model with elastic mantle and decoupled liquid core.

## 1. COUPLING BETWEEN EARTH'S ROTATION RATE AND ORIENTATION

The equatorial motion of the instantaneous Earth rotation vector  $\omega = \Omega(m_1 + im_2)$  in the terrestrial frame is given by the general Euler-Liouville equation which is developped to the second order :

$$\dot{m} - i\frac{C-A}{A}\Omega(1+m_3)m + \frac{\dot{c} + i\Omega c}{A} = \frac{\Gamma}{A\Omega} \quad (1)$$

where  $c = c_{13} + ic_{33}$  is the non-diagonal element of the deformation part of the inertia tensor and  $\Gamma$  is the equatorial tidal torque expressed in the terrestrial frame computed from the expression  $\Gamma'$  in the celestial frame by the relation :

$$\Gamma = \Gamma' e^{-i\Phi} \quad (2)$$

in which  $\Phi$  is the sidereal rotation angle.

Variations of Euler's angles are related to the instantaneous vector of rotation by the Euler's kinematical relations :

$$\begin{aligned} \dot{\theta} + i\dot{\Psi} \sin \theta &= -\Omega m e^{i\Phi} \\ \dot{\Phi} + \dot{\Psi} \cos \theta &= \Omega(1+m_3) \end{aligned} \quad (3)$$

Equations (1), (2) and (3) show two couplings between the axial and the equatorial components of the rotation vector of the Earth. One coupling comes from the product  $m \times m_3$  in the left hand side. Another coupling is in the torque  $\Gamma$  which is expressed in the terrestrial frame and is affected by the rotation of angle  $\Phi$  which depends on quantity  $m_3$ . Equation (1) can be solved analytically for a simplified Earth model in order to check the effects of these couplings. It appears to be negligible, at the level of a few microarcseconds.

## 2. VARIATIONS OF THE DYNAMICAL ELLIPTICITY

The tidal gravitationnal torque is derived from the sectorial part of the lunisolar potential  $\phi$  (Sasao et al. 1980) in the celestial reference frame :

$$\Gamma' = \Gamma'^{(1)} + \Gamma'^{(2)} = -i\Omega^2(C - A)\phi - i\Omega^2(\delta C - \delta A)\phi \quad (4)$$

where  $\Gamma'^{(1)}$  is the first-order part of the torque, depending on the lunisolar potential  $\phi$  and on the mean moments of inertia of the Earth, and  $\Gamma'^{(2)}$  is the second-order part of the torque, including the zonal deformations of the Earth :

Assuming that the trace of the inertia tensor is constant, let  $\delta(2A + C) = 0$ , we have :

$$\delta C - \delta A = \frac{3}{2}\delta C \quad (5)$$

The second order torque leads to the main effect on nutation angles. We implemented it considering variations of the dynamical ellipticity as given in the IERS Conventions 2003. In fact, it provides variations for the LOD induced by zonal tides which can be easily converted into variations of axial moment of inertia :

$$\frac{\delta C}{C} = \frac{\delta LOD}{LOD} = -m_3^z \quad (6)$$

where the subscript  $z$  indicates that these rotation rate variations come from zonal tides.  $\overline{LOD}$  is the duration of the mean solar day (86400 s).

## 3. NUMERICAL RESULTS

For the calculation we use the lunisolar potential computed from the lunar theory ELP2000 (Chapront-Touzé & Chapront 1983) and the solar system semi-analytical solution VSOP87 (Bretagnon & Francou 1988), and the use of the GREGOIRE software package developed by J. Chapront (Paris Observatory) and allowing manipulation of large Poisson's series. We develop the Earth model in order to include a liquid core. The main periodical effect is found on the 18.6-year nutation which amplitude should be changed by 208  $\mu$ as in longitude and -10  $\mu$ as in obliquity. The contribution to the precession rate in longitude is -5 mas per century. More details can be found in Lambert & Capitaine (2004).

## 4. REFERENCES

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