DOES PRECESSION DERIVED FROM FK5-HIPPARCOS AGREE WITH THE VLBI?

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ABSTRACT. Modern determinations based on VLBI observations [1,2] yield the correction $\Delta p = -3.0 \pm 0.1\ mas/y$ to the IAU (1976) value of general luni-solar precession in longitude $p = 5029.0966''/cy$, $J2000.0$. Nevertheless, extensive study of the FK5 spin with respect to HIPPARCOS yields the correction $\Delta p = -1.5 \pm 0.7\ mas/y$ [3,4] which is not consistent with the VLBI. Aiming at the explanation for this fact, the paper presents an examination of the differences FK5-HIPPARCOS treated by different numerical techniques.

It was found that the proper motions of the FK5 in Right Ascension are consistent with the VLBI value of correction to the precessional constant, whereas the proper motions in Declination are not. From this it follows that the precessional correction must be derived only from the differences $\Delta\mu\cos\delta$. To this one should add that the commonly used routines to derive the precessional correction are based on combined solution of the equations for $\Delta\mu'$ and $\Delta\mu\cos\delta$ which assigns to the Declination system the weight three times more than to the R.A. system. It is due to this reason the result comes wrong. At the same time, the differences $\Delta\mu\cos\delta$ taken separately yield the correction $\Delta p = -3.5 \pm 0.1\ mas/y$ which is in good agreement with the VLBI.

1. THE BASICS

The Helmholtz theorem applied to velocity field of stars [5] states that an individual velocity of a star is expressed as the sum of a translation $\bar{V_0}$ (Solar motion)), a divergence characterized by a deformation ellipsoid S, and a spin of the stellar system $\bar{\omega}$:

$$\bar{V} = \bar{V_0} + grad\ S + \bar{\omega} \times \bar{r}. \tag{1}$$

When the differences FK5-HIPPARCOS are used the only contribution to the difference $\Delta\bar{V}$ is expected from the spin since the Solar motion and the divergence terms vanish. Since the HIPPARCOS catalogue is free from precession and equinox motion, then in the differences FK5-HIPPARCOS the rigid body spin of the stellar system vanishes, and the spin is generated by the FK5 residual precession and non precessional motion of the equinox only:

$$\omega_1 = 0, \tag{2}$$

$$\omega_2 = -\Delta p\ \sin\epsilon, \tag{3}$$

$$\omega_3 = \Delta p\ cos\epsilon - (\Delta\lambda + \Delta e), \tag{4}$$

where ϵ - the tilt of ecliptic.

Thus we see that to find the correction to the constant of precession from the differences in proper motions FK5-HIPPARCOS one needs to solve the next equations of condition:

$$\begin{array}{rcllll} \Delta\mu\cos\delta & = & -\omega_1\sin\delta\cos\alpha & -\omega_2\sin\delta\sin\alpha & +\omega_3\cos\delta, \\ \Delta\mu' & = & \omega_1\sin\alpha & -\omega_2\cos\alpha, & \end{array} \tag{5}$$

where

$$\begin{array}{rcl} \Delta\mu & = & \mu_{FK5}-\mu_{HIP}, \\ \Delta\mu' & = & \mu'_{FK5}-\mu'_{HIP}. \end{array} \tag{6}$$

2. THE LEAST SQUARES SOLUTION

It is common practice of evaluating the unknowns ω_1, ω_2 and ω_3 from combined solution of equations (5). Nevertheless, the separate solutions are possible too. It is not difficult to show that if the results of separate solutions are $\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha$ and $\omega_1^\delta, \omega_2^\delta$, then the combined solution of Eqs. (5) looks like follows:

$$\begin{aligned} \omega_1^c &= \tfrac{1}{4}(\omega_1^\alpha + 3\omega_1^\delta), \\ \omega_2^c &= \tfrac{1}{4}(\omega_2^\alpha + 3\omega_2^\delta), \\ \omega_3^c &= \omega_3^\alpha. \end{aligned} \tag{7}$$

In other words: *each of ω_i^c is weighted average of ω_i^α and ω_i^δ with predominant contribution of the δ-components.*

From this it follows that the combined solution meets no objection if the systems of $\mu\cos\delta$ and μ' of both catalogues are free from systematic errors. Though the HIPPARCOS catalogue is claimed to have no systematic errors, it is not so in the case of FK5. In such situation the estimates of one-named parameters from separate solutions of equations (5) may differ dramatically giving evidence of large systematic errors in one (or both) systems. Since the result of combined solution is extremely sensitive to the errors of the declination system, the combined solution will give wrong result if the μ_δ system of the FK5 is worse than the $\mu\cos\delta$ system.

2. THE SOLUTION BY VECTORIAL HARMONICS

A new method to solve equations (5) was proposed in [6]. This approach is based on decomposition of the vector field

$$\tilde{\mathbf{V}} = V_\alpha\ \tilde{\mathbf{e}}_\alpha + V_\theta\ \tilde{\mathbf{e}}_\theta,$$

where $\theta = \pi/2 - \delta$, $\tilde{\mathbf{e}}_\alpha$,$\tilde{\mathbf{e}}_\theta$ – unit vectors, on a set of vectorial harmonics $\tilde{\mathbf{T}}_{lm}(\alpha,\theta)$ and $\tilde{\mathbf{S}}_{lm}(\alpha,\theta)$:

$$\tilde{\mathbf{V}}(\alpha,\theta) = \sum_{m=-l}^{m=+l}\sum_{l=1}^{\infty}(t_{lm}\ \tilde{\mathbf{T}}_{lm}(\alpha,\theta) + s_{lm}\ \tilde{\mathbf{S}}_{lm}(\alpha,\theta)). \tag{8}$$

The most attractive feature of this method is the fact that the rigid body rotational field

$$\tilde{\mathbf{V}} = (-\omega_1\cos\alpha\cos\theta - \omega_2\sin\alpha\cos\theta + \omega_3\sin\theta)\ \tilde{\mathbf{e}}_\alpha + (-\omega_1\sin\alpha + \omega_2\cos\alpha)\ \tilde{\mathbf{e}}_\theta \tag{9}$$

is determined only through the coefficients

$$t_{10} = \sqrt{\frac{8\pi}{3}}\ \omega_3, \tag{10}$$

$$t_{11} = \sqrt{\frac{4\pi}{3}}\ (-\omega_1 + i\omega_2). \tag{11}$$

It is this method that was used to find the parameters of mutual orientation and spin of the FK5 and HIPPARCOS [7]. Still, this property of vectorial functions must be treated with caution. Really, if the proper motions in $\alpha-$ and $\theta-$directions are generated by different spins with parameters ω_1^θ, ω_1^α and ω_2^θ, ω_2^α, i.e.

$$\tilde{\mathbf{V}} = (-\omega_1^\alpha \cos\alpha\cos\theta - \omega_2^\alpha \sin\alpha\cos\theta + \omega_3\sin\theta)\ \tilde{\mathbf{e}}_\alpha + (-\omega_1^\theta \sin\alpha + \omega_2^\theta \cos\alpha)\ \tilde{\mathbf{e}}_\theta, \tag{12}$$

then equation (11) is replaced by

$$t_{11} = \sqrt{\frac{\pi}{12}}\ [-(\omega_1^\alpha + 3\omega_1^\theta) + i(\omega_2^\alpha + 3\omega_2^\theta)]. \tag{13}$$

Now, from (11) and (13) we again get equations (7). This tells us that when the parameters of the spin are different, the vectorial functions have no advantages over the least square combined solution of equations (5). Nevertheless, if both components of proper motions are consistent with a model, the decomposition of proper motions on vectorial functions is promised to be very powerful tool. Recently the application of this method was made to all terms of equation (1). Besides the low order classical terms this approach revealed some higher order harmonics which are beyond the model [12].

2. THE SOLUTION BY SCALAR HARMONICS

Now we are in position to answer the question: which solution is reliable? This requires a more sophisticated method to penetrate into the essence of separate solutions. In this connection we propose to use the decomposition of each components $\Delta\mu\cos\delta$ and $\Delta\mu'$ on a set of the scalar (not vectorial) harmonics

$$\Delta\mu\cos\delta = \sum_{nkl} C_{nkl}\ \ Z_{nkl}(\alpha,\delta), \tag{14}$$

$$\Delta\mu' = \sum_{nkl} C'_{nkl}\ \ Z_{nkl}(\alpha,\delta), \tag{15}$$

where Z_{nkl} are the spherical functions. This technique was proposed by Brosche [8] for representing the systematic differences of two catalogues. Later on, it was elaborated by the author [9,10] for deriving rotation between two reference frames and for kinematical analysis of the proper motions [11]. The main idea of this approach may be explained as follows.

Suppose the decompositions (14) and (15) are made and the coefficients C_{nkl} and C'_{nkl} are derived. It is not difficult to show that in the case of the rigid spin of the frames there are three subsets of the C_{nkl} which are proportional to one of the components $\omega_1, \omega_2, \omega_3$ and two subsets of the C'_{nkl} which are proportional to one of ω_1, ω_2. This means, *and this is the crucial point of the method*, that each of the parameters ω_i may be evaluated at least twice (in the theory as many times as needed). Namely, from $\Delta\mu\cos\delta$ one may derive ω_1 from C_{211}, C_{411}; ω_2 – from C_{210}, C_{410}; ω_3 – from C_{001}, C_{201}, as well as from $\Delta\mu'$ one may calculate ω_1 – via C'_{110}, C'_{310} and ω_2 – via C'_{111}, C'_{311}.

If two estimates of, say ω_1, coincide within the limits of their errors we may be sure that the data contains spin, and this conclusion is made for each sets $\Delta\mu\cos\delta$ or $\Delta\mu'$ independently. We emphasize, that this approach in contrast to commonly used mathematical tools, provides a test that the model is (or not) compatible with the data. It is due to this ability of the scalar harmonics one can make a choice between two alternatives in case when the one-named parameters of equations (5) come different from the separate solutions of these equations.

3. NUMERICAL RESULTS

In this section we present results obtained by solutions of equations (5) from differences $\Delta\mu\cos\delta$ and $\Delta\mu'$ calculated for 1232 stars common to FK5 and HIPPARCOS catalogues.

Table 1: Spin and correction to the precession constant from separate and combined solutions, mas/y, 1232 stars.

	From $\Delta\mu\ \cos\delta$	From $\Delta\mu'$	From $\Delta\mu\ \cos\delta$ and $\Delta\mu'$
ω_1	0.32±0.20	-0.56±0.11	-0.32±0.14
ω_2	0.98±0.20	0.48±0.11	0.61±0.14
ω_3	0.80±0.11	-	0.80±0.14
Δp	-2.5± 0.5	-1.2±0.3	-1.5±0.4

The separate and combined solutions are shown in Table (1). From this table one can see that the estimates of the components ω_1 and ω_2 following from separate and combined solutions differ significantly. The values of the correction to the precession constant Δp following from each of solutions are different too, and nothing can be said what solution is preferable. Still, the separate solutions being discordant give evidence that something is wrong and the further analysis is needed.

This more penetrative analysis comes from the scalar harmonics method (Tables 2-3).

Table 2: Spin from $\Delta\mu\ \cos\delta$ by scalar harmonics, mas/y, 1232 stars.

		n	k	l	First value	n	k	l	Second value
	ω_1	2	1	1	0.06 ± 0.21	4	1	1	0.90 ± 0.57
	ω_2	2	1	0	**1.39 ± 0.20**	4	1	0	**1.17 ± 0.54**
	ω_3	0	0	1	0.62 ± 0.10	2	0	1	3.18 ± 0.34

Table 3: Spin from $\Delta\mu'$ by scalar harmonics, mas/y, 1232 stars.

		n	k	l	First value	n	k	l	Second value
	ω_1	1	1	0	-0.58 ± 0.11	3	1	0	-1.02 ± 0.43
	ω_2	1	1	1	0.37 ± 0.11	3	1	1	1.89 ± 0.44

Now we see that both estimates of ω_1 and ω_3 derived from either first or second equations (5) are discordant. The same result is stated for ω_2 obtained from $\Delta\mu'$. This is sufficient to make a conclusion that there is no rigid body rotation in the system of μ' of the FK5 with respect to

HIPPARCOS frame. On the contrary, both estimates of ω_2 derived from first equation (5) have good agreement, and this tells us that the only component of the FK5's proper motions suitable for determination of precession is $\Delta\mu\cos\delta$.

4. CONCLUSIONS

It may be argued that the results described above are due to specific properties of the sample under consideration. To see what happens when another sample is taken, we choosed the sample of 512 distant stars which were used by Fricke [13] for deriving the constant of precession IAU 1976. The differences FK5-HIPPARCOS of these stars have been treated in the same way as the sample of 1232 stars. The results are shown in Tables 4, 5 and 6.

Table 4: Spin and correction to the precession constant derived from 512 differences FK5-HIPPARCOS, mas/y.

	From $\Delta\mu$ cos δ	From $\Delta\mu'$	From $\Delta\mu$ cos δ and $\Delta\mu'$
ω_1	0.61±027	-0.68±0.11	-0.35±0.17
ω_2	0.76±0.26	0.53±0.12	0.60±0.17
ω_3	0.85±0.15	-	0.78±0.18
Δp	-1.9± 0.7	-1.3±0.3	-1.5±0.4

Table 5: Spin from 512 differences $\Delta\mu$ cos δ by scalar harmonics, mas/y.

		n	k	l	First value	n	k	l	Second value
	ω_1	2	1	1	-0.09 ± 0.38	4	1	1	0.46 ± 0.86
	ω_2	2	1	0	**1.46 ± 0.31**	4	1	0	**1.19 ± 0.65**
	ω_3	0	0	1	0.56 ± 0.21	2	0	1	2.95 ± 0.66

Table 6: Spin from 512 differences $\Delta\mu'$ by scalar harmonics, mas/y.

		n	k	l	First value	n	k	l	Second value
	ω_1	1	1	0	-0.86 ± 0.18	3	1	0	-0.41 ± 0.50
	ω_2	1	1	1	0.68 ± 0.27	3	1	1	1.46 ± 0.69

From these tables we see that the sample of 512 stars gave practically the same results. To this we must add that the situation does not change when the proper motions of the PPM are compared with those of the HIPPARCOS. Indeed, the separate LSM solutions based on 93387 differences PPM-HIPPARCOS, yield $\omega_2 = 1.59 \pm 0.04$ mas/y from $\Delta\mu\cos\delta$ and $\omega_2 = 0.63 \pm 0.02$ mas/y from $\Delta\mu'$. The scalar functions for both estimates of ω_2 from $\Delta\mu\cos\delta$ yield the values 1.43 ± 0.04 mas/y and 3.22 ± 0.15 mas/y. These estimates are discordant, but one must take into account that with respect to spin both hemispheres of the PPM are quite different [3] – and the method of scalar functions reveals this fact.

Summarizing, we can say that the rigid body rotation does exist only in the R.A. proper motions components of the differences FK5-HIPPARCOS and only this system is consistent with the VLBI if the precessional correction is concerned. Returning to the initial sample of 1232 stars we state:

- The Declination system of the FK5 proper motions shows no spin with respect to HIPPARCOS.
- The discordant value $\Delta p = -1.5 \pm 0.7$ mas/y is explained by too large weight that the combined solution assigns to the Declination system of proper motions

- The spin of the FK5 with respect to HIPPARCOS exists in the R.A. system of proper motions ONLY.
- This spin gives correction to the precession constant $\Delta p = -3.5 \pm 0.5$ mas/y which is consistent with the result obtained in the VLBI technique.

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6. REFERENCES

[1] Walter H.G., and Ma C. Correction to the luni-solar precession from very long baseline interferometry, Astron. Astrophys., Vol 284, pp. 1000-1006, 1994.

[2] Chapront J., Chapront-Touze M., Francou G., 1999, A&A 343,642.

[3] *Mignard F., Froeschle F.*, 2000, Global and local bias in the FK5 from the HIPPARCOS data. A&A, 354, pp.732-739.

[4] Zhu Z., Yang T., 1999, AJ 117,1103

[5] Clube S. V. M. Galactic rotation and the precession constant. Mon. Notic. Roy. Astron. Soc. 159, N3. pp.289-314.

[6] *Mignard F., Morando B.*, 1990, Analyse de catalogues stallaires au moyen des harmoniques vectorelles, Journees 90. Systemes de reference spatio-temporels. Paris, pp.151-158.

[7] European Space Agency, The Hipparcos and Tycho Catalogues, "ESA", 1997.

[8] *Brosche P.*, 1966, Representation of systematic differences in positions and proper motions of stars by spherical harmonics, Veroff, des Astron. Rechen-Inst. Heidelberg, N 17, p.1-27.

[9] Vityazev V.V., The ROTOR: a new method to derive rotation between two reference frames. Astron. and Astrophys. Trans., 1994, 4, pp. 195-218.

[10] Vityazev V.V., The ROTOR: Rotation of Frames via Representation of Systematic Differences in Terms of Spherical Functions. – Proc. of IAU Colloquium 165, 1997, Poland, pp. 464-474.

[11] Vityazev V.V., The MOTOR: Stellar kinematics via representation of proper motions by means of orthogonal functions. – Motion of Celestial bodies, Astrometry and Astronomical Reference Frames, Proc. of the JOURNEES 1999, Dresden, 2000, p. 59.

[12] *V. Vityazev, A. Shuksto*, Stellar Kinematics by Vectorial Harmonics, 2003 (in press).

[13] *Fricke W*, 1977, Basic material for the determination of precession and of Galactic rotation and a review of methods and results. – Veroffen Astron. Rechen-Inst., Heidelberg. N 28, p.52.