BODY TIDES IN THE EARTH-MOON SYSTEM AND THE EARTH'S ROTATION

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ABSTRACT. Differential equations of rotation of the Earth with the viscous fluid core are presented and applied to explanation of a number of observed effects in the Earth's rotation. The equations take into account some important effects ignored in the adopted theory of the Earth's rotation, namely the effects from the perturbing torques caused by interaction of the potentials, induced by the tidal deformations of the Earth and its fluid core, with the tide arousing bodies (including the dissipative cross interaction of the lunar tides with the Sun and the solar tides with the Moon). Perturbations of this kind could not be accounted in the adopted theory in which only those obtainable by the method of the transfer function are considered. The derived equations explicitly depend on two parameters characterizing the dissipation of energy of the Earth's rotation. These parameters are the effective tidal phase lag δ due to the dissipation by rotation of the Earth as a whole, and the tidal phase lag δ_c due to the dissipation by the differential rotation of the fluid core. The preliminary analysis has shown that the most noticeable of the dissipative effects - the excess of the observed secular obliquity rate compared with predictions of the rigid body model, and the large out-phase amplitudes of the 18.6-year and semi-annual nutations indeed may be explained as the combined effect of these two types of dissipative perturbations.

1. NUTATION IAU 2000 AND ITS DEFICIENCIES

Nutation IAU 2000, recently adopted as an international standard, has significantly improved fitting to the observed positions of the Celestial Pole determined by the VLBI techniques. Dynamical model behind this theory is founded basically on ideas of the work by Sasao, Okubo, and Saito (Sasao et al, 1980) in which the classic method developed by Poincare for the case of the rigid Earth with the fluid core has been generalized to account for effects of elasticity of the mantle (so called SOS model). In rigorous formulation the SOS model describes rotation of the Earth consisting of the elastic mantle and non-dissipative fluid core. When applying this model the dissipative effects of the Earth's rotation are usually treated in a formal way assuming that some constants of the SOS model (for example, the Love number k_2) have imaginary parts and estimating them from the observed positions of the Celestial Pole (see, for instance, Shirai & Fukushima, 2001). Such a semi-empirical approach is equivalent to incorporation of empirical terms into the differential equations of the SOS model. In fact, in order to describe satisfactory the dissipative effects in the nutations, at least five empirical terms must be incorporated. Physical meaning of these empirical parameters is rather uncertain. For instance, if the tidal phase lag δ is derived from them, its value appears to be roughly inconsistent with the reliable LLR estimate of δ. So the theoretical basis of Nutation IAU 2000 cannot be considered sound enough. Indeed, in (Dehant & Defraign, 1997) it has been shown that the observed out-phase amplitudes of the nutations cannot be satisfactory explained without introducing the empirical terms.

This defficiency arises mainly because of the SOS model (and thus Nutation IAU 2000) accounts only for a part of perturbations caused by the non-rigidity of the Earth, more exactly only for those from the tidal variations of the matrices of inertia of the mantle and the core, the rigid body approximation still being used to model the perturbing torques. In such a simplified approach the resulting perturbations can be expressed in terms of the rigid body nutations and thus it appears possible to present them as a linear differential operator (so called transfer function) applied to the rigid body nutations. However some important perturbations cannot be obtained in this way and that is why they are absent in Nutation IAU 2000. In brief their origin may be explained in the following way. The tidally deformed mantle and the fluid core, while interacting with the perturbing celestial bodies give rise to additional torques which are proportional to the static k_2 and dynamic k_2^d Love numbers, relatively. It is well known that for the elastic Earth the torques of the first type vanish; but that is not the case when the effective tidal phase lag δ is accounted to describe the impact of dissipation of energy by the body tides. The ignored dissipative torques rule evolution of the Earth-Moon system and so are very important. In particular they are responsible for a small (but informative for geophysics) part of the observed obliquity rate unexplainable in the rigid body model. It is easy to see that no secular rate in obliquity can be modeled in the frame of the mentioned above formal considerations of the imaginary parts of parameters of the SOS model. Other observable effects are caused by the ignored torques which are proportional to the dynamical Love number k_2^d and so depend on the differential rotation of the fluid core. These torques contribute on detectable level not only to obliquity rate and out-phase amplitudes of nutations but also to in-phase amplitudes which cannot be neglected but are not obtainable by the method of transfer function.

To derive the rigorous differential equations accounting for such effects, rather tedious analytical manipulations must be carrried out which cannot be presented in the frame of this paper due to the lack of space. Preliminary considerations with more details may be found in our paper (Krasinsky, 2003) available as the file *quasar.ipa.nw.ru/incoming/era/SOSMODEL.ps* by anoinymous FTP.

In the next sections the differential equations for the conventional SOS model as well as for the revised ones are presented (without proves) in the same notations to facilitate their comparison.

2. CONVENTIONAL SOS MODEL OF THE EARTH'S ROTATION

Parameters of the SOS model are so called compliances κ, γ, β. The compliances κ, γ may be expressed in terms of the static k_2 and dynamic k_2^d Love numbers, relatively. Loosely speaking, the dynamic Love number k_2^d scales perturbations in the moments of inertia of the fluid core caused by tides in the mantle, and vice versa. The compliance β can be expressed through a parameter k_2^c which plays part of the Love number of the fluid core. The compliances (or the corresponding Love numbers) may be obtained either theoretically making use of constants of the adopted up-to-date models of the Earth's interior, or from analysis of VLBI data by fitting the rotation theory. Instead of compliances τ, γ, β we here prefer to use the coefficients σ, ν, μ defined by the relations

$$\kappa = e\sigma,\ \gamma = e\frac{\nu}{\alpha},\ \beta = e\frac{\mu}{\alpha},$$

in which the constant α is the ratio of the main moment of inertia of the fluid core to that of the Earth as a whole, and e is the dynamical flattening.

If the 'secular' Love number k_s is defined in the stanard way by the expression

$$k_s = \frac{3Gm_E J_2}{R^3\omega^2} \approx 0.93831,$$

in which G is the gravitational constant, m_E, J_2, R, ω are the mass, the coefficient of the second zonal harmonics of the potential, the mean radius, and the rotational rate of the Earth, then the parameters $\sigma,\ \nu,\ \mu$ may be presented by the relations

$$\sigma = \frac{k_2}{k_s}, \nu = \frac{k_2^d}{k_s},\ \mu = \frac{k_2^c}{k_s}.$$

In these notations the standard SOS equations of the Earth's rotation (see Moritz & Mueller, 1987) may be written in the form

$$\dot{u}(1+e\sigma) - ie\omega(1-\sigma)u + (\alpha + e\nu)(\dot{v} + i\omega v) = L + i\frac{\sigma}{\omega}\frac{\partial L}{\partial t},$$
$$\dot{u} + \dot{v} + iv\omega\left(1 + e_c - \mu\frac{e}{\alpha}\right) = \frac{\nu}{\alpha}\left[L\,(1-e) - \frac{i}{\omega}\frac{\partial L}{\partial t}\right],$$

where at the right parts L is the rigid body torque normalized dividing it by the of main momentum of inertia A, and u, v are complex combinations of the components ω_1, ω_2 and v_1, v_2 of the vectors of angular velocity $\overline{\omega}$ of the mantle and that $\overline{v}$ of the differential rotation of the Earth:

$$u = \omega_1 + i\omega_2,$$
$$v = v_1 + iv_2.$$

The normalized rigid body torque L from the perturbed body is given by the expression

$$L = -ip\omega\xi\zeta,$$

where p is the parameter of the lunar or solar precession

$$p = \frac{3}{2}\frac{mG}{r^3}e,$$

e is the dynamical flattening, $\xi = \rho_1 + i\rho_2,\ \zeta = \rho_3$, and ρ_1, ρ_2, ρ_3 are the ecliptical rectangular coordinates of the geocentric unit vector to the tide arousing body. (In fact the rigid body torque l is the sum of the lunar and solar components: $L = L^1 + L^2$ where $L^k = -ip_k\xi^k\zeta^k$, $p = p_1 + p_2$, p_1, p_2 being parameters of the lunar and solar precession).

The normalized perturbing torque L implicitly depends on the three Euler's angles: the nutation angle θ, the angle of precession ϕ, and the rotational angle ψ. Making use of the Euler's kinematic equations that connect $\dot{\phi}, \dot{\theta}$ with the complex angular velocity u the close system of differential equations arises which presents the dynamical ground of Nutation IAU

2000. In these equations the rotational angle ψ is a known linear function of time and differs from the Greenwich Sidereal Time by the constant $\pi = 3.14..$

3. REVISED SOS EQUATIONS OF THE EARTH'S ROTATION In more rigorous formulation, the SOS equations have to be replaced by the following ones

$$\begin{aligned}
&\dot{u}\left(1+\frac{2}{3}e\sigma\right) - ie\omega(1-\sigma)u + \left(\alpha + \frac{2}{3}e\nu\right)(\dot{v}+i\omega v) + \\
&+iv\sum_{k=1,2}(1-3\zeta_k^2)p_k = L + (\delta+i)\frac{\sigma}{\omega}\frac{\partial L}{\partial t} + L^d + L_c^d, \\
&\dot{u}+\dot{v}+iv\omega\left[1+e_c-\frac{\mu e}{\alpha}(1+i\delta_c)\right] = \\
&= \frac{\nu}{\alpha}\left[L\left(1-\frac{2}{3}e\right)-\frac{i}{\omega}\frac{\partial L}{\partial t}\right] + i\delta\frac{\nu}{\alpha}\left[L(1-e)+i\left(\frac{2}{\omega}\right)\frac{\partial L}{\partial t}\right] = 0,
\end{aligned}$$

in which the normalized dissipative torque L^d consists of the lunar L_1^d and solar L_2^d components caused by the dissipation in the lunar and solar tides, and of the cross interaction torque $L_{1,2}^d$ of these tides

$$\begin{aligned}
L^d &= L_1^d + L_2^d + L_{1,2}^d, \\
L_k^d &= -4p_k\epsilon_k\sigma\delta\left[\omega\xi_k\zeta_k + i\left(\zeta_k\frac{\partial}{\partial t}\xi_k - \xi_k\frac{\partial}{\partial t}\zeta_k\right)\right] \; (k=1,2), \\
L_{1,2}^d &= 2\sigma\delta\omega(p_1\epsilon_2+p_2\epsilon_1)(\xi_2\zeta_1+\xi_1\zeta_2),
\end{aligned}$$

while L_c^d includes the terms due to the dissipation in the fluid core

$$L_c^d = \nu\delta_c\left[\frac{1}{2}pv(3\cos^2\theta - 2\cos\theta - 1) + ie\left(1-\frac{\nu}{\alpha}\right)L\right],$$

p_1, p_2 being the parameters of the lunar and solar precession, relatively, $\epsilon_1 = p_1/e\omega$, $\epsilon_2 = p_2/e\omega$, and $p = p_1 + p_2$.

These equations explicitly depend on the two dissipative parameters δ and δ_c. The parameter δ is the well-known tidal lag of the Earth as a whole that strongly affects the orbital motion of the Moon being responsible for the evolution of the Earth-Moon system. The parameter δ_c is the phase lag of the tides caused by the differential rotation of the fluid core and it plays important part in the Earth's rotation.

Setting the tidal lags δ, δ_c equal to zero one could expect that the standard and revised systems of the differential equations become equivalent. However it is easy to see that there is no full equivalence: in the revised equation for the variable u the factor $1+2e\sigma/3$ stands in place of the factor $1+e\sigma$ in the SOS equations. The origin of this discrepancy is traced as being due to the incomplete form of the centrifugal tidal potential used in the conventional SOS model in which only the tesseral components of this potential have been accounted for (more details are given in Krasinsky, 2003). Omission of the zonal components leads to minor errors of the second order with respect to e and probably does not deteriorates fitting to observations (though the theoretical interpretation of the results may be distorted indeed).

The rigorous equations show that any attempts to describe the dissipative effects by a formal consideration of imaginary parts of the Love numbers k_2, k_2^d (or of the compliances κ, γ) are of no physical meaning because the functional dependence of equations on the phase lags δ, δ_c

has another structure, excepting the single term in the right part of equation for u which is proportional to the derivative $\partial L/\partial t$. Only this term may be expressed in terms of the imaginary part of the Love number k_2.

4. APPLICATIONS

4.1. PRECESSION AND OBLIQUITY RATE

From the geophysical point of view it seems interesting to interpret the observed value $\dot{\theta}_{obs} = -24.08 \pm 0.017$ mas/cy of the obliquity rate based on VLBI data (see Shirai & Fukushima, 2001). The main part of this value is not the dissipative effect but is the result of omission of some terms in the adopted rigid body nutation, as it has been at first shown in (Williams, 1994). (In fact they are not secular but long periodic terms broken to the time series). In more detail, these terms are due to so called 'tilt' effect of perturbations in the tilt of the lunar orbit to ecliptic (that gives -25.4 mas/cy), and due to direct perturbations from planets with the resulted contribution -1.4 mas/cy, the total effect being -26.8 mas/cy. The value by Williams may be compared with those given by the more recent rigid body models of nutation: -26.5 in SMART97 (Bretagnon et all. 1998) and -27.2 in RDAN97 (Roosbeck & Dehant, 1998). Thus the observed obliquity rate $\dot{\theta}_{obs}$ that has to be explained as the dissipative effect varies in the range:

$$\dot{\theta}_{obs} = 2.7 \div 3.1 \text{ mas/cy}, \tag{1}$$

depending on the applied rigid body model.

From the revised version of SOS equations one can derive the following analytical expressions for the obliquity rate $\dot{\theta}_\delta$ induced by dissipation of the Earth as a whole and the rate $\dot{\theta}_{\delta_c}$ due to the dissipation caused by the differential rotation of the fluid core:

$$\begin{aligned} \dot{\theta}_\delta &= 2p\sigma\delta \sin\theta(\epsilon\cos\theta - 2\tilde{\epsilon}), & (2)\\ \dot{\theta}_{\delta_c} &= p\nu e\left(1 - \frac{\nu}{\alpha}\right)\delta_c \sin\theta\cos\theta, & (3) \end{aligned}$$

where

$$\begin{aligned} \epsilon &= \frac{p_1\epsilon_1 + p_2\epsilon_2}{p} = 2.04 \times 10^{-5},\\ \tilde{\epsilon} &= \frac{p_1 n_1\epsilon_1 + p_2 n_2\epsilon_2}{\omega p} = 6.27 \times 10^{-7}. \end{aligned}$$

The component $\dot{\theta}_\delta$ of the obliquity rate may be reliably evaluated making use of the estimate $\delta = 0.0376$ based on LLR data,p that gives the positive rates 0.675 mas/cy and 0.153 mas/cy as the impact of the Moon and Sun, relatively, with the total value 0.928 mas/cy. Then the remaining part of the observed obliquity rate (1) must be attributed to the effect of the fluid core:

$$\dot{\theta}_{\delta_c} = 1.8 \div 2.2 \text{ mas/cy}.$$

Applying the theoretical expression (3) we obtain the estimate

$$\delta_c \approx 0.010. \tag{4}$$

Much larger value of δ_c might be anticipated as the following reasoning seems to be plausible. The tidal lag δ of the Earth as a whole, obtained from LLR data, is the weighted sum of the contributions of the mantle and the fluid core. It is known that the dissipation in the mantle is weak and so its contribution to δ is small if any. Thus if only the core were responsible for the tidal lag δ then we could expect that the relation $\delta = \alpha\delta_c$ is valid which would give $\delta_c \approx 0.3$. Note that the derived estimate of δ_c is very sensitive to the ratio ν/α.

4.2. AMPLITUDES OF OUT-PHASE NUTATIONS

From the revised SOS equations it follows that the out-phase amplitudes of the main nutations (18.6 and half year periods) with the sufficient accuracy may be written in the form:

$$\begin{aligned} d\theta^f &= -R_f^{out} \sin\theta_0 d\phi_0^f, \\ \sin\theta d\phi^f &= R_f^{out} d\theta_0^f, \end{aligned}$$

where

$$R_f^{out} = \left[-\nu\delta + \sigma_f\left(\alpha - \nu\right)\delta_c\right]\frac{f}{f+f_c}(1-\alpha)^{-1},$$

and

$$f^c = \omega\left(e_c - \frac{\mu e}{\alpha}\right)(1-\alpha)^{-1}$$

has meaning of frequency of free oscillations of the fluid core in the non-rotating coordinate frame, so called Free Core Nutation, FCN.

In Table 2 the observed in-phase and out-phase amplitudes (in mas) are reproduced from the work (Shirai & Fukushima, 2001) for the three main nutations (including the fortnightly nutation). In this table there are also given the corresponding estimates of δ_c, obtained with the help of the given above analytical expression for the coefficient R_f^{out}. Note that the amplitudes presented in Table 2 are not really observed quantities but the theory-dependent ones because they are obtained by the mentioned above formal method estimating the imaginary parts of the coefficients of the transfer function as solve-for parameters from which the 'observed' out-phase amplitudes have been derived.

Table 1: Observed main nutations and estimates of δ_c.

Period	$d\phi_{in}$	$d\phi_{out}$	δ_c	μ^{dis}	$d\theta_{in}$	$d\theta_{out}$	δ_c	μ^{dis}
6798.38	17206	3.341	0.38	0.09	9205	-1.506	0.47	0.11
182.62	-1317	-1.717	0.46	0.11	579	-0.570	0.44	0.10
13.66	-228	0.286	-	-	98	0.148	-	-

One can see that the four independent estimation of δ_c presented in Table 2 are in a good accordance. They are rather large and marginally out the boundary of the physically meaningful range of this value ($\delta_c < \delta/\alpha \approx 0.35$). The fortnightly amplitudes desagree with the 'observed' ones, but reliability of the last (derived by the semi-empirical method) is disputable.

In the work (Dehant & Defraigne, 1997) it is shown that oceanic tides contribute to the out-phase amplitudes of both the 18.6 year and semi-annual nutations indirectly through action of the fluid core, and as the result about half of the observed values. Hence only the half of the estimated δ_c may be caused by the dissipation in the fluid core. The reduced value

$\delta_c \approx 0.15$ becomes more consistent with the lesser value derived in the previous subsection from the observed obliquity rate (which parameter is not affected by the ocean tides).

5. REFERENCES

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