FREE CORE NUTATION: STOCHASTIC MODELING VERSUS PREDICTABILITY

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ABSTRACT. The time series of the celestial pole offsets determined by VLBI contains the free core nutation (FCN) which is a pseudoharmonic oscillation with retrograde period of 430 days and variable amplitude between 0.1 and 0.3 milliarcseconds. This signal is significant at the assumed sub-milliarcsecond level of accuracy therefore needs explanation and modeling. In the first part of this study we recall our earlier results concerning the stochastic modeling of the observed FCN signal. Then we show how the model of the autoregressive process can be applied for prediction of the observed irregular component of nutation and compare results to the extrapolation based on the sinusoidal model of the FCN.

1. INTRODUCTION

The difference between the precession and nutation observed by the *very long baseline interferometry* (VLBI) technique and that predicted by the MHB2000 model (Mathews *et al.*, 2002) contains irregular variations which are significant at the sub-milliarcsecond level of accuracy (see, e.g., Dehant *et al.*, 2003) therefore needs explanation and modeling. There are at least two types of irregular motions:

1. The *free core nutation* (FCN), a pseudoharmonic oscillation with retrograde period of about 430 days and variable amplitude between 0.1 and 0.3 mas (milliarcseconds) as well as variable phase.

2. Atmospheric and nontidal oceanic contributions to nutation which are not strictly harmonic but contain a broad-band variability at the level of 0.1 mas (Bizouard *et al.*, 1998; Petrov *et al.*, 1998).

These irregular motions can be modeled by different methods. Here we will focus attention on the stochastic modeling by the *autoregressive integrated moving-average* (ARIMA) processes. First, we will recall briefly our earlier results on the ARIMA modeling of the FCN, which have been discussed in a series of papers (Brzeziński, 1994; 1996; 2000; Brzeziński and Petrov, 1998; Brzeziński *et al.*, 2002). The ARIMA model is particularly suitable for determination of the parameters of the FCN resonance, the period T, the quality factor Q and the excitation power S needed to maintain the observed free motion. Then, we will consider the possibility of applying this model to predict the future values of the FCN signal. We will also describe first numerical experiment comparing the autoregressive prediction of the observed time series of the celestial pole offsets to the extrapolation based on the sinusoidal model of the FCN.

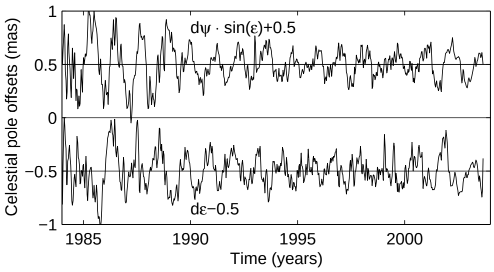
2. FREE CORE NUTATION MODE IN EARTH ROTATION

The FCN belongs to the catalogue of the solid Earth modes; see Figure 3 of Eubanks (1993). It influences Earth rotation in two different ways: 1) through resonant enhancement of the amplitudes of those lunisolar nutation waves which are close to the frequency of resonance (indirect effect), and 2) it gives rise to the free oscillation of the pole in response to the irregular excitations, atmospheric, oceanic, etc. (direct effect). In addition, the FCN resonance influences also the tidal gravity variations, but only indirect effect has been detected so far.

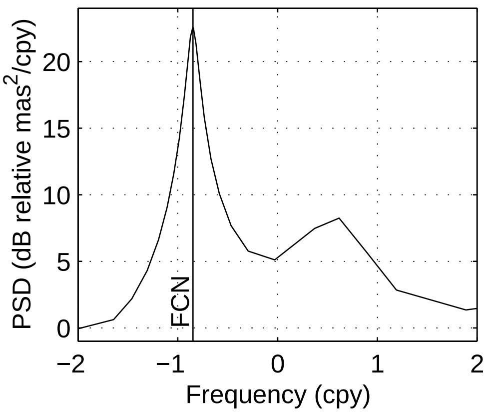
2.1. *Observations of the FCN*

The FCN was predicted and explained theoretically already at the end of 19th century (Hough, 1895). Many attempts were made in the past to detect the FCN oscillation in the astrometric observations of Earth rotation, e.g. by Popov (1963), Yatskiv *et al.* (1975), but only the recent measurements by the VLBI have been precise enough both to verify theoretical models of the indirect effect on nutation and to reveal the FCN signal in the celestial motion of the pole (Herring *et al.*, 1986). The FCN oscillation contributes to the time series of the celestial pole offsets which are routinely provided by the International Earth Rotation and Reference Systems Service (IERS). Several series based on the VLBI observations are available from 1979 till now, but for the purpose of tracking the FCN signal it is necessary to reject the data before 1984.

As it can be seen from Figure 1b, the irregular variability of nutation shown in Figure 1a consists mostly of the FCN oscillation which contributes more than 60% to the total power in the series. The period $T = -429.6$ days determined from the *maximum entropy method* (MEM) of spectral analysis (Brzeziński, 1995) agrees quite well with the value $T = -430.2$ days adopted by the MHB2000 model (Mathews *et al.*, 2002), marked in Figure 1b by the vertical line. The amplitude of the FCN varies in time but it does not show any permanent decaying trend. The maximum values, around 0.3 mas, occur before 1990, then become significantly smaller in 1990-ties, and increase again after 2000.



a)



b)

Figure 1: Celestial pole offsets observed by the VLBI (IERS combination series C04), after removal of the empirical corrections to the IAU2000 precession/nutation model a) time domain representation, b) frequency domain representation – maximum entropy power spectrum.

2.2. *Modeling of the FCN*

The MHB2000 precession-nutation model adopted as an official IAU standard model and designated as IAU2000 (IERS, 2003), does not include the FCN because it is a free motion that cannot be predicted rigorously. However, the Fortran program `IAU2000A.f` evaluating the nutation model, which is attached to Chapter 5 of the IERS Conventions 2003 (*ibid.*), contains the subroutine `FCN_nut` which can be optionally used in computations. This subroutine implements the FCN model which is recommended by the IERS. The model assumes a constant dissipationless value of the FCN frequency and uses the empirical values of the sine- and cosine- amplitudes (equivalently, the sine- and cosine- amplitudes can be expressed as the amplitude and phase with respect to the selected reference epoch) pre-estimated for the subsequent two-year intervals. The amplitudes are interpolated linearly to the epoch of computation and these are the instantaneous values used to evaluate the FCN signal. In case when the evaluation epoch is larger than the time of the last pre-estimated value, the most recent amplitudes are used in the computation.

An alternative model of the observed FCN signal proposed by Shirai and Fukushima (2001) is a piecewise damped sinusoidal oscillation with a number of the excitation impulses. After assuming the values of the FCN period $T = -431$ days and the quality factor $Q = 15300$, they found that an adequate representation of the FCN signal during the period 1979 to 2000 can be obtained when assuming 4 different excitation epochs: 1989.39, 1994.47, 1994.76 and 1998.99, and determining five complex amplitudes by the least-squares method. This model has a physical explanation, namely it is an implementation of the hypothesis stating that the FCN is excited by large earthquakes.

Another approach for modelling the free oscillations in Earth rotation, originally proposed by Jeffreys (1940) as an adequate representation of the Chandler wobble, is a stochastic modeling by the ARIMA processes. The model assumes that the free signal which is subject to damping, is continuously excited by a certain random process, or a combination of processes. Such excitation corresponds to the retrograde quasi diurnal variation in the atmospheric and oceanic angular momentum driven by the daily solar heating cycle.

The ARIMA modeling of the FCN has been discussed extensively by Brzeziński (1994, 1996, 2000), Brzeziński and Petrov (1998), Brzeziński *et al.* (2002). We proceeded according to the following scheme:

1. Introduce possible simplifications to the equation of motion.
2. Solve equation of motion analytically and perform discretization by applying the trapezoidal rule of integration.
3. Assume equidistant sampling of all variables and introduce simple stochastic models for the excitation process and for the measurement noise.

If the excitation process is modeled as a white noise and the measurement errors as a random walk, then the resulting stochastic model for the observed FCN signal is

$$d_l - \varphi_1 d_{l-1} = v_l - \theta_1 v_{l-1} - \theta_2 v_{l-2}, \qquad (1)$$

where $d_l = z_l - z_{l-1}$ is the first difference of the observation $z_l = P_l + n_1$ of nutation, $P_l = d\psi_l \sin\varepsilon_o + i d\varepsilon_l$ denotes the nutation expressed as complex combination of the celestial pole offsets in longitude $d\psi$ and obliquity $d\varepsilon$, with $\varepsilon_o = \varepsilon(\mathrm{J2000})$ and $i = \sqrt{-1}$, n_l is complex measurement noise assumed to be realization of the random walk process, the complex coefficients φ_1, θ_1, θ_2

are known functions of the FCN period T and quality factor Q, and $\{v_l\}$ is a zero-mean sequence of uncorrelated complex-valued random impulses. Eq.(1) describes the ARIMA(1,1,2) model; see, e.g., (Marple, 1987) for theoretical basis. Its parameters can be determined by applying the maximum likelihood algorithm to the time series of observations.

The ARIMA model expressed by eq.(1) which follows from the physics of the FCN, provides the optimum representation in a sense of the criterion of parsimony: it is fully describes by the three complex coefficients φ_1, θ_1, θ_2 and one real coefficient expressing the standard deviation of the driving white noise $\{v_l\}$. An equivalent from the point of view of mathematics, representation of the ARIMA process is provided by the pure *autoregressive* (AR) model

$$z_l - \varphi_1 z_{l-1} - \varphi_2 z_{l-2} - \ldots\ldots - \varphi_p z_{l-p} = v_l. \tag{2}$$

This model involves usually more parameters than the optimum ARIMA counterpart, eq.(1). In case of the nutation time series with a 10-day sampling, shown in Figure 1a, we found the optimum AR order $p = 21$ (see next section for details). It means that the number of parameters increased seven times. But such a pure AR model offers important advantages for our applications. First, there exist simple algorithms for determining its parameters, the coefficients φ_j, $j = 1, \ldots, p$, and standard deviation of the driving noise $\{v_l\}$. In the computations described in the next section we apply the least-squares version of the MEM algorithm to estimate the AR coefficients, with the Akaike *final prediction error* (FPE) criterion for finding the optimum order p; see (Brzeziński, 1995) for computational details. Second, the application of the AR model for prediction is straightforward. One needs only to replace in eq.(2) the unknown future random impulses v_l by their expected values equal to zero. Finally, this model takes into account not only the FCN signal but also other irregular components of the observed time series, such as that expressed by the broad peak at prograde frequencies between 0 and 1 cycles per year, shown in Figure 1b.

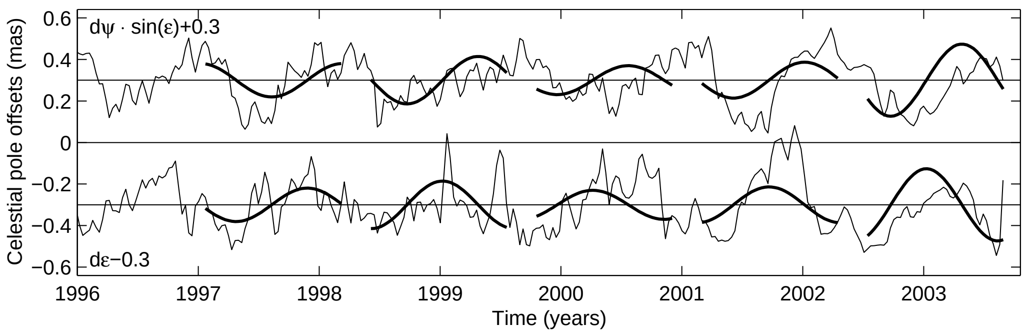
3. PREDICTION OF THE IRREGULAR VARIATIONS IN NUTATION

We used in computations the IERS combination series C04 related to the IAU2000 precession-nutation model (file `EOPC04_IAU2000.62-now` downloaded from the following website address `http://hpiers.obspm.fr/eoppc/eop/eopC04`). The series used here contains 7192 daily values of the celestial pole offsets $X = d\psi \sin\varepsilon_o$, $Y = d\varepsilon$, spanning the period between 1984.0 and 2003.7. First, we removed from the input series the model comprising the linear trend and corrections to the important nutation terms. We found by the least- squares fit the following corrections which are in some cases surprisingly large:

1-st order polynomial		$d\psi \sin\varepsilon_o$	$d\varepsilon$
	constant (μas)	59 ± 6	32 ± 7,
	slope (μas/yr)	8 ± 1	9 ± 1,
Periodical terms (μas)		retrograde	prograde
	18.6 yr	28 ± 8	68 ± 6,
	9.3 yr	28 ± 6	33 ± 6,
	1.0 yr	5 ± 6	10 ± 6,
	0.5 yr	19 ± 7	9 ± 6.

Then, we smoothed the residual nutation series by the Gaussian filter with full width at a half of maximum equal to 20 days, and interpolated at 10-day intervals, with the first modified Julian date 45710.0. The reduced time series of the nutation residuals, which has a length of 718, is shown in Figure 1a. The MEM power spectrum of $P = X + iY$ computed for the AR order $p = 21$ (cf. eq.(2)) shows the FCN peak containing more than 60% power of the series. The MEM spectral analysis yields the FCN period of $T = -429.6$ days (Figure 1b), the quality factor $Q = 2995$ and the mean amplitude, that is the square root of the power of oscillation, $A = 179$ μas. If we compute the FCN parameters assuming over the entire time interval 1984.0 to 2003.7 the model comprising both the retrograde and prograde complex sinusoids with a period of 430 days, the result is $A = 107 \pm 6$ μas for the retrograde component and $A = 11 \pm 6$ μas for the prograde one.

Next, we computed the one-year forecasts of the reduced nutation series using two different methods. The first method, designated below as sinusoidal prediction, corresponds to the FCN model recommended by the IERS. It consists in fitting to the last two years of data of the model comprising a sum of the complex sinusoid with a period of -430 days (that describes a uniform circular motion of P in the clockwise direction) and the constant, and then extrapolating this model into the future. Hence, this model allows the time variation of the FCN amplitude and phase while assuming that the motion is purely retrograde. The other method is the AR prediction with the autoregressive coefficients φ_l, eq.(2), estimated by using the last eight years of data. The optimum AR order p was determined as that corresponding to the first local minimum of the FPE value, for which $p \geq 20$.



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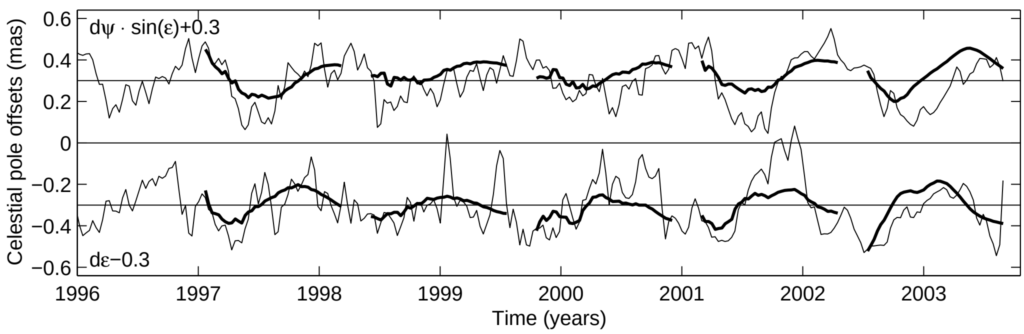
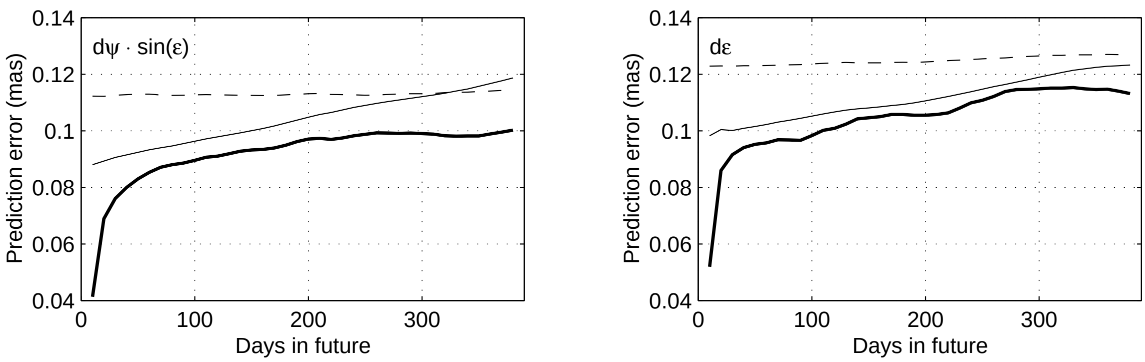


b)

Figure 2: One year predictions (thick line) of the observed nutation residuals (thin line), computed at different starting epochs a) by the extrapolation of the model: constant plus complex sinusoid with a period of -430 days, b) by the use of the autoregressive model.

We considered also the third prediction method assuming constant future values of the series, where the constant was determined as a mean over the last 2.5 years of data that is roughly two full FCN cycles. This model serves us as a reference when estimating the efficiency of the other two methods. The mean prediction error for m days in future was computed as a root mean square difference between prediction and true value, averaged over the whole set of possible starting prediction times. Figure 2 shows sample one-year predictions by the first two methods, computed for different starting prediction epochs, while the mean prediction errors for m between 10 and 380 days in the future, are compared in Figure 3. For other prediction methods applied to the observations of polar motion see the paper by Kosek *et al.* (this volume) and the references therein.

From the inspection of Figures 2 and 3 we can draw several conclusions. All predictions are generally worse for $Y = d\varepsilon$ which indicates that this component of nutation contains higher noise than the other component $X = d\psi \sin \varepsilon_o$. The mean prediction error of the forecast by constant does not depend on the prediction time, while in case of the sinusoidal model the error increases approximately linearly. As could be expected, the best results are obtained by the AR model. The prediction error growths rapidly for the prediction time m between 10 and 50 days, then increases roughly linearly, and finally becomes almost constant. Again, there is a difference between the X and Y components of nutation. In case of the X component the initial growth of the error is slower, and the stability is reached already for m equal to about 200 days and at the level of 0.10 mas. In case of Y, the corresponding quantities are 270 days and about 0.12 mas. The advantage over the sinusoidal extrapolation is particularly well seen for short and long prediction times. As it can be seen from Figure 2, one reason for better performance of the AR prediction is that the underlying stochastic model does not only attempt to express the FCN but also other fine features of the irregular variability of nutation.



a) b)

Figure 3: The mean prediction error of the observed nutation residuals, assuming the following models: the constant (dashed line), the constant plus complex sinusoid with a period of −430 days (thin solid line) and the autoregressive process (thick solid line).

An important measure of the efficiency of a forecast method is the "predictability" coefficient introduced by Chao (1985)

$$R = 1 - \frac{\sigma_{pred}}{\sigma}\,, \tag{3}$$

where σ_{pred} denotes the estimated mean prediction error (Figure 3), while σ denotes the standard deviation of the series. We replace here σ by the mean error of the prediction by constant, as expressed by the dashed line in Figure 3. For the autoregressive model the predictability takes the following values

days in future	$X = d\psi \sin \varepsilon_o$	$Y = d\varepsilon$
10	63%	58%,
20	39%	30%,
30	32%	25%,
40	29%	23%,
...	...	...,
360	13%	10%,

Hence, in case of the observed celestial pole offsets the predictability is significant only for short prediction lengths. For one-year prediction the efficiency is much lower than in case of polar motion, where it is of the order of 80% to 90%. There are two reasons for such difference: 1) the signal-to-noise ratio is up to 3 orders of magnitude higher for polar motion than for the celestial pole offsets; 2) the FCN is less stable than the Chandler wobble because its dissipation time estimated from observations is several times shorter.

4. SUMMARY AND CONCLUSIONS

Modeling the FCN signal in the time series of the celestial pole offsets observed by VLBI, is an important task of the sub-milliarcsecond astrometry. Its variability (Figure 1) is typical for the randomly excited free oscillator with damping. A similar observation concerning the Chandler wobble led Jeffreys (1940) to the concept of describing the free motion as a realization of the stochastic process, the ARIMA process. We followed this concept and applied the ARIMA model for description of the FCN; see, e.g., (Brzeziński, 1996) for details.

The application of the ARIMA processes in the investigations concerning the FCN offers several advantages. Such model has a good physical explanation since it can be derived directly from the equation of motion. The underlying assumption is that the excitation function behaves as a random process, which is reasonable as far as we consider the influence of the atmosphere and of the nontidal oceanic variability on nutation. The application of the ARIMA model enables determination of the parameters of the FCN mode (Brzeziński and Petrov, 1998) which is independent from the estimation based on the indirect effect (Mathews *et al.*, 2002). The ARIMA model can also be used for the time domain comparison between the FCN signal and the atmospheric and/or oceanic excitation. Unfortunately, such comparisons could not be conclusive so far because we do not have adequate estimates of the atmospheric and oceanic excitation functions (Petrov *et al.*, 1997).

In this research we considered the application of the AR model for prediction of the observed irregular component of nutation including the FCN signal. First computation using the available time series of the celestial pole offsets demonstrated clear advantage of the AR-based prediction over the extrapolation of the sinusoidal model. This advantage is particularly large for short term predictions, up to about 1 month. For longer prediction times, between 1/2 and 1 year, the AR model yields the rms errors of about 0.10 mas for $X = d\psi \sin \varepsilon_o$ and 0.11 mas for $Y = d\varepsilon$.

However, even in case of the AR prediction its efficiency is low. For the prediction length of 10 days, the predictability is (63%,58%) for (X,Y), then it decreases rapidly reaching (32%,25%) for $m = 30$ days, and for $m = 1$ year it takes rather low value of (13%,10%).

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