A NUMERICAL METHOD FOR THE ANALYSIS OF THE SYSTEMATIC ERRORS IN REFERENCE SYSTEMS FROM NON REGULAR SAMPLES

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ABSTRACT. The main aim of this paper is the analysis of the systematic errors contained in a star catalogue. These errors can be obtained by means of the analysis of the residuals observation minus calculus taken from a set of observed positions in a spatial domain.

The classical methods do not work fine if the sample is non homogeneously distributed in the spatial domain. In this paper a new method for the errors analysis from non homogeneous samples is proposed.

1. INTRODUCTION

The main difficulty of this method is the non-ortogonality of the set of functions over the discrete points of the sphere given by the observations because the sample is usually non-homogeneous. This problem can be approached using a previous filter by means of a suitable couple composed by a previos smoothing proces and a reconstruction operator.

The smoothing algorithm provide estimators for the values of the $\Delta \lambda(\lambda, \beta)$ and $\Delta \beta(\lambda, \beta)$ from the sample for a lattice over the ecliptic band, and using an exact reconstruction operator until an appropiate order $k$ the error values $(\Delta \lambda, \Delta \beta)$ are reached. The non-ortogonality problem can be solved using this method.

The calculated position of the asteroids can be improved by means of a previous orbital elements correction process. The solution of the normal system obtained from the minimum condition for the residual function provides the values of parameters of the bias model.

2. FILTERING MODEL

Let $D$ be the spherical domain where the observed positions are distributed. In this work we the domain $D$ has been taken as a band arround the ecliptic $D = \{ (\lambda, \beta) | \lambda \in [0, 2\pi], \beta \in [-\beta_{max}, \beta_{max}] \}$. Let $(\alpha_i, \delta_i)$ be a set of observed positions of minor planets reduced from a star catalogue, and $(\lambda_i, \delta_i)$ their ecliptic coordinates. The differences between observed an calculated positions $(\Delta \lambda_i, \Delta \delta_i)$ can be developed [1] by means the series expansions:

$$\Delta \lambda \cos \beta = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \epsilon_{n,m} K_{n,m}(\lambda, \beta) \quad \Delta \beta = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \eta_{n,m} K_{n,m}(\lambda, \beta)$$

where $K_{n,m}(\lambda, \beta), n, m = 0, 1, \ldots$ is a suitable orthogonal base of $L^2(D)$ space.
The coefficients of the analytical expansion can be obtained from the minimum condition of the residual function:

\[ R\{\{\epsilon_{ij}, \eta_{ij}\}\}_{i=1, \ldots, N; j=1, \ldots, M} = \sum_{i=1}^{NE} \left[ r_{\beta_i}^2 + r_{\lambda_i}^2 \cos^2 \beta_i \right] \]

where NE is the size of the sample.

This method is appropriate if the stars distribution is homogeneous in the spherical domain D. Unfortunately when the sample is taken from minor planets observations the sample in non homogeneously distributed in the band D. In this case it is more appropriate [2] take the minimum condition from the integrals:

\[ R\{\{\epsilon_{ij}, \eta_{ij}\}\}_{i=1, \ldots, N; j=1, \ldots, M} = \int \int_D \left[ r_{\beta_i}^2 + r_{\lambda_i}^2 \cos^2 \beta_i \right] d\sigma \]

To evaluate this integral we can discretize the spatial domain by means of a rectangular lattice. The control volume \( D_i \) is determined by the center of the four adjacent rectangles at the node \( i \) (except for the nodes in the boundary) and the values of the \( \Delta \lambda, \Delta \beta \) is replaced by the mean values:

\[ \overline{\Delta \lambda_i} = \frac{1}{V_i} \int \int_{D_i} \Delta \lambda d\sigma, \quad \overline{\Delta \beta_i} = \frac{1}{V_i} \int \int_{D_i} \Delta \beta d\sigma \]

where \( V_i \) is the area of the control volume \( D_i \), this domain.

The integrals can be calculated from a numerical formulae built for the points of the sample. The values of the \( (\Delta \lambda, \Delta \beta) \) can be calculated from a bidimensional reconstruction operator exact until \( k \) order in lattice size step \( h \). The bidimensional reconstruction operator can be built as composition of two unidimensional operators [4] (Casper, Atkins) one in \( \lambda \) direction and other in \( \beta \) direction.

3. CONCLUDING REMARKS

The classical method is not appropriate for use a dynamical correction model to study catalogue bias because the sample is not homogeneous in the domain D so the orthogonality of set of basic functions over the sample is not guaranteed.

The proposed method developed in this paper is more suitable in these situations.

4. ACKNOWLEDGMENTS

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5. REFERENCES

