HIGH-PRECISION NUMERICAL ANALYSIS OF THE RIGID EARTH ROTATION PROBLEM WITH USING A HIGH-PERFORMANCE COMPUTER

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ABSTRACT. The present investigation is a development of the previous research on the rigid Earth rotation problem (Eroshkin *et al.*, 2002). The problem is studied numerically by using a high-performance computer Parsytec CCe20. All the calculations are carried out with the quadruple precision. The problem is solved both for the Newtonian case (dynamical case) and for the relativistic case (kinematical case) in which the geodetic perturbations in the Earth rotation are taken into account. Over the whole time interval of the numerical integration the solutions are compared with the corresponding solutions of the semi-analytical theory SMART97 (Bretagnon *et al.*, 1998), corrected in accordance with (Brumberg and Bretagnon, 2000). The behaviour of the secular and periodical terms in the residuals is discussed.

1. INTRODUCTION

In the previous investigation (Eroshkin *et al.*, 2002) a high-precision numerical solution of the rigid Earth rotation problem, accounting for the most essential relativistic perturbations — geodetic perturbations, was constructed by using a personal computer Pentium III 450 MHz. All the calculations were performed with a double precision. The precision of the numerical solution was at the level of several microarcseconds (μas) over the time interval of 1500 years. Such precision was achieved by means of using the regularizing variables, the Rodriges-Hamilton parameters, from which the main part of the secular variation of the angle of the proper rotation was excluded. An important point of the algorithm was a supplementary normalization of the values of Rodrigues-Hamilton parameters evaluated by means of the iterative procedure of the integrator HIPPI (Eroshkin, 2000).

Nevertheless it was considered quite necessary to repeat all the calculations with a quadruple precision in order to verify the earlier obtained results. It is also interesting to construct a numerical solution not only for the relativistic case but also for the Newtonian one. This paper describes the results of the numerical solution of the rigid Earth rotation problem obtained with the use of a high-performance computer Parsytec CCe20, which is a supercomputer of massive-parallel architecture with a separated memory at the Center for Supercomputing Applications of the Institute for High-Performance Computing and Data Bases in St.Petersburg, Russia (http://www.csa.ru/). All the calculations were performed with a quadruple precision. The mathematical model of the present investigation is identical to that used in (Eroshkin *et al.*, 2002). The kinematical solution SMART97 was initially corrected by the change of signs of all the geodetic terms in the angle of the proper rotation, in accordance with (Brumberg and Bretagnon, 2000). The main purposes of the present research are the verification and improvement of the algorithms of constructing the numerical solutions and additional testing the method of numerical integration HIPPI. The integration of the differential equations is carried out over the time interval from AD 1000 to 3000. It starts from the standard initial epoch January 1, 2000, goes forward to 3000 and then backward to 1000. The numerical integration, carried out forth and back in time, checks a reliability of the method HIPPI. It is performed with 1-day constant step size and with 24-th degree Chebyshev polynomials approximating the right hand sides of the differential equations of the problem.

2. RESULTS

The analysis of the problem is performed over 2000 year time interval from AD 1000 to 3000, with the initial epoch January 1, 2000 (JD=2451545.0). The results of the numerical integration are compared with the semi-analytical solution SMART97 in Euler angles: the longitude of the ascending node ψ (Figures 1a, b), the proper rotation angle ϕ (Figures 2a, b) and the inclination angle θ (Figures 3a, b), (a — Kinematical case, b — Dynamical case). Tables 1–3 contain the results of the least squares adjustment of the behaviours of the discrepancies of the secular character $\Delta_s\psi$, $\Delta_s\phi$, $\Delta_s\theta$ and the secular terms in the corresponding angles of SMART97. In the present paper T denotes the time expressed in thousand Julian years (tjy), counted from the fundamental reference epoch J2000, and the coefficients are expressed in microarcseconds (μas). The correcting polynomials of degree 6 provide the best fitting to the approximating curves.

First of all, it is necessary to notice that Figure 1a and Figure 3a are very similar to the corresponding pictures of Figure 2 of the previous investigation (Eroshkin *et al.*, 2002). However the behaviour of the residuals in the proper rotation angle, depicted in Figure 2a, differs essentially from the corresponding picture in Figure 2 of the previous paper. It is discovered that this discrepancy is connected with the chosen criterion determining the process of the numerical integration. Namely, if HIPPI integrator is used then the convergence of the iterative procedure at every nodal point has to be controlled by the values of the relative errors and not of the absolute ones. The numerical experiments demonstrate that if one uses the relative error control then the results of the quadruple precision integration and double precision integration are quite close over one thousand year time interval.

The comparison of the secular trends in the residuals, corresponding to the kinematical and dynamical cases, depicted in Figures 1–3, is also of interest. The behaviour of the residuals for the angles ψ and ϕ in the kinematical case (Figure 1a and Figure 2a, respectively) is very similar to that in the dynamical case (Figure 1b and Figure 2b, respectively).

From the Tables 1–2 follows that for these angles the essential part of the residuals arises when constructing the dynamical solution SMART97. The geodetic rotations, which are used to derive the kinematical solution of SMART97, introduce relatively small part of the residuals. For the inclination angle θ the behaviour of the residuals in the kinematical and dynamical cases, depicted in Figure 3, differ significantly. Table 3 shows that the basic distinction in the representation of the interpolating curves comes from the quadratic and cubic terms. There is no satisfactory explanation of this phenomenon yet.

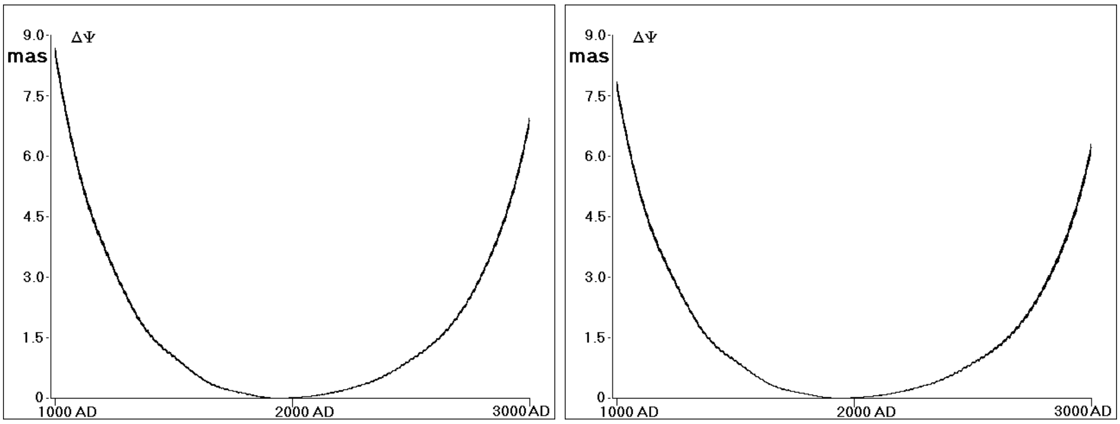

Figure 1: Numerical integration minus solution SMART97 in the angle ψ.

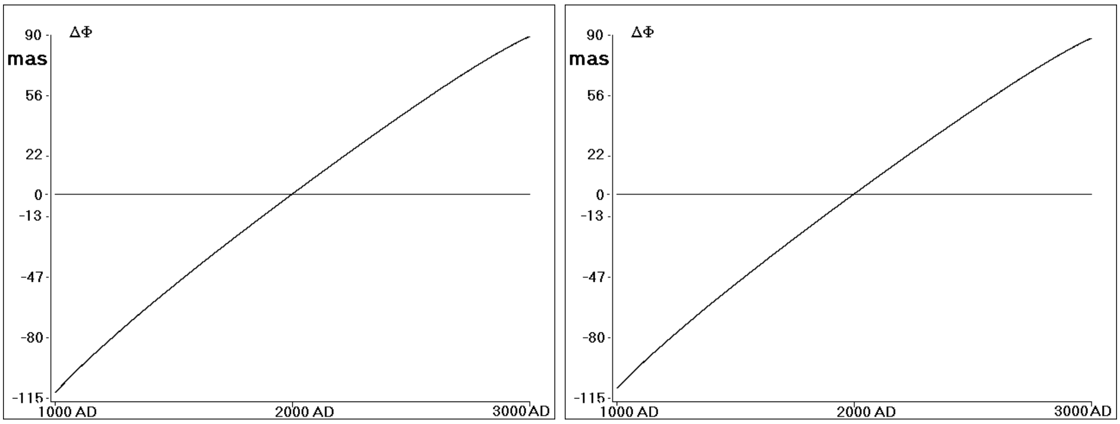

Figure 2: Numerical integration minus solution SMART97 in the angle ϕ.

a) — Kinematical case b) — Dynamical case

Table 1: Secular parts of the angle ψ and interpolating polynomials $\Delta_s\psi$.

	Kinematical case		Dynamical case	
	$\psi_{smart97}(\mu\text{as})$	$-\Delta_s\psi(\mu\text{as})$	$\psi_{smart97}(\mu\text{as})$	$-\Delta_s\psi(\mu\text{as})$
T^0		7.00		6.89
T	50384564881.3693	−206.50	50403763708.8052	−206.90
T^2	−107194853.5817	−3451.30	−107245239.9143	−3180.80
T^3	−1143646.1500	1125.00	−1144400.2282	1048.00
T^4	1328317.7356	−788.00	1329512.8261	−306.00
T^5	−9396.2895	−57.50	−9404.3004	−65.50
T^6		−3415.00		−3421.00

a) — Kinematical case b) — Dynamical case

Table 2: Secular parts of the angle ϕ and interpolating polynomials $\Delta_s\phi$.

	Kinematical case		Dynamical case	
	$\phi_{smart97}(\mu\text{as})$	$\Delta_s\phi(\mu\text{as})$	$\phi_{smart97}(\mu\text{as})$	$\Delta_s\phi(\mu\text{as})$
T^0	1009658226149.3691	6.58	1009658226149.3691	6.53
T	474660027824506304.0000	99598.30	474660027824506304.0000	97991.40
T^2	−98437693.3264	−7182.30	98382922.2808	−6934.40
T^3	−1217008.3291	1066.80	−1216206.2888	1004.00
T^4	1409526.4062	−750.00	1408224.6897	−226.00
T^5	−9175.8967	−30.30	−9168.0461	−37.80
T^6		−3676.00		−3682.00

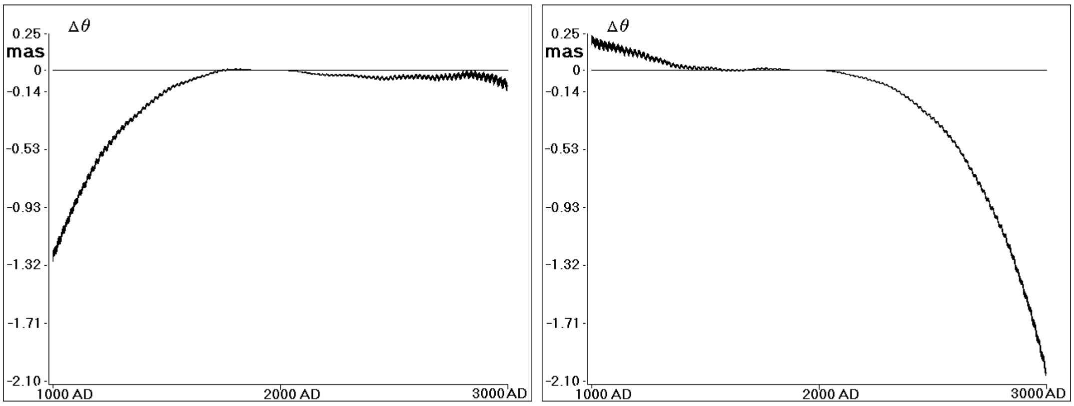

Figure 3: Numerical integration minus solution SMART97 in the angle θ.

a) — Kinematical case b) — Dynamical case

Table 3: Secular parts of the angle θ and interpolating polynomials $\Delta_s\theta$.

	Kinematical case		Dynamical case	
	$\theta_{smart97}(\mu\text{as})$	$\Delta_s\theta(\mu\text{as})$	$\theta_{smart97}(\mu\text{as})$	$\Delta_s\theta(\mu\text{as})$
T^0	84381409000.0000	1.42	84381409000.0000	1.39
T	−265011.2586	−96.61	−265001.7085	−96.73
T^2	5127634.2488	−353.10	5129588.3567	−595.90
T^3	−7727159.4229	771.50	−7731881.2221	−945.10
T^4	−4916.7335	−84.50	−4930.2027	−76.50
T^5	33292.5474	−86.00	33330.6301	−70.00
T^6		−247.50		−247.80

Several harmonics $\Delta_p\psi$, $\Delta_p\phi$, $\Delta_p\theta$ in the residuals are determined by the least squares method with the arguments chosen from the arguments of SMART97 theory. In Tables 4–6 the same harmonics are presented which were determined in (Eroshkin *et al.*, 2002). The standard errors of the coefficients shown in Tables 4–6 have the magnitude of about two orders smaller than the coefficients themselves. The argument λ_3+D-F equals $\Omega+180^o$; λ_2, λ_3, λ_4, λ_5, λ_6 are the mean longitudes of Venus, the Earth, Mars, Jupiter and Saturn, respectively; D is the difference between the mean longitudes of the Moon and the Sun; Ω is the mean longitude of the ascending node of the lunar orbit; F is the mean argument of the Moon's latitude.

Table 4: The periodical terms $\Delta_p\psi$ and the corresponding harmonics in SMART97.

argument, period		Kinematical case		Dynamical case	
		Coefficients of $\psi_{smart97}(\mu\text{as})$	Coefficients of $-\Delta_p\psi(\mu\text{as})$	Coefficients of $\psi_{smart97}(\mu\text{as})$	Coefficients of $-\Delta_p\psi(\mu\text{as})$
$2\lambda_5-5\lambda_6$ 800.9 years	cos	−667.3427	10.05	−667.6674	10.02
	T cos	−291.6144	−5.23	−291.7531	−5.22
	T^2 cos	17.9597	−10.85	17.9848	−10.82
	sin	−512.8209	−0.31	−513.0837	−0.32
	T sin	219.7644	−4.53	219.9043	−4.56
	T^2 sin	98.4238	−3.25	98.4621	−3.24
$3\lambda_2-7\lambda_3+4\lambda_4$ 302.4 years	cos	−3.4630	−8.46	−3.4630	−8.48
	T cos	0.1067	28.06	0.1067	28.06
	T^2 cos		−41.87		−41.82
	sin	5.1808	2.29	5.1808	2.30
	T sin	0.1046	6.29	0.1046	6.31
	T^2 sin		−27.87		−27.88
λ_3+D-F $(\Omega+180^o)$ 18.6 years	cos	−439.4833	−0.97	−439.4833	−0.98
	T cos	13416.3947	−27.28	13416.3960	−26.68
	T^2 cos	35849.0443	4.7	35842.8827	2.9
	sin	17280773.3737	−0.08	17280776.3805	−0.07
	T sin	84107.0763	−0.30	84107.0745	−0.37
	T^2 sin	−1178.0123	−5.7	−1178.3901	−5.2

Table 5: The periodical terms $\Delta_p\theta$ and the corresponding harmonics in SMART97.

argument, period		Kinematical case		Dynamical case	
		Coefficients of $\theta_{smart97}(\mu as)$	Coefficients of $\Delta_p\theta(\mu as)$	Coefficients of $\theta_{smart97}(\mu as)$	Coefficients of $\Delta_p\theta(\mu as)$
$2\lambda_5 - 5\lambda_6$ 800.9 years	cos	-237.8839	2.66	-237.9787	2.649
	T cos	214.2387	2.35	214.3301	2.35
	T^2 cos	21.5893	-3.77	21.6003	-3.76
	sin	437.0537	-2.118	437.2269	-2.110
	T sin	106.7941	2.86	106.8398	2.85
	T^2 sin	-61.1330	2.94	-61.1586	2.92
$3\lambda_2 - 7\lambda_3 + 4\lambda_4$ 302.4 years	cos	0.2135	-1.72	0.2135	-1.72
	T cos	-0.0256	-7.37	-0.0256	-7.37
	T^2 cos		2.40		2.42
	sin	0.0549	-2.75	0.0549	-2.752
	T sin	-0.0729	4.19	-0.0729	4.20
	T^2 sin		7.04		7.05
$\lambda_3 + D - F$ $(\Omega + 180^o)$ 18.6 years	cos	-9227885.6279	0.23	-9227886.9315	0.22
	T cos	-9273.2018	0.02	-9273.2018	0.05
	T^2 cos	11.5998	0.95	11.5998	0.87
	sin	32.1530	-0.46	32.1530	-0.46
	T sin	4413.0849	-14.35	4413.0855	-14.02
	T^2 sin	25501.6281	0.74	25499.8022	1.19

Table 6: The periodical terms $\Delta_p\phi$ and the corresponding harmonics in SMART97.

argument, period		Kinematical case		Dynamical case	
		Coefficients of $\phi_{smart97}(\mu as)$	Coefficients of $\Delta_p\phi(\mu as)$	Coefficients of $\phi_{smart97}(\mu as)$	Coefficients of $\Delta_p\phi(\mu as)$
$2\lambda_5 - 5\lambda_6$ 800.9 years	cos	-618.7215	9.16	-618.3585	9.14
	T cos	-269.1674	-4.78	-269.0474	-4.77
	T^2 cos	17.3920	-10.13	17.3622	-10.10
	sin	-474.1121	-0.25	-473.8684	-0.26
	T sin	204.8051	-4.05	204.6460	-4.08
	T^2 sin	90.6361	-3.08	90.6058	-3.07
$3\lambda_2 - 7\lambda_3 + 4\lambda_4$ 302.4 years	cos	-3.1772	-7.97	-3.1772	-7.98
	T cos	0.0979	25.56	0.0979	25.55
	T^2 cos		-37.34		-37.29
	sin	4.7532	2.09	4.7532	2.09
	T sin	0.0959	5.71	0.0959	5.74
	T^2 sin		-25.51		-25.51
$\lambda_3 + D - F$ $(\Omega + 180^o)$ 18.6 years	cos	-403.2789	-0.92	-403.2789	-0.92
	T cos	12309.0477	-24.80	12309.0462	-24.25
	T^2 cos	32869.9705	4.4	32875.6240	2.7
	sin	15852158.3146	0.02	15852155.0374	0.03
	T sin	77184.1368	-0.86	77184.1388	-0.90
	T^2 sin	-1252.4257	-5.5	-1252.0139	-5.0

If one compares the coefficients for Kinematical case, shown in Tables 4–6, with the corresponding quantities in (Eroshkin *et al.*, 2002) then certain differences can be discovered. They have two explanations. Firstly, the signs of all the geodetic terms in the kinematical solution for the proper rotation angle ϕ in SMART97 were changed in accordance with (Brumberg and Bretagnon, 2000). As a result, the coefficients of the harmonic with the argument $2\lambda_5 - 5\lambda_6$ in $\Delta_p\phi$ became essentially smaller. Secondly, in the present investigation the time interval of the adjustment is 2000 years instead of 1500 years, that is 1/3 larger than in the previous work.

As it was previously explained (Eroshkin *et al.*, 2002), the secular and periodical terms determined from the residuals of comparison of the numerical solution and semi-analytical solution SMART97 are the corrections to the corresponding terms of SMART97. As a final step of the present investigation, the kinematical solution SMART97 is modified by adding the secular correction terms from Tables 1–3. The numerical integration is performed anew with the initial conditions determined by the modified kinematical solution SMART97. The results of

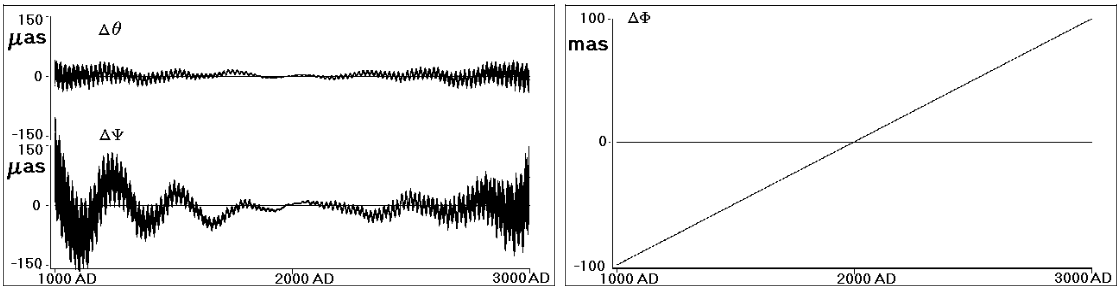

Figure 4: Numerical integration minus kinematical solution SMART97 supplemented by the secular corrections in Euler angles.

comparison of the numerical and modified semi-analytical solutions are presented in Figure 4. The secular trend in the proper rotation angle ϕ does not change practically with respect to that depicted in Figure 2a. In fact, there are differences which can be seen if one compares the polynomial $\Delta_s\phi = 6.62 + 99787.58T - 0.65T^2 - 0.39T^3 + 1.42T^4 + 0.47T^5 - 0.75T^6$, representing the secular behaviour of the residuals in the proper rotation angle from the Figure 4 with the corresponding polynomial from Table 2. This comparison shows that the corrections to the secular terms in the angle of the proper rotation are the real corrections to the second and higher order terms, whereas the linear part of correction is a fictitious one.

It was earlier discovered that the small variation of the initial moment of the numerical integration can change the value of the secular trend. It is quite possible that the reason for this phenomenon is connected with the appearance of the fictitious free nutation (fictitious Euler nutation) in the process of the numerical integration.

3. CONCLUSIONS

The results of the present investigation confirm the validity of the previous study, based on the double precision calculations. The semi-analytical theory of the rigid Earth rotation SMART97, corrected in accordance with the conclusions in (Brumberg and Bretagnon, 2000), represents high-precision semi-analytical solutions of the problem over the time interval of several centuries. The precision of this theory can be improved, for example, numerically by the method proposed by the authors. The time interval of its validity can be also extended in this way.

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