# GENERALIZED MEAN OF INDIVIDUAL EOP SERIES BY LEAST-SQUARES COLLOCATION TECHNIQUE

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ABSTRACT. To obtain combined solution of the Earth orientation parameters, least-squares collocation method is used. Auto- and crosscovariance functions of signals were calculated. The result corrections achive $\pm 0.25$ mas of arc for polar motion and nutation, and $\pm 0.02$ msec for UT1 – UTC.

## 1. INTRODUCTION

At present observations of the Earth orientation parameters (EOP) are carried out using different observation techniques. The main part of them are VLBI, VLBI-intensives, SLR, LLR and GPS. Every observational program has different set of EOP, different time steps or nonequidistant, properly errors, etc. Therefore combined solution of EOP is needed. This solution has to be equidistant and take into account possible correlations between observational series. The existing reference system IERS (EOP) C04 which is obtained using Allan variance analysis of the difference between series without any reference to combined series (Gambis, 2001). This solution does not take into account possible correlations in observational series and between different observational series, that will be demonstrated in this paper.

At 1997 Prof. Gubanov in his monography (Gubanov, 1997) demonstrated that least-squares collocation technique may be applied to improvement of combined solution of EOP. This method gives two different algorithms of estimation: least-squares collocation and generalized average. The former is used in case of we know reference covariance function and the latter used otherwise. The stochastic approach developed at (Moritz, 1987) allows to takes into account possible correlations in observational series and between each others.

## 2. STRUCTURE OF OBSERVATIONS

There are two types of observational series that are used to calculate combined solution EOP(IERS)C04: longterm and operational available at `http://hpiers.obspm.fr/eop-pc/` Longterm series are updated on this WWW every year. Almost all observations are covers time interval since 1993 up to 2002. Two last years until now are covered by operational EOP series. They are updated a weekly twice. In order to remove low frequency harmonics, e.g. Chandler wobble in polar motion, annual and semiannual components in EOPs, etc. from observational series, time series of differences between observational series and improved reference system were calculated and values of EOP were resulted on the observational time lags by linear interpolation. These differences have linear shift which is due to tidal variations, tectonics movement etc. and white noise which is due to random errors of observations. This trend was calculated and removed from all differences. Let us consider set of observational series in more details.

The time series of polar motion are produced by GPS, VLBI and SLR data centers. The GPS series have 1 day time step and reduced to the midnight of the date. The long term GPS series are calculated by Universitaet Bern GeoForschung Zentrum, and Jet Propulsion Laboratory. The operational GPS series are obtained by the same data centers and some others, but the others series were observed on too short time interval. VLBI time series are calculated by Bundesamt fuer Kartographie und Geodaesie, Goddard Space Flight Center, Institute of Applied Astronomy, Observatoire de Paris and Shangai Observatory. These series are not equidistant and have to be interpolated. The SLR observational series are produced by Space Geodesy Centre, Central Laboratory for Geodesy Analysis, Institute of Applied Astronomy and Glavniy Observatorya Ukraine.

UT1 – UTC is observed by the same Data Centers as polar motion, but there is VLBI-intensive observation technology that calculate only UT1 – UTC. These series have 1 day time step and short series can be processed in combined solution. More detailed descriptions of these series are available on the IERS web-site. The VLBI-intensive programs carry out every day observations, but these are have more low accuracy and precision than other programs. But intensive series need to be interpolated anyway because time lags of observations do not coincide with time lags of EOP(IERS)C04 system.

The observations of nutation are carried out by VLBI techniques only. Because on the IERS website files with observations are sorted by Data Analysis Center, therefore one observational file may contains some different EOPs. These observations are carried out with varying time lags, therefore time step between two observations in the same series may be from 1 day upto 7 days and interpolation is demanded in order to reduce to equidistant time lags.

## 3. TIME SERIES INTERPOLATION BY LEAST-SQUARES COLLOCATION

Let us consider the problem of observations interpolation onto equidistant time grid. On input we have many time series, everywhere have its proper observation program. Some of these programs have time step approximately equal to 1 day, any others — 3 days or 7 days. In general almost all correspond time series are not equidistant, but in order to obtain combined solution we have to reduce these series to uniform time lags. This problem may be solved by interpolation procedure. There are many interpolation techniques, such as polynomial interpolation, interpolation by rational function, spline interpolation, etc. These techniques process series, but the least-squares collocation makes interpolation of statistical characteristic of series — autocovariance function.

Let us consider more detail the least-squares collocation (LSC) technique for individual series. We have to evaluate signal vector $\boldsymbol{s}$ from observational vector $\boldsymbol{l}$, that has random white noise $\boldsymbol{r}$

$$\boldsymbol{l} = \boldsymbol{s} + \boldsymbol{r} \tag{1}$$

If signal and noise does not correlate between each other and we know autocovariance matrix of noise $\boldsymbol{Q}_{rr}$ that problem can be solved by main formula of LSC (Gubanov, 1997)

$$\hat{\boldsymbol{s}} = \boldsymbol{Q}_{sl}\boldsymbol{Q}_{ll}^{-1}\boldsymbol{l}, \tag{2}$$

where $\boldsymbol{Q}_{ll}$ — autocovariance matrix of individual data. Matrices $\boldsymbol{Q}_{ll}, \boldsymbol{Q}_{sl}$ can be estimated from autocovariance function of process

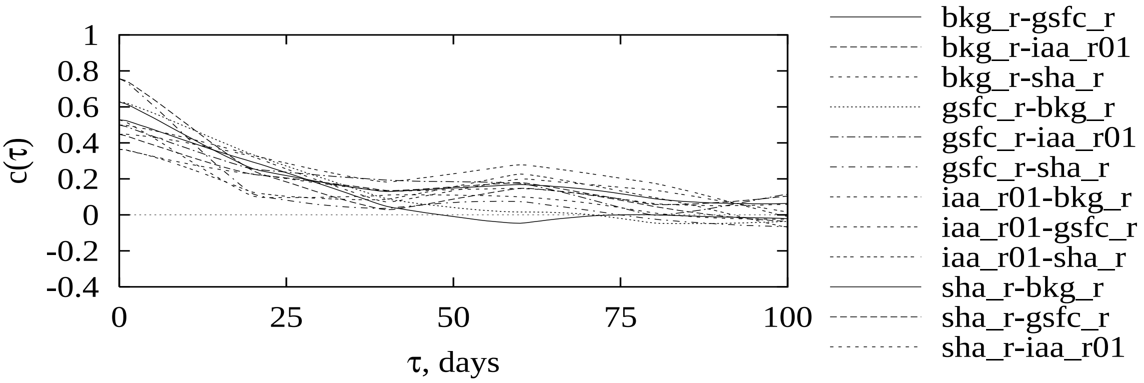


Figure 1: Some cross-correlation functions for $X$-coordinate of pole.

$$\boldsymbol{Q}_{sl} = \begin{pmatrix} q(t'_0 - t_0) & q(t'_0 - t_1) & \dots & q(t'_0 - t_n) \\ q(t'_1 - t_0) & q(t'_1 - t_1) & \dots & q(t'_1 - t_n) \\ \dots & \dots & \ddots & \dots \\ q(t'_{n'} - t_0) & q(t'_{n'} - t_1) & \dots & q(t'_{n'} - t_n) \end{pmatrix}, \tag{3}$$

$$\boldsymbol{Q}_{ll} = \begin{pmatrix} q(t_0 - t_0) & q(t_1 - t_0) & \dots & q(t_n - t_0) \\ q(t_0 - t_1) & q(t_1 - t_1) & \dots & q(t_n - t_1) \\ \dots & \dots & \ddots & \dots \\ q(t_n - t_1) & q(t_n - t_1) & \dots & q(t_n - t_n) \end{pmatrix} + \sigma_n^2 \boldsymbol{I}, \tag{4}$$

(5)

where $t_0$, $t_1$, ..., $t_n$ — time lags of input data, $t'_0$, $t'_1$, ..., $t'_{n'}$ — time lags of output data, $\sigma_n^2$ — noise dispersion, $q(\tau)$ — estimations of autocovariance function of signal. If signal $\boldsymbol{s}$ and noise $\boldsymbol{r}$ does not correlate with each other then covariation matrix between signal and observations $\boldsymbol{Q}_{sl}$ is equal to matrix of signal autocovariation $\boldsymbol{Q}_{ss}$. problem. As we can improvable time series and reference system. differences by equal to zero that is

## 4. OBTAINING COMBINED SOLUTION BY GENERALIZED AVERAGE ALGORITHM

Interpolated and filtered series were interpreted as vectors $\boldsymbol{l}_k$. If values of cross-correlation functions near $\tau = 0$ are approximately equal to zero for any pair of observational series then the reference system IERS (EOP) C04 is not need of any improvement, but this is not take place in real processing. At Figs. 1–5 some cross-correlation functions between individual series are presented.

Because we do not know apriory covariance function of corrections to reference system EOP (IERS) C04 therefore we have to average by generalized average algorithm (Moritz, 1987, Gubanov, 1997 pp.97–106). The stochastic corrections to reference system EOP (IERS) C04 can be calculated by formula

$$\hat{\boldsymbol{s}} = \left( \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{\boldsymbol{Q}}_{ij} \right)^{-1} \left( \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{\boldsymbol{Q}}_{ij} \boldsymbol{l}_i \right), \tag{6}$$

where $\boldsymbol{l}_i$ are vectors of individual interpolated and filtered series, block-matrices $\tilde{\boldsymbol{Q}}_{ij}$ may be built from auto- and crosscovariation functions between series $\boldsymbol{l}_i$ and $\boldsymbol{l}_j$. After that we have

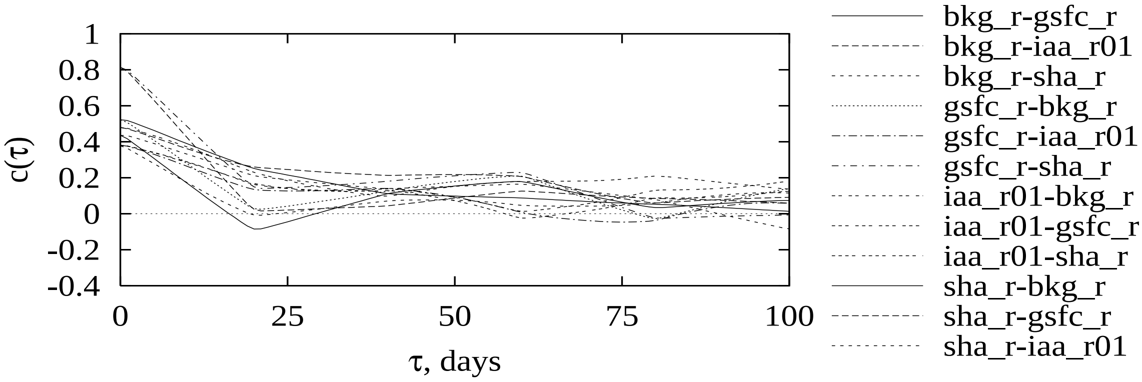


Figure 2: Some cross-correlation functions for $Y$-coordinate of pole.

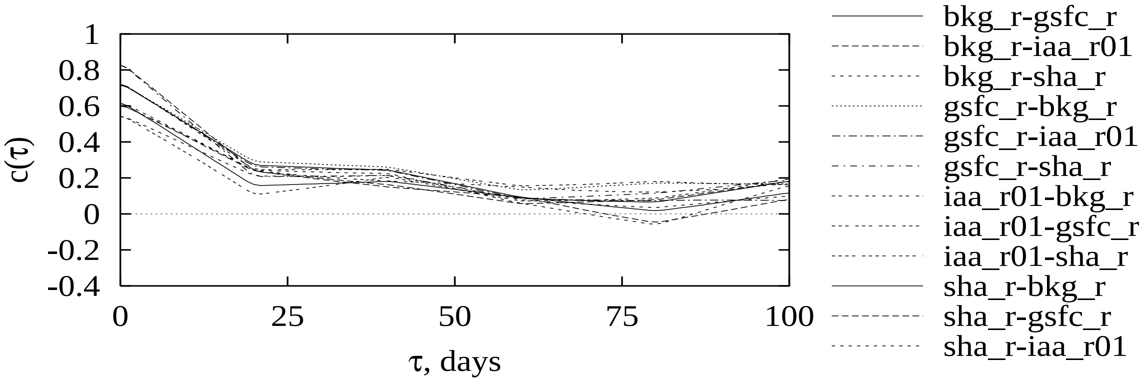


Figure 3: Some cross-correlation functions for UT1 − UTC.

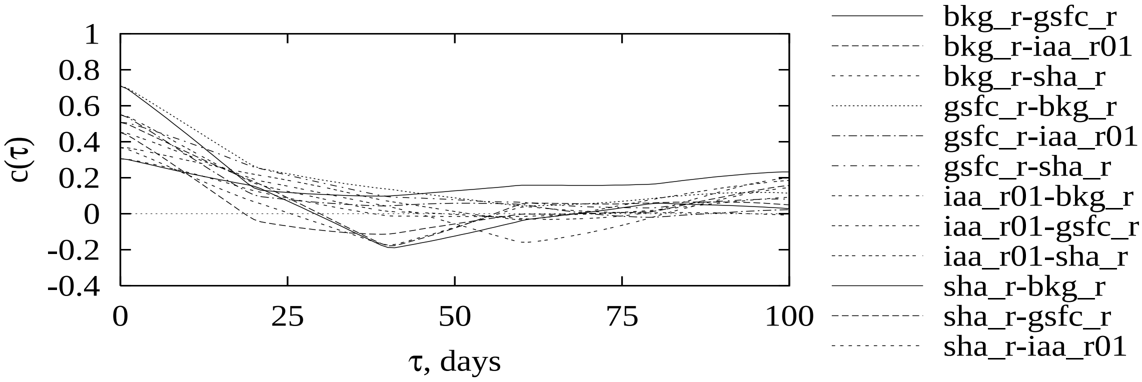


Figure 4: Some cross-correlation functions for $d\Psi$.

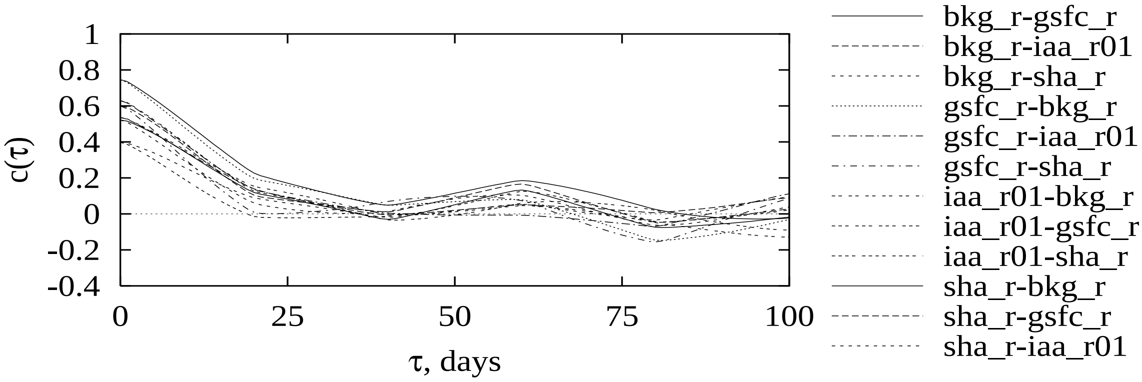


Figure 5: Some cross-correlation functions for $d\epsilon$.

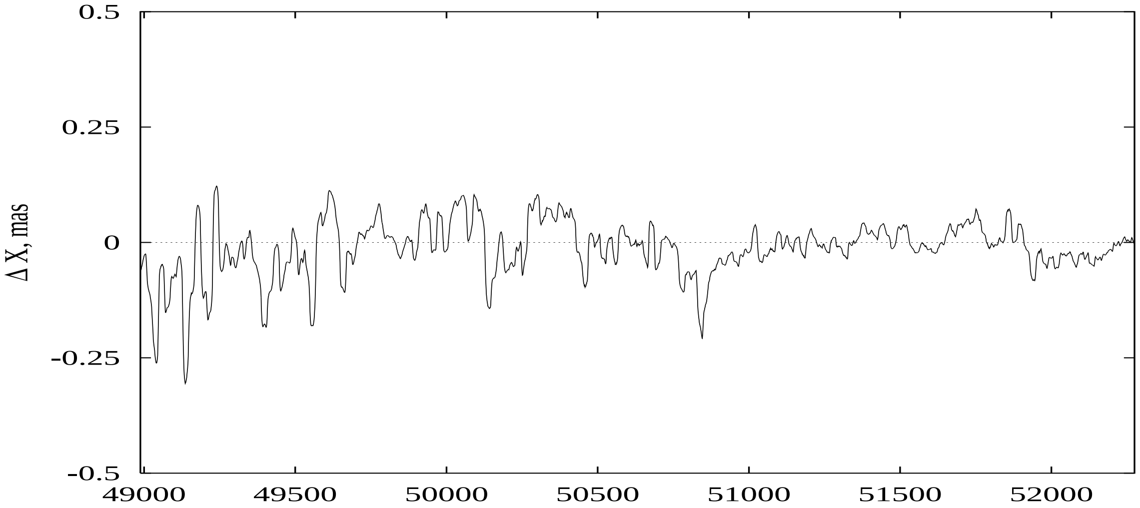


Figure 6: Corrections to reference system EOP (IERS) C04 for $X$-coordinate of pole.

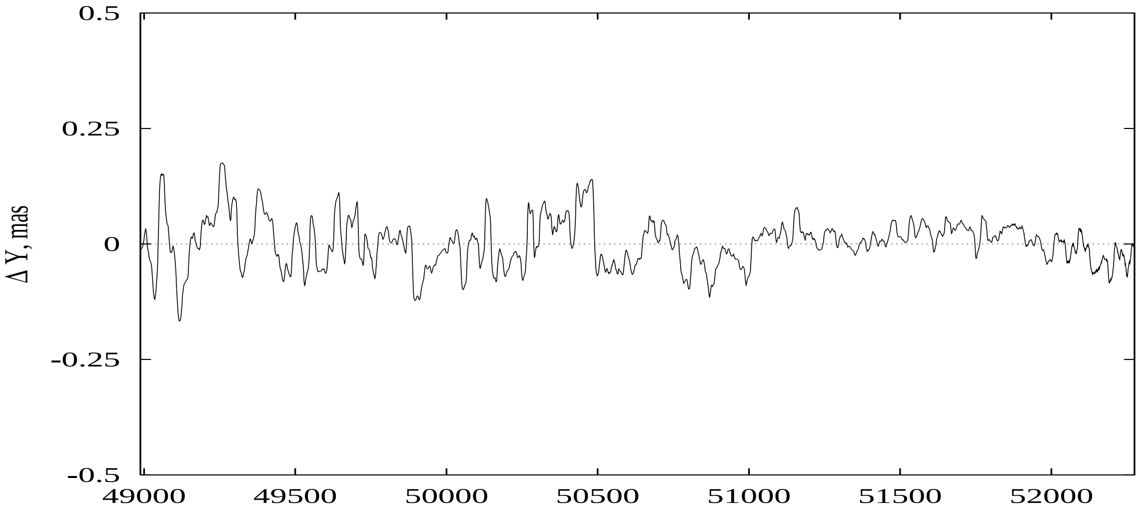


Figure 7: Corrections to reference system EOP (IERS) C04 for $Y$-coordinate of pole.

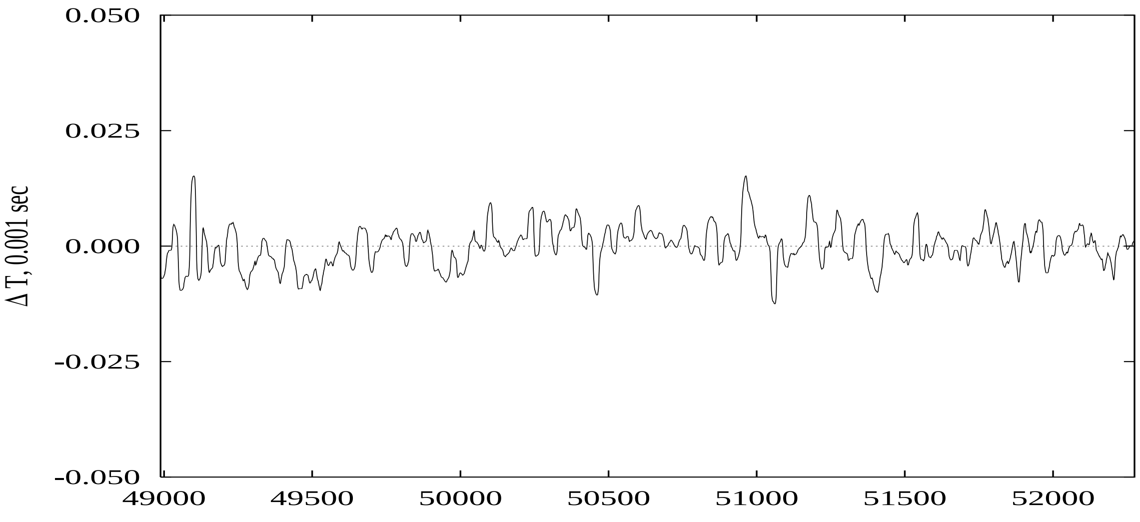


Figure 8: Stochastic corrections to reference system EOP (IERS) C04 for UT1 – UTC.

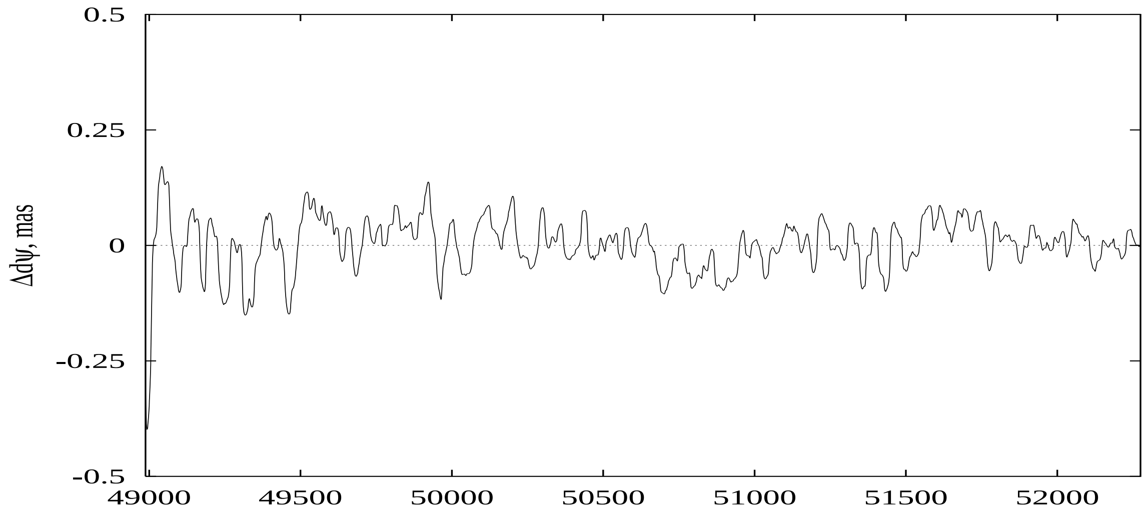


Figure 9: Stochastic corrections to reference system EOP (IERS) C04 for $d\Psi$.

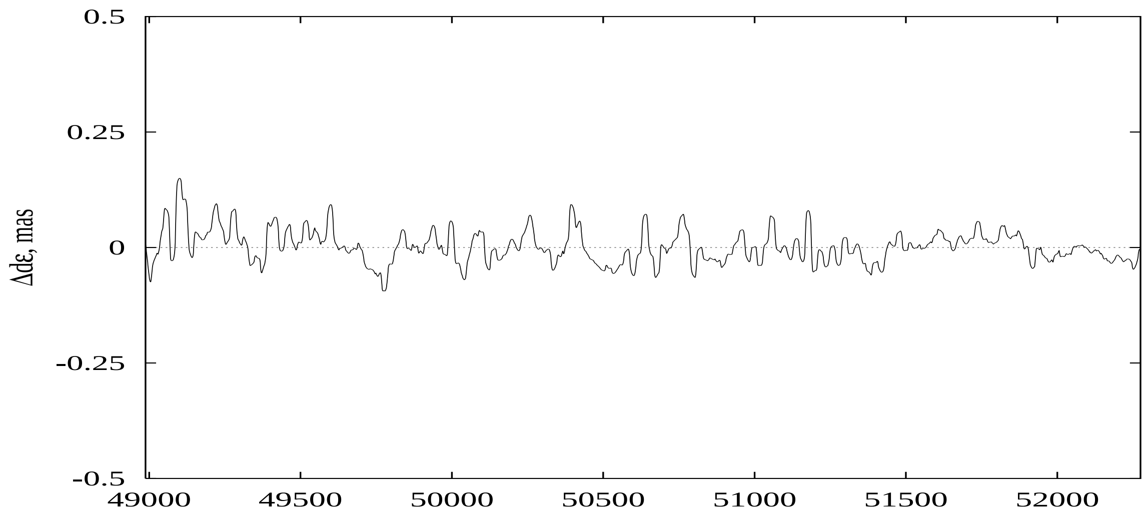


Figure 10: Stochastic corrections to reference system EOP (IERS) C04 for $d\epsilon$.

built general covariance matrix of data $\boldsymbol{Q} = [\boldsymbol{Q}_{ij}]$ and its inverse matrix $\boldsymbol{Q}^{-1} = \tilde{\boldsymbol{Q}} = [\tilde{\boldsymbol{Q}}_{ij}]$. Thus using matrices $\tilde{\boldsymbol{Q}}_{ij}$ and vectors $\boldsymbol{l}_j$ we can obtain stochastic corrections using generalized average algorithm. These stochastic corrections to series EOP (IERS) C04 are available at Figs. 6–10.

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## 5. REFERENCES

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