NEW APPROACH TO DEVELOPMENT OF MOON ROTATION THEORY

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The external shell of the Moon (its mantle) is considered as non-spherical, rigid layer. Inner shell is liquid core, which occupies ellipsoidal cavity of the Moon. The motion of two-layer Moon is considered in gravitational fields of the Moon and Sun. For study we use combination of two methods. First method was developed earlier and applied for study of interaction in the Earth core-mantle system due to Moon and Sun attraction and was applied in Earth rotation theory (Ferrandiz, Barkin, 2001). Another method is an analytical method of construction of the resonant rotational motion of synchronous satellites and Mercury, considered as non-spherical rigid bodies. This method have been applied to construction of analytical theory of Moon rotation (Barkin, 1989) and here we modified this method with purpose to apply it to study of resonance rotation of two-layer Moon.

Main resonant properties of the Moon motion are described by Cassini's laws (Cassini, 1693) and studied by the famous scientists on the base of rigid and non-spherical Moon models. And here in first we have been obtained and formulated these laws and their generalization for two-layer Moon model. New fine resonant effect was studied and perturbations of the first and second order have been described in analytical form. Our purpose also was to obtain evaluations of periods of free core nutation for some model of the Moon. Canonical equations of motion in Andoyer and Poincare variables were constructed for considered problem in form the more convenient for application of mentioned methods. Force function of considered mechanical system (we take into account second and third harmonics) was constructed. Obtained results illustrate important phenomena of core-mantle interaction of the Moon.

Generalized Cassini's laws for two-layer model of the Moon were described as specially-constructed generating solution on the base of average equations of the first order. They consist additional regularities to classical Cassini's laws connected with liquid core.

1. Vectors of angular velocities and angular momentums of the core and Moon coincide with its polar axis of inertia.

2. The mantle-core system of the Moon rotates as one rigid body about polar axis of inertia in direction of its orbital motion with constant angular velocity equal to mean orbital motion with respect to geocentric ecliptic reference system connected and rotated with mean node of lunar orbit on ecliptic plane. One from the equatorial axes of inertia in mean motion is oriented to the Earth centre.

3. Mean ascending node of the lunar orbit on ecliptic coincides with mean descending node of general plane orthogonal to vectors of angular momentums of the core and Moon, to angular velocity of the Moon. This plane coincides with lunar equator. Normal to ecliptic plane, to equator plane, vectors of angular momentums of the Moon and its core, vector angular velocity

of the Moon are situated in one plane orthogonal to ecliptic plane.

4. Angular momentums of the Moon and its core (and also angular velocity of the Moon) form constant angle with normal to ecliptic plane which equal $\rho = 1^o 32' 48''$ and it is determined from equation (12), (13) in dependence from precession of lunar orbit plane, from fine resonant properties of perturbations in translatory-rotary motion of the Moon and from dynamical oblatenesses of the Moon.

On the next step we have analyzed frequencies of free oscillations of core-mantle system of the Moon. On the base of known data about Moon structure (we have used model of axisymmetrical lunar core of Williams et al. (2003)) we have been obtained the following model values (evaluations) of moments of inertia of the Moon and its core:A=0.393751244, B=0.39384103, C=0.3940000;A_c=0.000236211, C_c=0.000236258(1 unit=mR^2,m and R is a mass and a mean radius of the Moon). Corresponding periods of oscillations were determined on the base of analytical formulae of a developed theory. They are equal:T_1=53855 days and T_2=0.99980095 T_F (T_F=27.212231 days is a sinodic period). Last period determines long period of relative oscillation of the core and mantle T_r=136683 days. The mentioned periods are equal:T_1=147.5 years, T_r=374.2 years. Difference in periods of rotational motion and free oscillation for the Moon consists 7.800 min. For the Earth system this difference consists 4.464 min.

A new effect of a splitting of vectors of angular momentums of the Moon and its core has been established in result of analysis of the role of third harmonic of the force function. The small mutual inclination of angular momentum vectors of the mantle and liquid core (angular splitting in 0.0327 arcsec) was discovered and described analytically.

Developed method let us consecutively determine constant and pure conditionally-periodic perturbations of the first, second and more higher orders for all Poincare variables. Analytical formulae for perturbations in rotation of two-layer Moon were obtained for Andoyer and Poincare variables and their amplitudes were evaluated. General scheme of construction of perturbations of second and higher orders was described. Next stage presents a more difficult problem on construction of solution in neighborhood of discussed in this paper solution. For rigid model of the Moon last problem has been solved earlier and perturbations (including perturbations of the fifth order) were constructed (Barkin, 1989).

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