A NUMERICAL METHOD FOR THE ANALYSIS OF THE SYSTEMATIC ERRORS IN REFERENCE SYSTEMS FROM NON REGULAR SAMPLES

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ABSTRACT. The main aim of this paper is the analysis of the systematic errors contained in a star catalogue. These errors can be obtained by means of the analysis of the residuals observation minus calculus taken from a set of observated positions in a spatial domaine.
The clasical methods do not work fine if the sample is non homogeneously distributed in the spatial domain. In this paper a new method for the errors analysis from non homogeneous samples is proposed.

1. INTRODUCTION

The main difficulty of this method is the non-ortogonality of the set of functions over the discrete points of the sphere given by the observations because the sample is usualy non-homogeneous. This problem can be aproached using a previous filter by means of a suitable couple composed by a previois smoothing procses and a reconstruction operator.
The smoothing algorithm provide estimators for the values of the $\triangle\lambda(\lambda,\beta)$ and $\triangle\beta(\lambda,\beta)$ from the sample for a lattice over the ecliptic band, and using an exact reconstruction operator until an appropiate order k the error values $(\triangle\lambda, \triangle\beta)$ are reached. The non-ortogonality problem can be solved using this method.
The calculated position of the asteroids can be improved by means of a previous orbital elements correction process. The solution of the normal system obtained from the minimun condition for the residual function provides the values of parameters of the bias model.

2. FILTERING MODEL

Let D be the spherical domain where the observated positions are distributed. In this work we the domain D has been token as a band arround the ecliptic $D = \{(\lambda,\beta)|\lambda \in [0,2\pi], \beta \in [-\beta_{max}, \beta_{max}\}$.
Let (α_i, δ_i) be a set of observated positions of minor planets reducted from a star catalogue, and (λ_i, δ_i) their ecliptic coordinates. The differences betwen observated an calculated positions $(\triangle\lambda_i, \triangle\beta_i)$ can be developed [1] by means the series expansions:

$$\triangle\lambda\cos\beta = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \epsilon_{n,m} K_{n,m}(\lambda,\beta) \quad \triangle\beta = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \eta_{n,m} K_{n,m}(\lambda,\beta)$$

where $K_{n,m}(\lambda,\beta), n, m = 0, 1, ...$ is a suitable orthogonal base of $l^2(D)$ space.

The coeficients of the analytical expansion can be obtained from the minimum condition of the residual function:

$$R(\{\epsilon_{i,j},\eta_{i,j}\}_{i=1,..,N;j=1,..,M}) = \sum_{i=1}^{NE} \left[r_{\beta_i}^2 + r_\lambda^2 \cos^2\beta_i\right]$$

where NE is the size of the sample.
This method is apropiated if the stars distribution is homogeneous in the spherical domain D. Unfortunately when the sample is token from minor planets observations the sample in non homogeneously distributed in the band D. In this case it is more apropiated [2] take the minimun condition from the integral:

$$R(\{\epsilon_{i,j},\eta_{i,j}\}_{i=1,..,N;j=1,..,M}) = \int\int_D \left[r_{\beta_i}^2 + r_{\lambda_i}^2 \cos^2\beta_i\right] d\sigma$$

To evaluate this integral we can discretize the spatial domaine by means of a rectangular lattice. The control volume D_i is determinated by the center of the four adyacent rectangles at the node i (except for the nodes in the boundary) and the values of the $\triangle\lambda$, $\triangle\beta$ is repalced by the mean values:

$$\overline{\triangle\lambda_i} = \frac{1}{V_i}\int\int_{D_i} \triangle\lambda d\sigma, \quad \overline{\triangle\beta_i} = \frac{1}{V_i}\int\int_{D_i} \triangle\beta d\sigma$$

where V_i is the area of the contol volume D_i.
this domain.
The integrals can be calculated from a numerical formulae built for the points of the sample. The values of the $(\triangle\lambda, \triangle\beta)$ can be calculated from a bidimensional reconstruction operator exact until k order in lattice size step h. The bidimensional reconstruction operator can be built as composition of two unidimensional operators [4](Casper, Atkins) one in λ direction and other in β direction.

3. CONCLUDING REMARKS

The clasical method is not appropiated for use a dymamical correction model to study catalogue bias because the sample is not homogeneous in the domain D so the ortogonality of set of basic functions over the sample is not guaranted.
The proposed method developed in this paper is more suitable in these situations.

4. ACKNOWLEDGMENTS

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5. REFERENCES

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