A NEW FORMULATION OF THE DAMPING EFFECT IN EARTH'S AND MARS' FREE POLAR MOTION

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ABSTRACT. In this paper, we undertake firstly an extension of Kubo's procedure (1991) for the torque-free rotational motion of a triaxial and non-rigid body, neglecting therefore the external forces and taking into account only the deformation caused by the centrifugal force due to the rotation. Then, the straightforward application of such extension to the axially and elastic Earth and Mars is demonstrated.

The results are shown in terms of Andoyer's variables and its canonically conjugate variables since the corresponding variational equations expressed in terms of these variables seem to be more convenient and permit to separate clearly the different aspects considered in the study of the free polar motion, that is to say, the triaxiality, elasticity and time lag.

Finally, and from our mathematical modelling of the problem, a detailed discussion of the damping of the free polar motion is presented for a better understanding of the Chandler wobble and its maintenance in presence of such dissipation.

1. ANALYTICAL SOLUTION FOR THE FREE POLAR MOTION OF A DEFORMABLE BODY

We have considered an Hamiltonian formulation to express the analytical solution for the free polar motion of an elastic and triaxial body. The explicit form of such solution is:

$$l = -l_1 t + l_2 \sin(2l_1 t) \tag{1}$$

$$J = J_0 \times \exp\{-J_1 t - J_2 \cos(2l_1 t)\} \tag{2}$$

where the coefficients l_i and J_i are the general form:

$$
\begin{aligned}
l_1 &= \frac{C_0\Omega}{\bar{A}}\left[\frac{C_0-\bar{A}}{C_0} - \rho\cos\delta\right] \quad ; \quad l_2 = \frac{C_0\Omega\varepsilon}{2\bar{A}l_1} \\
J_0 &= 0.''5\times\exp\left\{\frac{C_0\Omega\varepsilon}{2\bar{A}l_1}\right\} \quad ; \quad J_1 = \frac{C_0\Omega\rho}{\bar{A}}\sin\delta \quad ; \quad J_2 = \frac{C_0\Omega\varepsilon}{2\bar{A}l_1}
\end{aligned} \tag{3}
$$

and being A_0, B_0 and C_0, the principal moments of inertia of a rigid body, $\bar{A} = \frac{A_0+B_0}{2}$, ε a triaxial parameter, ρ an elastic parameter and δ the phase shift.

From the above equations, we can deduced that in addition to the classical Chandlerian component of the polar motion, an oscillation exists, with frequency twice the frequency of the Chandler wobble and an amplitude proportional to the triaxial coefficient. The tables 1 and 2 show how the phase shift, responsible of the damping effect, combined with parameters related to the triaxiality and elasticity are influencing to the polar motion.

$\delta(°)$	l_1	l_2	$J_0\times 10^5$	J_1	J_2
0	0.01425	-0.00277	0.24174	0.0	-0.00277
1	0.01425	-0.00277	0.24174	0.00011	-0.00277
5	0.01427	-0.00276	0.24174	0.00056	-0.00277

Table 1: Variations of the coefficients l_1, l_2, J_0, J_1 and J_2 for the Earth with changing δ. The units are in radians.

k	$\delta(°)$	l_1	l_2	$J_0\times 10^5$	J_1	J_2
0.15	0	0.02862	-0.037289	0.23353	0.0	-0.03729
0.15	1	0.02862	-0.037288	0.23353	0.00008	-0.03729
0.15	5	0.02863	-0.037267	0.23354	0.00038	-0.03727
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0.25	0	0.02568	-0.041553	0.23254	0.0	-0.04155
0.25	1	0.02568	-0.041551	0.23254	0.00013	-0.04155
0.25	5	0.02571	-0.041509	0.23255	0.00064	-0.04151

Table 2: Variations of the coefficients l_1, l_2, J_0, J_1 and J_2 for Mars with changing δ and the love number k. The units are in radians.

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